# Economics of Banking
# Lecture 12

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# The circular city

There are $n$ banks, situated on a circle of total length 1:

Distance between neighboring banks $1/n$.

Depositors spread evenly over circle, each coming with 1 unit.

Transportation cost of $t$ per unit of distance.

Fixed cost of keeping a bank $f$.

# Optimum for society

Minimize sum of (a) travel cost and (b) the fixed cost:

(a) Cost of serving the customers of a given bank is

$$2\int_0^{\frac{1}{2n}} tx\,dx = \frac{t}{4n^2}.$$

Total cost to society with $n$ banks is

$$C = \frac{t}{4n} + nf.$$

First order conditions for a minimum:

$$\frac{dC}{dn} = -\frac{t}{4n^2} + f = 0$$

with solution

$$n^* = \frac{1}{2}\sqrt{\frac{t}{f}},$$

# Market solution

Assume that our bank has $r_D$ and all the others have $r'_D$.

Find the borderline depositor $x$ from

$$r_D - tx = r'_D - t\left(\frac{1}{n} - x\right),$$

so that the demand $D = 2x$ is

$$D = \frac{1}{n} + \frac{r_D - r'_D}{t}.$$

Use standard monopoly price rule

$$r - r_D = \frac{1}{2}\left[\frac{t}{n} + (r - r'_D)\right]$$

In symmetric equilibrium, $r_D = r'_D$,

$$r_D = r - \frac{t}{n}.$$

# Free entry

Bank profits are

$$\frac{1}{n}\frac{t}{n} - f = \frac{t}{n^2} - f,$$

New banks enter as long as there are nonzero profits, so

$$\frac{t}{n^2} - f = 0$$

Solving for $n$:

$$n^0 = \sqrt{\frac{t}{f}}.$$

Too many banks in equilibrium!

# Deposits and loans

Assume: Each individual wants to borrow $L < 1$.
Transport cost $t_L$ in loan business.

With the same reasoning as before, we get a symmetric equilibrium with loan rate

$$r_L = r + \frac{t_L}{nL}.$$

Profits are $\frac{t_L}{n^2}$ in loan business.

# An application

We use the model to consider *deposit rate regulation:*
Keeping deposit rates low in order to obtain lower loan rates

For simplicity: $r_D$ is set to 0

Then deposit business is more profitable, loan as before. No change in loan rates.

Alternative: Allow banks to use contingent contracts (no loans without deposits). Then

$$r_L^* = \left(r - \frac{r}{L}\right) + \frac{t_D + t_L}{nL} = \left(r + \frac{t_L}{nL}\right) - \frac{1}{L}\left(r - \frac{t_D}{n}\right)$$

Loan rates are lower!

# Risk-taking in banks

Consider a bank where all engagements run over a single period.

Expected value of loans is fixed at $\mu$ but varies with a parameter $\sigma$

Loans $L$ at rate $\tilde{r}_L$ are funded by deposits $D$ with rate $r_D(\sigma)$ (assume $L = D$)

Bank fails if $r_L < r_D(\sigma)$

Probability of success is $p(\sigma) = \mathrm{P}\{\tilde{r}_L > r_D(\sigma)\}$

Value of future bank business (franchise value) $V$.

# Perfect information

Depositors can observe $\sigma$

Funding condition is

$$E[\min\{\tilde{r}_L, r_D(\sigma)\}|\sigma] = 1$$

Expected profit of bank

$$\Pi(\sigma) = D\, E[\max\{0, \tilde{r}_L - r_D(\sigma)\}|\sigma],$$

Bank chooses $\sigma$ so as to maximize $\Pi(\sigma) + p(\sigma)V$.

Use $\max\{0, r_L - r_D\} + \min\{r_L, r_D\} = r_L$ to get

$$\Pi(\sigma) + D = E[\tilde{r}_L|\sigma]D = \mu D,$$

# Profits and survival

Under **perfect information:** $\Pi(\sigma) = (\mu - 1)D$ independent of $\sigma$ but $\sigma$ matters for $p(\sigma)$

Consequence: for $V > 0$, the bank chooses minimal $\sigma$.

**Imperfect information:** Depositors expect $\hat{\sigma}$ and will demand a rate $r_D(\hat{\sigma})$ giving at least zero expected return. Banks find

$$\max_{\sigma}\{\Pi(\sigma) + p(\sigma)V\},$$

and profits

$$\Pi(\sigma) = D\,\mathrm{E}[\max\{0, \tilde{r}_L - r_D(\hat{\sigma})\}|\sigma]$$

increase in $\sigma$: Trade-off between profits and survival!

# Oligopoly with insured depositors

There are $n$ banks choosing investment:

Investment with parameter $s$ yields a payoff $s$ with probability $p(s)$.

Deposit rate $r_D$ depends on $\sum_{i=1}^{n} D_i$.

Deposits are insured, banks pay a rate $\alpha$ for this.

Bank $i$ chooses $(s_i, D_i)$ so as to maximize expected profits

$$p(s_i)\left(s_i - r_D\left(D_i + \sum_{j\neq i} D_j\right) - \alpha\right) D_i.$$

# Riskiness and number of banks

In a symmetric equilibrium, all choose the same $D_i (= D)$.

First order conditions

$$p'(s)(s - r_D(nD) - \alpha) + p(s) = 0,$$
$$s - r_D(nD) - r_D'(nD)D - \alpha = 0.$$

can (in principle) can be solved for $D$ and $s$ when $n$ is fixed.

We are interested in what happens to $s$ when $n$ increases.

First order conditions is an equations system which gives $s$ as implicit function of $n$.

It can be shown that $\frac{\partial s}{\partial n} > 0$.

# Banks choosing loan rate

But banks do not invest, they offer loans to entrepreneurs!

Entrepreneurs maximize $p(s)(s - r_L)$, with first order condition

$$s + \frac{p(s)}{p'(s)} = r_L.$$

Gives rise to demand relationship of the form $r_L(L)$

Now banks select $D_i$

$$p(s)\left[r_L\left(\sum_{j=1}^{n} L_j\right) - r_D\left(\sum_{j=1}^{n} D_j\right) - \alpha\right] D_i.$$

under the constraint $L_i = D_i$.

# Equilibrium with many banks

Define $Z = \sum_{j=1}^{n} D_j$.
$s$ is increasing in $r_L$ which decreases in $Z$, so $s$ is in $\sum_{j=1}^{n} D_j$.

The first order condition is

$$\begin{aligned}(r_L(Z) - r_D(Z) - \alpha)\left[p'(s(Z))s'(Z)D_i + p(s(Z))\right] \\ + (r_L'(Z) - r_D'(Z))p(s(Z))D_i = 0.\end{aligned}$$

In a symmetric equilibrium we get

$$r_L(Z) - r_D(Z) - \alpha = \frac{(r_D'(Z) - r_L'(Z))p(s(Z))Z}{p'(s(Z))s'(Z)Z + p(s(Z))n}. \quad (1)$$

Let $\Phi$ be right-hand side in (1).

Then implicit function theorem applied to (1) gives

$$\frac{\partial Z}{\partial n} = -\frac{\frac{\partial \Phi}{\partial n}(Z, n)}{r_L'(Z) - r_D'(Z) - \frac{\partial \Phi}{\partial n}(Z, n)} > 0.$$

Use now that $s$ is decreasing in $Z$: More banks reduce risk!