Economics of Banking
Lecture 14

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Bank runs

Diamond-Dybvig again

Investment project I with payoff $R > 1$ per unit.

Individuals subject to liquidity shock at date $t = 1$ with probability π.

Optimal arrangement: Deposit contract.

Bank delivers either c_1^0 at $t = 1$ or c_2^0 at $t = 2$, such that

$$\max_{c_1, c_2, I} \pi u(c_1) + (1-\pi) u(c_2)$$

subject to constraints

$$\begin{aligned} \pi c_1 &= 1 - I, \\ (1-\pi) c_2 &= RI. \end{aligned}$$

Sunspot equilibria

Suppose that *patient* depositors become afraid that bank will not pay at $t = 2$:

Then they they will prefer c_1 at $t = 1$ to nothing at $t = 2$.

Bank run: Bank cannot satisfy its obligations and is bankrupt.

The initial beliefs of turned out to be justified!

(Extended model: Add a mechanism producing signal $s \in \{0, 1\}$.

If the depositors believe that $[s = 1] \Rightarrow [c_2 = 0]$, then running the bank is optimal choice, and:

Beliefs sustained: This is an example of a *sunspot equilbrium.*)

How can bank runs be prevented?

We shall be concerned with several proposals for preventing than bank runs happen:

(1) **Suspension of payment:** Bank declares that it will pay out only the amount corresponding to c_1 times number of impatient depositors.

Effect: Patient depositors can see that the bank will survive to $t = 2$.

Problem: The share π of impatients must be observed with high precision.

Works only in theory. Points to insurance of deposits (to be treated later).

Deposit certificates instead of deposits

(2) **Deposit certificates**

Certificates with dividend payment d at $t = 1$, giving $R(1 - d)$ at $t = 2$.

Impatient individuals sell the certificate at price p and gets

$$c_1 = d + p,$$

Patient individuals buy $\frac{d}{p}$ and get

$$c_2 = \left(1 + \frac{d}{p}\right) R(1 - d).$$

Finding the optimal dividend

The price p is found by supply=demand,

$$\pi = (1-\pi)\frac{d}{p} \text{ or } p = \frac{1-\pi}{\pi}d,$$

so

$$c_1 = \frac{d}{\pi}, \quad c_2 = \frac{R(1-d)}{1-\pi}.$$

Dividend payment can be specified to achieve optimum.

Problem: Depends on the correct setting of d.

Does away with traditional banking (shareholders instead of depositors).

Narrow banking

(3) **Narrow banking**

Banks taking deposits should invest only in safe securities (sold at full value at any time). so that the bank runs cannot occur.

In the context of the D-D model: The constraints are

$$\begin{aligned} c_1 &\leq 1 - I \\ c_2 &\leq RI, \end{aligned}$$

Maximizing expected utility here is inferior to the D-D solution, preventing banking as we understood it so far.

Too simple formalization – narrow banking is about separating the activities of banks.

Liquidity risk

Liquidity can be considered as an inventory problem:

Assume: Bank gets r_L on loans but must pay penalty rate r_p if reserves R too small.

Depositors' demand $\tilde{x}$ has probability distribution function $F(x)$ and density $f(x)$.

Expected profit

$$\pi(R) = (D - R)r_L + Rr - r_p \int_R^{\infty} (x - R) f(x) \, \mathrm{d}x.$$

Optimal reserves

First order conditions for a maximum:

$$-r_L + r + r_p \int_R^\infty f(x)\,\mathrm{d}x = 0$$

and since $\int_R^\infty f(x)\,\mathrm{d}x = 1 - F(x)$,

$$1 - F(R) = \mathrm{P}\{\tilde{x} \geq R\} = \frac{r_L - r}{r_p}.$$

Model not realistic: Suppose that $r_L - r = 4\%$ and $r_p = 20\%$.

Then the probability of being short of cash is $1/5$.

Basel III

Liquidity risk was **not** considered in Basel I and II.

Basel III introduces:

- a *liquidity coverage ratio:* High quality assets must cover one month net cash outflow.
- a ratio between long-term assets, suitably risk weighted, and *net stable funding* (deposits, long-term loans and equity).

Criticism: financial supervisors may not be the right institutions for dealing with questions of liquidity (better to use lenders of last resort).

Many banks with different depositors

Now there are n, banks, and bank j has depositors with probability π_j of being impatient. Average probability is $\overline{\pi}$.

Bank j proposes deposit contract (c_1^j, c_2^j).
In the social optimum the contracts should maximize

$$\frac{1}{n}\sum_{j=1}^{n}[\pi_j u(c_1^j) + (1-\pi_j)u(c_2^j)]$$

under the constraints

$$\frac{1}{n}\sum_{j=1}^{n}\pi_j c_1^j = 1 - I,$$

$$\frac{1}{n}\sum_{j=1}^{n}(1-\pi_j)c_2^j = RI,$$

$$0 \leq \ I \ \leq 1.$$

Properties of optimum

By concavity of u (risk aversion):

All individuals get the same contract (c_1^*, c_2^*), independent of bank.

If I^* is the investment supporting the contract, then at $t = 1$ we have:

$$\text{Average need for liquidity} = \frac{n_1}{n} \sum_{j:\pi_j > \overline{\pi}} \pi_j c_1^* - \frac{n_1}{n}(1 - I^*),$$

$$\text{Average surplus of liquidity} = \frac{n_2}{n}(1 - I^*) - \frac{1}{n} \sum_{j:\pi_j \leq \overline{\pi}} \pi_j c_1^*,$$

with n_1 and $n_2 = n - n_1$ are the numbers of banks with deficit resp. surplus.

The interbank rate

At $t = 2$, the loans are paid back with an interbank interest rate r.

Surplus (at $t = 1$) banks can use this when paying back at $t = 2$:

$$(1 + r)(1 - I^* - \pi_k c_1^*) = (1 - \pi_k)c_2^* - RI^*,$$

and borrowers can use the surplus at $t = 2$ to pay back debt,

$$(1 + r)[\pi_j c_1^* - (1 - I^*)] = RI^* - (1 - \pi_j)c_2^*,$$

Insert $1 - I^* = \overline{\pi}c_1^*$ and $RI^* = (1 - \overline{\pi})c_2^*$:

$$(1 + r)(\overline{\pi} - \pi_j)c_1^* = [(1 - \pi_j) - (1 - \overline{\pi})]c_2^* = (\overline{\pi} - \overline{\pi}_j)c_2^*,$$

so that $(1 + r) = \dfrac{c_2^*}{c_1^*}$ or

$$(1 + r) = \left(\frac{\overline{\pi}}{1 - \overline{\pi}}\right)\left(\frac{I^*}{1 - I^*}\right)R.$$

Interbank market with uncertainty

As before, probability of impatience differ, here: either π_h or π_l,

As always, investment can be liquidated at $t = 1$ at a value $L < 1$ per unit.

New feature: The investment is subject to uncertainty:

Outcome is R with some probability u, 0 otherwise, where

$$u = \begin{cases} u_s \\ u_l \end{cases} \quad u_S > u_r$$

The probability is revealed to the relevant bank only at $t = 1$.;

Full information

Size of deposit withdrawals $j \in \{h, l\}$ and quality of investment $i \in \{r, s\}$ can be observed by all banks at $t = 1$.

Then there will be different interest rates, r_s and r_r, for to the two types of borrowers.

Lenders get the expected payoff $u_s(1 + r_s)$ from s-borrowers and $u_r(1 + r_r)$ from r-borrowers, so

$$u_s(1 + r_s) = u_r(1 + r_r).$$

Equilibrium

A bank with a large π_h needs a loan of size $\pi_h c_1 - (1 - I^*)$.

The rates r_s and r_r can be found from the condition to be satisfied at $t = 2$:

$$(1 + r_i)(\pi_h c_1 - (1 - I^*)) = u_i R I^* - (1 - \pi_h) c_2,$$
$$(1 + r_i)\left[(1 - I^*) - \pi_l c_1\right] = (1 - \pi_l) c_2 - u_i R I^*$$

$i = s, r$, and the average rate (given the proportion of safe and risky projects) is

$$\rho_0 = \frac{\hat{\pi}}{1 - \hat{\pi}} \frac{I^*}{1 - I^*} \hat{u} R$$

(much as before)

Asymmetric information

If lenders cannot observe whether borrower is s or r, then there is only one rate ρ_1.

(1) Let u' be average probability. If $u'\rho_1 < 1$, there will be no lenders.

(2) If both r and s banks are borrowing, then $u' = \hat{u}$, and the repayment rate ρ_1 is found as in the B-G model

$$\rho_1 = \frac{\hat{\pi}}{1-\hat{\pi}} \frac{I^*}{1-I^*} \hat{u} R.$$

(3) In this case, it expected profit for a borrower is

$$u(RI^* - (1-\pi_h)c_2 - \rho_1(\pi_h c_1 - (1-I^*))) - (1-I^*).$$

and *not* using the market, it is

$$u(R(I^* - \frac{1}{L}(\pi_h c_1 - (1-I^*)) - (1-\pi_h)c_2) - \pi_h c_1$$

The drying out of the interbank market

Rewriting the two equations as

$$u(RI^* - (1-\pi_h)c_2) - [u\rho_1\pi_h c_1 + (1-u\rho_1)(1-I^*)]$$

and

$$u(RI^* - (1-\pi_h)c_2) - \left[\left(1+\frac{uR}{L}\right)\pi_h c_1 - \frac{uR}{L}(1-I^*))\right].$$

If $u_s \approx 1$ and u_s small so that ρ_1 is large (remember, $u'\rho_1 > 1$), then *not* borrowing is better for s-borrowers if $\dfrac{uR}{L}$ is close to 1.

Consequence: *Adverse selection*, only bad borrowers, lenders may stay away.