Economics of Banking
Lecture 16

April 2025

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A simple model of risky banking

Investment projects may have a liquidation value:

Two banks in two periods $t = 0, 1, 2$.

Investment: R in next period with probability π, 0 otherwise.

Payoffs independent over time, but industry-dependent.

Banks have specific skills: When the investment is done by a non-bank, it yields $R - \Delta$.

Banks have fully insured deposits.

Bank failures

Expected net outlays of the insurer:

(1) FF (two failures): Banks sold to private investors, paying at most

$$\underline{p} = \pi(R - \Delta - 1)$$

(2) SF or FS (one fails and the other succeeds):, The other bank which take over the assets and can get

$$\overline{p} = \pi(R - 1)$$

but may pay less, namely p^* with

$$\underline{p} < p^* < \overline{p}.$$

(3) SS (both banks succeed): no liquidation.

Insurance premium

The premium depends on the correlation between banks

If they invest in the same industry, then probability of FF at $t = 1$ is π.

Expected net outlays net outlays of the insurance:

$$q_s = (1 - \pi)(1 - \underline{p}),$$

If they invest in different industries, then expected net outlays are instead

$$q_d = \pi(1 - \pi)(1 - p^*) + (1 - \pi)^2(1 - \underline{p}) = q_s - \pi(1 - \pi)(p^* - \underline{p}).$$

Fair premium depends on correlation!

Other factors influencing premium

Suppose that bank 1 is small and bank 2 is large:

If 1 fails, 2 may take over at some price p° with $\underline{p} < p^\circ < \overline{p}$, and the expected net outlays of the insurance is

$$q^\circ = \pi(1-\pi)(1-p^\circ) + (1-\pi)^2(1-\underline{p}) < (1-\pi)(1-\underline{p}),$$

If 2 fails, 1 is too small to take over its investments, and the fair premium for 2 is $(1-\pi)(1-\underline{p})$.

Thus, the fair premium is smaller for 1 than for 2.

Possibility of bailout

Suppose that there is a lender of last resort (next chapter), supplying credits to banks at social cost c.

Bailing out if FF gives $\pi R - 1 - c$ per bank, liquidating gives a loss of $\pi\Delta$.

Premium if banks choose the same industry are

$$\overline{q}_s = (1-\pi)\left[(1-\underline{p}) + \min\{\pi\Delta, c\}\right]$$

and if different industries

$$\overline{q}_d = \pi(1-\pi)(1-p^*) + (1-\pi)^2\left[(1-\underline{p}) + \min\{\pi\Delta, c\}\right].$$

We have $\overline{q}_s > q_s$, $\overline{q}_d > q_d$, and $\overline{q}_s > \overline{q}_d$. Finally:

$$\overline{q}_s - \overline{q}_d > q_s - q_d,$$

Difference becomes greater when all costs are taken into consideration.

Incentive compatibility

Premiums $\hat{q}_s$ and $\hat{q}_d$ designed so that banks to choose uncorrelated investments:

Profits if the banks invest in the same industry:

$$\Pi_s = \pi \mathrm{E}\left[\widetilde{\Pi}_2 \middle| SS\right] + (1-\pi)\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right] - \hat{q}_s,$$

and if they invest in different industries

$$\Pi_d = \pi \mathrm{E}\left[\widetilde{\Pi}_2 \middle| SS\right] + \pi(1-\pi)\mathrm{E}\left[\widetilde{\Pi}_2 \middle| SF\right] + (1-\pi)^2\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right] - \hat{q}_d.$$

We have

$$\mathrm{E}\left[\widetilde{\Pi}_2 \middle| SF\right] = \mathrm{E}\left[\widetilde{\Pi}_2 \middle| SS\right] + (\overline{p} - p^*),$$

so that

$$\Pi_d = \pi \mathrm{E}\left[\widetilde{\Pi}_2 \middle| SS\right] + \pi(1-\pi)(\overline{p} - p^*) + (1-\pi)^2\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right] - \hat{q}_d.$$

Incentive compatibility II

Then

$$\Pi_s - \Pi_d = \pi(1-\pi)\left[\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right] - (\overline{p} - p^*)\right] + \hat{q}_d - \hat{q}_s.$$

and we can solve to get

$$\hat{q}_s = \pi(1-\pi)\left[\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right] - (\overline{p} - p^*)\right] + \hat{q}_d.$$

Surcharge for correlated investments depends on the subsidy received through i a bailout, which is

$$\mathrm{E}\left[\widetilde{\Pi}_2 \middle| FF\right]$$

and on discount given to the surviving bank if only one failure.

Another model of banking

There are N households and μ banks, each endowed with 1 unit of money.

Investment gives a payoff $R > 0$ with probability p_L, 0 otherwise.

Households may invest on their own or sign a (deposit) contract with a bank taking care of investment.

Banks can choose to be *sound:* Success probability is $p_H = p_L + \triangle p > p_L$ at a cost C,

The households cannot observe whether a bank is sound or unsound.

Banks are allowed to invest their own unit and $k - 1$ units of deposit money.

Incentive compatibility

Banks get a fee Q from the depositors.

Banks taking deposits k will be sound only if

$$[R + Q(k-1)]p_H - Ck \geq [R + Q(k-1)]p_L,$$

or equivalently

$$Q \geq \frac{Ck - R\triangle p}{(k-1)\triangle p}$$

Household demand for deposits

If households believe that the fraction of sound banks is g, then deposits used only if

$$Q \leq \frac{Rg\triangle p}{p_L + g\triangle p} = R - \frac{Rp_L}{p_L + g\triangle p}. \tag{1}$$

We get a region of feasible (k, Q)-pairs.

Introducing deposit insurance

Deposit insurance collects taxes from banks, depositors and individual investors, tax rates are τ_j, for $j = B, D, S$.

Tax revenue is

$$T = \mu\tau_B + N\rho\tau_D + N(1-\rho)\tau_S.$$

with ρ fraction of households being depositors.

Each bank has $1-\tau_B$ and can initiate investments to the amount of $k(1-\tau_B)$, $(k-1)\mu(1-\tau_B)$ comes from depositors and from T, assume $T < (k-1)\mu(1-\tau_B)$.

After-tax wealth of depositors is $N(1-\tau_D)$ and not all consumers can be depositors, so

$$\rho = \frac{(k-1)\mu(1-\tau_B) - T}{N(1-\tau_D)} = \frac{\mu[k(1-\tau_B)-1]}{N(1-\tau_S)} - \frac{\tau_S}{1-\tau_S}.$$

Expected returns

Expected return from investing T, $T(R-Q)(p_L + g\triangle p)$, can be transferred to depositors who get

$$\frac{T}{\rho N}(R-Q)(p_L + g\triangle p) \tag{2}$$

if investment failed.

Bank incentives are as before (measured per unit invested), but household condition for bank use is

$$\rho\left[\frac{T}{\rho N} + (1-\tau_D)\right](R-Q)(p_L+g\triangle p)+(1-\rho)(1-\tau_S)Rp_L \geq Rp_L(1-\tau_S).$$

New region of feasible contracts

This condition can be rearranged to

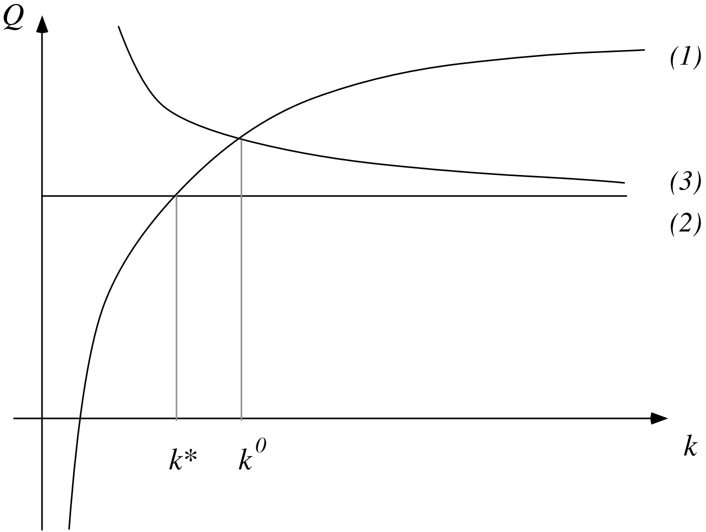
$$Q \leq R - \frac{Rp_L}{p_L + g\triangle p}\,\frac{\mu(k(1-\tau_B)-1) - N\tau_S}{\mu(k(1-\tau_B)-1) + \mu\tau_B}.$$

Here

$$\frac{\mu(k(1-\tau_B)-1) - N\tau_S}{\mu(k(1-\tau_B)-1) + \mu\tau_B} \leq 1$$

increases in k towards 1.

Tax financed deposit insurance increases investment



A case where deposit insurance is suboptimal

As usually, there are three dates, $t = 0, 1, 2$.

Investments:

1. A storage technology,
2. A risky technology, one unit at $t = 0$ yields $\widetilde{R}$ at $t = 2$, with

$$\widetilde{R} = \begin{cases} \frac{1}{q}R & \text{with probability } q \\ 0 & \text{with probability } 1 - q \end{cases}$$

   where q can be chosen by the bank (private information).

Expected payoff is R and variance is

$$\sigma^2\left(\tilde{R}\right) = q\left(R - \frac{1}{q}R\right)^2 + (1-q)R^2 = \frac{1-q}{q}R^2,$$

decreasing in q, 0 when $q = 1$.

Depositors' choices

Households are impatient with probability π.

If the contracts satisfy standard constraints

$$\begin{aligned} \pi c_1 &= 1 - I, \\ (1-\pi)c_2 &= I\widetilde{R}, \end{aligned} \tag{3}$$

then date 2 expected utility is

$$qu\left(\frac{I}{1-\pi}\frac{1}{q}R\right) \leq u\left(q\frac{I}{1-\pi}\frac{1}{q}R\right) = u\left(\frac{I}{1-\pi}R\right)$$

by concavity of u, so $q = 1$ is optimal.

Socially optimal investment is found by maximizing expected utility subject to (3).

Optimal contracts

Assume that banks maximize the utility of the representative consumer.

The contract will specify withdrawal c_E at date 1 and withdrawal c_L at date 2, outcome risk must be absorbed by the bank.

There is a bank run if depositors observe a signal (a "sunspot") at $t = 1$, probability of this signal is p.

If the fraction of depositors observing the signal is μ, then we maximize

$$(1 - \mu p)\left[\pi u(c_E) + (1 - \pi)u(c_L)\right] + \mu p \hat{\pi} u(c_E).$$

with $\hat{\pi}$ is the probability that bank can satisfy the fraction μ of depositors,

$$\hat{\pi} = \frac{1 - I(1 - L)}{c_E}. \tag{4}$$

Banks will have no incentive to choose risky projects, they compete for depositors who are risk averse.

Introducing deposit insurance in this model

Deposit insurance promises D to depositors in bankrupt banks.

Then bank runs are prevented if D is large enough, but:

Bankruptcy may occur also if the bank has a risky project which fails.

The insurance scheme must be financed, assume that this is done by taxing depositors.

The tax T collected should satisfy

$$TR = (1 - \pi)\mu D.$$

If there no or few banks fail, surplus B will be distributed among depositors in surviving banks.

A deviating bank

Consider a case where all banks have chosen the safe project with $q = 1$.

If a bank chooses $q < 1$, then the deposit contract should be suitably revised.

Assume that the bank proposes a contract with a markup $1/q$ on the repayment at $t = 2$.

The expected utility of a patient depositor is

$$qu\left(\frac{1}{q}\left[c_L - \frac{TR}{1-\pi}\right] + \frac{B}{1-\pi}\right) + (1-q)u(D).$$

Since we consider a deviation from $q = 1$, bankruptcy will occur only due to failed projects, so probability probability $1 - q$.

Also, since only one bank deviates, we have that $B = TR$.

Deviation may be attractive

The partial derivative of expected utility wrt. q is

$$\left.\frac{\partial U}{\partial q}\right|_{q=1} = -u'(c_L)\left[c_L - \mu D\right] - u(D) + u(c_L),$$

which is < 0 when $\mu < 1$ and D is close to c_L.

Thus $q = 1$ is not an equilibrium choice when there is deposit insurance.

Alternative: **Liquidity provision**

Banks can borrow specially issued money with their assets as collateral.

In next period, banks can change back to their original assets, if not able to pay back, it defaults.