Economics of Banking
Lecture 2

February 2023

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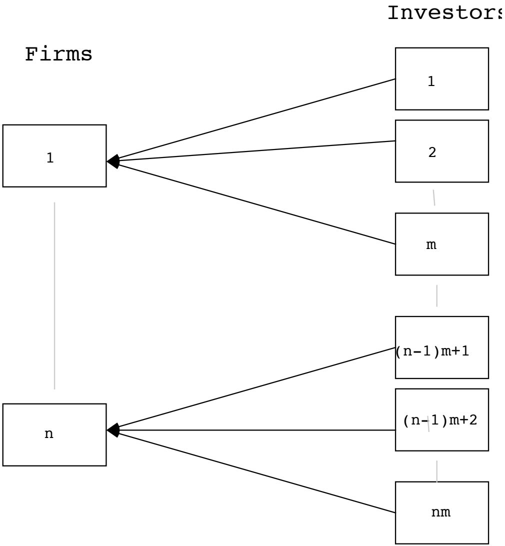
Delegated monitoring

There are m investors each with 1 unit
Each of them invests in n firms (spreading risk).
Outcome is $\tilde{y}$ (identically and independently distributed among firms).
Assume that $\bar{y} = \mathrm{E}\tilde{y}$ is large enough,

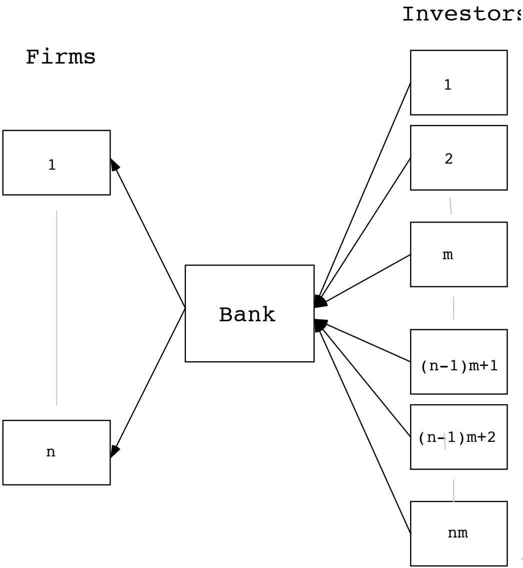
$$\bar{y} - K > 1 + r$$

(on average investment in firms is better than investment in bonds).

Investors and firms



With centralized monitoring



What about incentives?

Will the put up the necessary monitoring effort in monitoring the investments?
Organize the monitor as an independent **bank** taking deposits from investors:
Deposit rate r_D
The bank collects the outcomes and repays depositors with interest
But the bank may go bankrupt, namely if

$$\sum_{j=1}^{n} \tilde{y}_j - nK < (1 + r_D)n$$

What is the cost of having a bank?

Average bankruptcy cost with n firms is

$$C_n = n\gamma \, \mathrm{P}\left[\sum_{j=1}^{n} \tilde{y}_j - nK < (1 + r_D)n\right],$$

What happens when n becomes large?
Step 1: The deposit rate r_D:
Using the bank must be as good for investors as investing in bonds:

$$\mathrm{E}\left[\min\left\{\sum_{j=1}^{n} \frac{\tilde{y}_j}{n} - K, 1 + r_D\right\}\right] = 1 + r.$$

When $n \to \infty$, we have $\sum_{j=1}^{n} \frac{\tilde{y}_j}{n} \to \bar{y}$ by law of large numbers.
It follows that $1 + r_D \to 1 + r$ so that $r_D \to r$.

Bank is cost saving

Step 2: Return to the bankruptcy cost which on average is

$$\frac{C_n}{n} = \gamma \, \mathrm{P}\left[\frac{\sum_{j=1}^{n} \tilde{y}_j}{n} - K < 1 + r_D\right]$$

when $n \to \infty$, we use that $\sum_{j=1}^{n} \frac{\tilde{y}_j}{n} \to \bar{y}$ and $r_D \to r$, so that

$$\frac{C_n}{n} \to 0$$

In particular

$$K + \frac{C_n}{n} < mK$$

for large enough n if $m > 1$.

The moral hazard model

Two investment projects:

	I_G	I_B
Payoff if success	G	B
Prob. of success	π_G	π_B

Assume:
$B > G$ but $\pi_G > \pi_B$ and

$$\pi_B B < 1 < \pi_G G.$$

Incentive problem

Money market has repayment rate R.

Investor will choose I_G if

$$\pi_G(G - R) \geq \pi_B(B - R)$$

or

$$R \leq R^* = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B}.$$

Money market may not work

Savers will demand that lending money is at least as good as storing it:

$$\pi_G R \geq 1$$

Since $R \leq R^*$ when the market works, we have

$$\pi_G R^* \geq 1 \qquad \left(\text{or } \pi_G \geq \frac{1}{R^*} \right)$$

or

$$\pi_G G - \pi_B B \geq 1 - \frac{\pi_B}{\pi_G}.$$

(moral hazard not too important)

Monitoring saves the market

Assume that a **bank** can monitor that I_G is chosen (at a cost C_m). Funding condition is then

$$\pi_G R \geq 1 + C_m.$$

Then (assuming zero profits)

$$R = \frac{1 + C_m}{\pi_G}.$$

Since $R \leq G$, we get

$$\pi_G \geq \frac{1 + C_m}{G}.$$

Investment can be carried through using the bank even though we may have that $\pi_G < 1/R^*$.

A model of borrower behavior

Assume that a number of potential borrowers are all endowed with an investment project:
Outcome $\tilde{y}(\theta)$ is normal with mean θ and variance σ^2.
Variance σ^2 is the same for all, θ is individual information, not observable to others.
Investor-borrowers are risk-averse and assess their own project in such a way that their expected utility is

$$\mathrm{E}u(W + \tilde{y}(\theta)) = u\left(W + \theta - \frac{1}{2}\rho\sigma^2\right).$$

Some technicalities

Where did this expression come from?

If we assume that the utility function of the borrower-investor has the form

$$u(y) = -e^{-\rho y},$$

and the variable w is normally distributed with mean θ and variance σ^2, then one gets

$$\begin{aligned} \mathrm{E}[-e^{-\rho y}] &= \int_{-\infty}^{+\infty} -e^{-\rho y} \left(\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(y-\theta)^2}{2\sigma^2}} \right) \mathrm{d}y, \\ &= -e^{-\rho(\theta - \frac{1}{2}\rho\sigma^2)} \end{aligned}$$

The quantity $\theta - \frac{1}{2}\rho\sigma^2$ is the *certainty equivalent* of the risky project $\tilde{y}(\theta)$.

Selling projects

The market cannot observe project quality θ, so all have a common price P.

Investor will sell only if

$$\theta \leq P - \frac{1}{2}\rho\sigma^2.$$

This is the *adverse selection:* good projects are not offered in the market, the price is too low.

Given that only bad projects are in the market, this must be reflected in the equilibrium price:

$$P = \mathsf{E}[\theta \mid \theta \leq \theta^*],$$

where θ^* is the best project in the market.

Two types

Assume only two types $\theta_1 < \theta_2$ with probability π_1 and $\pi_2 = 1 - \pi_1$.
Case 1: If both can be sold , then $P = \pi_1\theta_1 + (1 - \pi_1)\theta_2$ and

$$\pi_1\theta_1 + (1 - \pi_1)\theta_2 - \frac{1}{2}\rho\sigma^2 \geq \theta_2,$$

so that

$$\pi_1(\theta_2 - \theta_1) \leq \frac{1}{2}\rho\sigma^2.$$

This shows that adverse selection must be small.

Case 2: If adverse selection is large, then only bad projects are in the market and $P = \theta_1$.
Investors with good projects must develop them alone even though risk averse.

Keeping a share

Sell only *part* of the project, keeping α.
The signal of quality is the participation of the project owner. This signal is trustworthy when

$$u(W + \theta_1) \geq u(W + (1 - \alpha)\theta_2 + \alpha\theta_1 - \frac{1}{2}\rho\alpha^2\sigma^2)$$

or equivalently if

$$\theta_1 \geq (1 - \alpha)\theta_2 + \alpha\theta_1 - \frac{1}{2}\rho\alpha^2\sigma^2$$

or

$$\frac{\alpha^2}{1 - \alpha} \geq \frac{2(\theta_2 - \theta_1)}{\rho\sigma^2}.$$

Let $\hat{\alpha}$ be the smallest share satisfying this inequality (with $=$).

Loss to seller

The financing cost to the project owner (compared with selling at full value θ_2), is the risk premium

$$C_f = \frac{1}{2}\rho\hat{\alpha}^2\sigma^2$$

This loss depends on σ^2. Indeed, we have

$$C_f = \frac{1}{2}\rho\hat{\alpha}^2\sigma^2 = (\theta_2 - \theta_1)(1 - \hat{\alpha})$$

and $\hat{\alpha}$ is a decreasing function of σ^2.

Coalitions of borrowers

Let n owners of good projects go together and create an investment pool.

Pooled project has outcome $\sum_{j=1}^{n} \tilde{y}_j(\theta_2)$,

Variance is $n\sigma^2$

Per unit invested the outcome is $\frac{1}{n}\sum_{j=1}^{n} \tilde{y}_j(\theta_2)$, and variance is

$$\left(\frac{1}{n}\right)^2 (n\sigma^2) = \frac{\sigma^2}{n}.$$

Cost to each investor is reduced.