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**The circular city**

There are $n$ banks, situated on a circle of total length 1:

Distance between neighboring banks $1/n$.

Depositors spread evenly over circle, each coming with 1 unit.

Transportation cost of $t$ per unit of distance.

Fixed cost of keeping a bank $f$.

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Optimum for society

Minimize sum of (a) travel cost and (b) the fixed cost:

(a) Cost of serving the customers of a given bank is

$$2 \int_{0}^{\frac{1}{n}} tx \, dx = \frac{t}{4n^2} \cdot \left[ \frac{1}{2} x^2 \right]_{0}^{\frac{1}{n}}$$

Total cost to society with $n$ banks is

$$C = \frac{t}{4n} + nf.$$  

First order conditions for a minimum:

$$\frac{dC}{dn} = -\frac{t}{4n^2} + f = 0$$

with solution

$$n^* = \frac{1}{2} \sqrt{\frac{t}{f}}.$$
Market solution

Assume that our bank has \( r_D \) and all the others have \( r_D' \).

Find the borderline depositor \( x \) from

\[
 r_D - tx = r_D' - t \left( \frac{1}{n} - x \right),
\]

so that the demand \( D = 2x \) is

\[
 D = \frac{1}{n} + \frac{r_D - r_D'}{t} \cdot \frac{1}{n} + \frac{(r - r_D') - (r - r_D)}{t}.
\]

Use standard monopoly price rule

\[
 b \cdot r_D - r_D = \frac{1}{2} \left[ \frac{t}{n} + (r - r_D') \right]
\]

In symmetric equilibrium, \( r_D = r_D' \),

\[
 r_D = r - \frac{t}{n}.
\]
Monopolistic competition

Free entry

Bank profits: New banks enter as long as there are nonzero profits, i.e.

\[ \frac{1}{n} t - f = \frac{t}{n^2} - f, \]

New banks enter as long as there are nonzero profits, so

\[ \frac{t}{n^2} - f = 0 \]

Solving for \( n \):

\[ n^0 = \sqrt{\frac{t}{f}}. \]

Too many banks in equilibrium!

Monopolistic competition

Deposits and loans

Assume: Each individual wants to borrow \( L < 1 \).

Transport cost \( t_L \) in loan business.

With the same reasoning as before, we get a symmetric equilibrium with loan rate

\[ r_L = r + \frac{t_L}{nL}. \]

Profits are \( \frac{t_L}{n^2} \) in loan business.
Monopolistic competition

An application

We use the model to consider deposit rate regulation:
Keeping deposit rates low in order to obtain lower loan rates

For simplicity: $r_D$ is set to 0

Then deposit business is more profitable, loan as before. No change in loan rates.

Alternative: Allow banks to use contingent contracts (no loans without deposits). Then

$$r^*_L = \left( r - \frac{r}{L} \right) + \frac{t_D + t_L}{nL} = \left( r + \frac{t_L}{nL} \right) - \frac{1}{L} \left( r - \frac{t_D}{n} \right)$$

Loan rates are lower!

Risk-taking in banks

Consider a bank where all engagements run over a single period.

Expected value of loans is fixed at $\mu$ but varies with a parameter $\sigma$

Loans $L$ at rate $\tilde{r}_L$ are funded by deposits $D$ with rate $r_D(\sigma)$ (assume $L = D$)

Bank fails if $r_L < r_D(\sigma)$

Probability of success is $p(\sigma) = P\{\tilde{r}_L \geq r_D(\sigma)\}$

Value of future bank business (franchise value) $V$. 
Perfect information

Depositors can observe $\sigma$

Funding condition is $\mathbb{E}[\min\{\tilde{r}_L, r_D(\sigma)\}] \geq 1$

Expected profit of bank

$\Pi(\sigma) = D \mathbb{E}[\max\{0, \tilde{r}_L - r_D(\sigma)\}]|\sigma]$

Bank chooses $\sigma$ so as to maximize $\Pi(\sigma) + p(\sigma)V$.

Use $\max\{0, \tilde{r}_L - r_D\} + \min\{\tilde{r}_L, r_D\} = \tilde{r}_L$ to get

$\Pi(\sigma) + D = \mathbb{E}[\tilde{r}_L]|D = \mu D$

Profits and survival

Under perfect information: $\Pi(\sigma) = (\mu - 1)D$ independent of $\sigma$ but $p(\sigma)$

Consequence: for $V > 0$, the bank chooses minimal $\sigma$.

Imperfect information: Depositors expect $\hat{\sigma}$ and will demand a rate $r_D(\hat{\sigma})$ giving at least zero expected return. Banks find

$\max_{\sigma}\{\Pi(\sigma) + p(\sigma)V\}$

and profits

$\Pi(\sigma) = D \mathbb{E}[\max\{0, \tilde{r}_L - r_D(\hat{\sigma})\}]|\sigma]$

increase in $\sigma$: Trade-off between profits and survival!
Oligopoly with insured depositors

There are \( n \) banks choosing investment:

Investment with parameter \( s \) yields a payoff \( s \) with probability \( p(s) \).

Deposit rate \( r_D \) depends on \( \sum_{i=1}^{n} D_i \).

Deposits are insured, banks pay a rate \( \alpha \) for this.

Bank \( i \) chooses \( (s_i, D_i) \) so as to maximize expected profits

\[
p(s_i) \left( s_i - r_D \left( D_i + \sum_{j \neq i} D_j \right) - \alpha \right) D_i.
\]

Riskiness and number of banks

In a symmetric equilibrium, all choose the same \( D_i(= D) \).

First order conditions

\[
p'(s)(s - r_D(nD) - \alpha) + p(s) = 0,
\]

\[
s - r_D(nD) - r'_D(nD)D - \alpha = 0.
\]

can (in principle) can be solved for \( D \) and \( s \) when \( n \) is fixed.

We are interested in what happens to \( s \) when \( n \) increases.

First order conditions is an equations system which gives \( s \) as implicit function of \( n \).

It can be shown that \( \frac{\partial s}{\partial n} > 0 \).
Banks choosing loan rate

But banks do not invest, they offer loans to entrepreneurs!

Entrepreneurs maximize $p(s)(s - r_L)$, with first order condition

$$s + \frac{p(s)}{p'(s)} = r_L.$$ 

Gives rise to demand relationship of the form $r_L(L)$

Now banks select $D_i$

$$p(s) \left[ r_L \left( \sum_{j=1}^{n} L_j \right) - r_D \left( \sum_{j=1}^{n} D_j \right) - \alpha \right] D_i.$$ 

under the constraint $L_i = D_i$.

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Equilibrium with many banks

Define $Z = \sum_{j=1}^{n} D_j$.  
$s$ is increasing in $r_L$ which decreases in $Z$, so $s$ is in $\sum_{j=1}^{n} D_j$.

The first order condition is

$$\left( r_L(Z) - r_D(Z) - \alpha \right) \left[ p'(s(Z))s'(Z)D_i + p(s(Z)) \right]$$

$$+ \left( r'_L(Z) - r'_D(Z) \right) p(s(Z))D_i = 0.$$ 

In a symmetric equilibrium we get

$$r_L(Z) - r_D(Z) - \alpha = \frac{(r'_D(Z) - r'_L(Z))p(s(Z))Z}{p'(s(Z))s'(Z)Z + p(s(Z))n}.$$  (1)
Let $\Phi$ be right-hand side in (1).

Then implicit function theorem applied to (1) gives

$$\frac{\partial Z}{\partial n} = -\frac{\partial \Phi(Z, n)}{\frac{\partial Z}{\partial n}(Z, n)} > 0.$$ 

Use now that $s$ is decreasing in $Z$: More banks reduce risk!