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Delegated monitoring

There are $m$ investors each with 1 unit
Each of them invests in $n$ firms (spreading risk).
Outcome is $\tilde{y}$ (identically and independently distributed among firms).
Assume that $\tilde{y} = E\tilde{y}$ is large enough,

$$\tilde{y} - K > 1 + r$$

(on average investment in firms is better than investment in bonds).

Investors and firms

![Diagram showing the relationship between investors and firms.](image)
Why banks 2

With centralized monitoring

What about incentives?

Will the put up the necessary monitoring effort in monitoring the investments?
Organize the monitor as an independent bank taking deposits from investors:
Deposit rate $r_D$
The bank collects the outcomes and repays depositors with interest
But the bank may go bankrupt, namely if

$$\sum_{j=1}^{n} \tilde{y}_j - nK < (1 + r_D)n$$
Why banks 2

What is the cost of having a bank?

Average bankruptcy cost with \( n \) firms is

\[
C_n = n \gamma P \left[ \sum_{j=1}^{n} \tilde{y}_j - nK < (1 + r_D)n \right],
\]

What happens when \( n \) becomes large?

Step 1: The deposit rate \( r_D \):

Using the bank must be as good for investors as investing in bonds:

\[
E \left[ \min \left\{ \frac{1}{n} \sum_{j=1}^{n} \tilde{y}_j - K, 1 + r_D \right\} \right] \geq 1 + r.
\]

When \( n \to \infty \), we have \( \sum_{j=1}^{n} \frac{\tilde{y}_j}{n} \to \bar{y} \) by law of large numbers.

It follows that \( 1 + r_D \to 1 + r \) so that \( r_D \to r \).

Bank is cost saving

Step 2: Return to the bankruptcy cost which on average is

\[
\frac{C_n}{n} = \gamma P \left[ \frac{\sum_{j=1}^{n} \tilde{y}_j}{n} - K < 1 + r_D \right]
\]

when \( n \to \infty \), we use that \( \sum_{j=1}^{n} \frac{\tilde{y}_j}{n} \to \bar{y} \) and \( r_D \to r \), so that

\[
\frac{C_n}{n} \to 0
\]

In particular

\[
K + \frac{C_n}{n} < mK \quad \text{or} \quad nK < \lambda mK
\]

for large enough \( n \) if \( m > 1 \).
Moral hazard

The moral hazard model

Two investment projects:

<table>
<thead>
<tr>
<th>Payoff if success</th>
<th>$I_G$</th>
<th>$I_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob. of success</td>
<td>$\pi_G$</td>
<td>$\pi_B$</td>
</tr>
</tbody>
</table>

Assume:

$B > G$ but $\pi_G > \pi_B$ and

$$\pi_B B < 1 < \pi_G G.$$ 

Incentive problem

Money market has repayment rate $R$.

Investor will choose $I_G$ if

$$\pi_G (G - R) \geq \pi_B (B - R)$$

or

$$R \leq R^* = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B}.$$
Moral hazard

Money market may not work

Savers will demand that lending money is at least as good as storing it:

\[ \pi_G R \geq 1 \]

Since \( R \leq R^* \) when the market works, we have

\[ \pi_G R^* \geq 1 \quad \text{or} \quad \pi_G \geq \frac{1}{R^*} \]

or

\[ \pi_G G - \pi_B B \geq 1 - \frac{\pi_B}{\pi_G} \]

(moral hazard not too important)

Monitoring saves the market

Assume that a bank can monitor that \( I_G \) is chosen (at a cost \( C_m \)). Funding condition is then

\[ \pi_G R \geq 1 + C_m. \]

Then (assuming zero profits)

\[ R = \frac{1 + C_m}{\pi_G}. \]

Since \( R \leq G \), we get

\[ \pi_G \geq \frac{1 + C_m}{G}. \]

Investment can be carried through using the bank even though we may have that \( \pi_G < 1/R^* \).
A model of borrower behavior

Assume that a number of potential borrowers are all endowed with an investment project:
Outcome $\tilde{y}(\theta)$ is normal with mean $\theta$ and variance $\sigma^2$.
Variance $\sigma^2$ is the same for all, $\theta$ is individual information, not observable to others.
Investor-borrowers are risk-averse and assess their own project in such a way that their expected utility is

$$E u(W + \tilde{y}(\theta)) = u \left( W + \theta - \frac{1}{2} \rho \sigma^2 \right).$$

Some technicalities

Where did this expression come from?

If we assume that the utility function of the borrower-investor has the form

$$u(y) = e^{\rho y},$$

and the variable $w$ is normally distributed with mean $\theta$ and variance $\sigma^2$, then one gets

$$E[e^{-\rho y}] = \int_{-\infty}^{+\infty} e^{-\rho y} \left( \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(y-\theta)^2}{2\sigma^2}} \right) dy,$$

$$= e^{-\rho (\theta - \frac{1}{2} \rho \sigma^2)}.$$

The quantity $\theta - \frac{1}{2} \rho \sigma^2$ is the certainty equivalent of the risky project $\tilde{y}(\theta)$. 
Selling projects

The market cannot observe project quality $\theta$, so all have a common price $P$. Investor will sell only if

$$\theta \leq P - \frac{1}{2} \rho \sigma^2.$$  

This is the *adverse selection*: good projects are not offered in the market, the price is too low.

Given that only bad projects are in the market, this must be reflected in the equilibrium price:

$$P = E[\theta \mid \theta \leq \theta^*],$$

where $\theta^*$ is the best project in the market.
Adverse selection

Two types

Assume only two types $\theta_1 < \theta_2$ with probability $\pi_1$ and $\pi_2 = 1 - \pi_1$.

Case 1: If both can be sold, then $P = \pi_1 \theta_1 + (1 - \pi_1) \theta_2$ and

$$\pi_1 \theta_1 + (1 - \pi_1) \theta_2 - \frac{1}{2} \rho \sigma^2 \geq \theta_2,$$

so that

$$\pi_1 (\theta_2 - \theta_1) \leq \frac{1}{2} \rho \sigma^2.$$

This shows that adverse selection must be small.

Case 2: If adverse selection is large, then only bad projects are in the market and $P = \theta_1$.
Investors with good projects must develop them alone even though risk averse.

Keeping a share

Sell only part of the project, keeping $\alpha$.
The signal of quality is the participation of the project owner. This signal is trustworthy when

$$u(W + \theta_1) \geq u(W + (1 - \alpha) \theta_2 + \alpha \theta_1 - \frac{1}{2} \rho \alpha^2 \sigma^2)$$

or equivalently if

$$\theta_1 \geq (1 - \alpha) \theta_2 + \alpha \theta_1 - \frac{1}{2} \rho \alpha^2 \sigma^2$$

or

$$\frac{\alpha^2}{1 - \alpha} \geq \frac{2(\theta_2 - \theta_1)}{\rho \sigma^2}.$$

Let $\hat{\alpha}$ be the smallest share satisfying this inequality (with $=.$).
Loss to seller

The financing cost to the project owner (compared with selling at full value $\theta_2$), is the risk premium

$$C_f = \frac{1}{2} \rho \hat{\alpha}^2 \sigma^2$$

This loss depends on $\sigma^2$. Indeed, we have

$$C_f = \frac{1}{2} \rho \hat{\alpha}^2 \sigma^2 = (\theta_2 - \theta_1)(1 - \hat{\alpha})$$

and $\hat{\alpha}$ is a decreasing function of $\sigma^2$.

Coalitions of borrowers

Let $n$ owners of good projects go together and create an investment pool. Pooled project has outcome $\sum_{j=1}^{n} \bar{y}_j(\theta_2)$,

Variance is $n \sigma^2$

Per unit invested the outcome is $\frac{1}{n} \sum_{j=1}^{n} \bar{y}_j(\theta_2)$, and variance is

$$\left(\frac{1}{n}\right)^2 (n \sigma^2) = \frac{\sigma^2}{n}.$$ 

Cost to each investor is reduced.