Economics of Banking
Lecture 12

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The circular city

There are n banks, situated on a circle of total length 1:

Distance between neighboring banks $1/n$.

Depositors spread evenly over circle, each coming with 1 unit.

Transportation cost of t per unit of distance.

Fixed cost of keeping a bank f.

Optimum for society

Minimize sum of (a) travel cost and (b) the fixed cost:

(a) Cost of serving the customers of a given bank is

$$2\int_0^{\frac{1}{2n}} tx\,dx = \frac{t}{4n^2}.$$

Total cost to society with n banks is

$$C = \frac{t}{4n} + nf.$$

First order conditions for a minimum:

$$\frac{dC}{dn} = -\frac{t}{4n^2} + f = 0$$

with solution

$$n^* = \frac{1}{2}\sqrt{\frac{t}{f}},$$

Market solution

Assume that our bank has r_D and all the others have r_D'.

Find the borderline depositor x from

$$r_D - tx = r_D' - t\left(\frac{1}{n} - x\right),$$

so that the demand $D = 2x$ is

$$D = \frac{1}{n} + \frac{r_D - r_D'}{t}.$$

Use standard monopoly price rule

$$r - r_D = \frac{1}{2}\left[\frac{t}{n} + (r - r_D')\right]$$

In symmetric equilibrium, $r_D = r_D'$,

$$r_D = r - \frac{t}{n}.$$

Free entry

Bank profits are

$$\frac{1}{n}\frac{t}{n} - f = \frac{t}{n^2} - f,$$

New banks enter as long as there are nonzero profits, so

$$\frac{t}{n^2} - f = 0$$

Solving for n:

$$n^0 = \sqrt{\frac{t}{f}}.$$

Too many banks in equilibrium!

Deposits and loans

Assume: Each individual wants to borrow $L < 1$.
Transport cost t_L in loan business.

With the same reasoning as before, we get a symmetric equilibrium with loan rate

$$r_L = r + \frac{t_L}{nL}.$$

Profits are $\frac{t_L}{n^2}$ in loan business.

An application

We use the model to consider *deposit rate regulation:*
Keeping deposit rates low in order to obtain lower loan rates

For simplicity: r_D is set to 0

Then deposit business is more profitable, loan as before. No change in loan rates.

Alternative: Allow banks to use contingent contracts (no loans without deposits). Then

$$r_L^* = \left(r - \frac{r}{L}\right) + \frac{t_D + t_L}{nL} = \left(r + \frac{t_L}{nL}\right) - \frac{1}{L}\left(r - \frac{t_D}{n}\right)$$

Loan rates are lower!

Risk-taking in banks

Consider a bank where all engagements run over a single period.

Expected value of loans is fixed at μ but varies with a parameter σ

Loans L at rate $\tilde{r}_L$ are funded by deposits D with rate $r_D(\sigma)$ (assume $L = D$)

Bank fails if $r_L < r_D(\sigma)$

Probability of success is $p(\sigma) = \mathrm{P}\{\tilde{r}_L > r_D(\sigma)\}$

Value of future bank business (franchise value) V.

Perfect information

Depositors can observe σ

Funding condition is

$$E[\min\{\tilde{r}_L, r_D(\sigma)\}|\sigma] = 1$$

Expected profit of bank

$$\Pi(\sigma) = D\, E[\max\{0, \tilde{r}_L - r_D(\sigma)\}|\sigma],$$

Bank chooses σ so as to maximize $\Pi(\sigma) + p(\sigma)V$.

Use $\max\{0, r_L - r_D\} + \min\{r_L, r_D\} = r_L$ to get

$$\Pi(\sigma) + D = E[\tilde{r}_L|\sigma]D = \mu D,$$

Profits and survival

Under **perfect information:** $\Pi(\sigma) = (\mu - 1)D$ independent of σ but σ matters for $p(\sigma)$

Consequence: for $V > 0$, the bank chooses minimal σ.

Imperfect information: Depositors expect $\hat{\sigma}$ and will demand a rate $r_D(\hat{\sigma})$ giving at least zero expected return. Banks find

$$\max_{\sigma}\{\Pi(\sigma) + p(\sigma)V\},$$

and profits

$$\Pi(\sigma) = D\,\mathrm{E}[\max\{0, \tilde{r}_L - r_D(\hat{\sigma})\}|\sigma]$$

increase in σ: Trade-off between profits and survival!

Oligopoly with insured depositors

There are n banks choosing investment:

Investment with parameter s yields a payoff s with probability $p(s)$.

Deposit rate r_D depends on $\sum_{i=1}^{n} D_i$.

Deposits are insured, banks pay a rate α for this.

Bank i chooses (s_i, D_i) so as to maximize expected profits

$$p(s_i)\left(s_i - r_D\left(D_i + \sum_{j \neq i} D_j\right) - \alpha\right) D_i.$$

Riskiness and number of banks

In a symmetric equilibrium, all choose the same $D_i (= D)$.

First order conditions

$$p'(s)(s - r_D(nD) - \alpha) + p(s) = 0,$$
$$s - r_D(nD) - r_D'(nD)D - \alpha = 0.$$

can (in principle) can be solved for D and s when n is fixed.

We are interested in what happens to s when n increases.

First order conditions is an equations system which gives s as implicit function of n.

It can be shown that $\frac{\partial s}{\partial n} > 0$.

Banks choosing loan rate

But banks do not invest, they offer loans to entrepreneurs!

Entrepreneurs maximize $p(s)(s - r_L)$, with first order condition

$$s + \frac{p(s)}{p'(s)} = r_L.$$

Gives rise to demand relationship of the form $r_L(L)$

Now banks select D_i

$$p(s)\left[r_L\left(\sum_{j=1}^{n} L_j\right) - r_D\left(\sum_{j=1}^{n} D_j\right) - \alpha\right] D_i.$$

under the constraint $L_i = D_i$.

Equilibrium with many banks

Define $Z = \sum_{j=1}^{n} D_j$.
s is increasing in r_L which decreases in Z, so s is in $\sum_{j=1}^{n} D_j$.

The first order condition is

$$(r_L(Z) - r_D(Z) - \alpha)\left[p'(s(Z))s'(Z)D_i + p(s(Z))\right] + (r_L'(Z) - r_D'(Z))p(s(Z))D_i = 0.$$

In a symmetric equilibrium we get

$$r_L(Z) - r_D(Z) - \alpha = \frac{(r_D'(Z) - r_L'(Z))p(s(Z))Z}{p'(s(Z))s'(Z)Z + p(s(Z))n}. \tag{1}$$

Let Φ be right-hand side in (**??**).

Then implicit function theorem applied to (**??**) gives

$$\frac{\partial Z}{\partial n} = -\frac{\frac{\partial \Phi}{\partial n}(Z, n)}{r_L'(Z) - r_D'(Z) - \frac{\partial \Phi}{\partial n}(Z, n)} > 0.$$

Use now that s is decreasing in Z: More banks reduce risk!