Table of contents

We shall be concerned with several aspects of the loan contract:

- What is the loan contract?
- Loan contracts under perfect information
- Loan contracts with asymmetric information I: hidden information
  - Costly monitoring
  - Threat of non-renewal
- Loan contracts with asymmetric information II: hidden action
What is in the loan contract?

Simple view: a loan contract specifies when and how much to repay

Less simple view: Contract specifies:

- Repayment
- What happens if borrower cannot repay this amount

Loan contracts with full information

Borrower gets \( \tilde{y} \), is observable and contractable

But \( \tilde{y} \) is random, so:

Borrower has utility \( u \), lender has \( v \), and expected outcomes are

\[
E[u(\tilde{y} - R(\tilde{y}))] = \int u(y - R(y))f(y) \, dy, \quad E[v(R(\tilde{y}))] = \int v(R(y))f(y) \, dy
\]

Contracts should be Pareto optimal: none of the parties can be made better off without the other party becoming worse off.
Pareto optimal contracts

We want to characterize such contracts:

PO contracts maximize a weighted sum of the expected utilities of borrowers and lender:

$$\lambda_B E[u(\bar{y} - R(\bar{y}))] + \lambda_L E[v(R(\bar{y}))]$$

is maximized for some positive numbers $\lambda_B, \lambda_L$.

First order conditions are

$$\frac{\partial}{\partial y} \left( \lambda_B u'(y - R(y)) - \lambda_L v'(R(y)) \right) = 0,$$

for each value $y$ of the random variable $\bar{y}$.

Implicit function

This equation gives us $R$ as a function of $y$.

We now use the implicit function theorem to get

$$\frac{dR}{dy} = \frac{\lambda_B u''}{\lambda_B u'' + \lambda_L v''}.$$
Using the result:

\[ R(y) \]

If the bank is risk neutral, so that \( v'' = 0 \), we get

\[ \frac{dR}{dy} = 1 \]

If both are risk averse, then \( R'(y) < 1 \) (risk-sharing)
Asymmetric information I: Hidden information  
Costly monitoring  

Truthful reporting

If the outcome of $\tilde{y}$ is observed only by borrower, there is an incentive problem.

Assume that true $y$ can be inspected at a cost (not specified here).

We want truth-telling to be optimal for the borrower, but to use as little inspection as possible.

Let $A$ be the reports from the borrower which will be audited.

Properties of such a repayment function

1. If $y_1 < y_2$ both are not audited, then we cannot have $R(y_1) < R(y_2)$

Thus, repayment is constant, say $R(y) = \bar{R}$, in the no-auditing region.

2. If $y_1$ is audited, and $R(y_2) < R(y_1)$, then also $y_2$ must be audited. In particular,

$$R(y) = \bar{R}, \quad y \notin A,$$
$$R(y) \leq \bar{R}, \quad y \in A.$$
Incentive compatible contract

A repayment function with these properties may look as this:

\[ R \]
\[ y \]

Minimizing cost

We now add a condition of efficiency: contract maximizes expected repayment for given probability of audit. Then we get the standard contract.
Threat of no renewal

Model of repeated engagements: 2 periods, in each period outcome $y_H$ with probability $p$, otherwise $y_L$. No discounting.

We assume $y_L < 1$.

At $t = 2$, borrower reports $y_L$.

Rule: New engagement at $t = 1$ only if reported outcome is $y_H$.

Conditions for feasibility

Incentive compatibility condition for the borrower (at $t = 1$)

$$-R + p(y_H - y_L) \geq -y_L$$

Present value for the bank is nonegative if

$$-1 + (1 - p)y_L + p(R - 1 + y_L) = p(R - 1) - 1 + y_L \geq 0$$

Combine them to get

$$1 - y_L \leq p(Ey - 1)$$
Special case: sovereign debt

Simple (Solow) model of a country:

Country borrows $I$, invests one-period production with output $f(I)$.

Repayment after one period $(1 + r)I$.

Optimal level of investment $I^*$ maximizes $f(I) - (1 + r)I$, first order condition

$$f'(I^*) = 1 + r.$$
Asymmetric information I: Hidden information

**Repudiating debt**

What if debt is not paid back? Borrowers reply: no new debt any more.

Future loss (at discount rate $\beta$) is

$$\sum_{t=1}^{\infty} \beta^t [f(l) - (1 + r)l] = \frac{\beta}{1 - \beta} [f(l) - (1 + r)l].$$

Debt is repaid if loss $\geq$ gain from repudiating debt:

$$(1 + r)l \leq \beta f(l).$$

Asymmetric information II: Hidden information

**The model**

Outcome $\tilde{y}$ has density function $f(y, e)$ which depends on effort $e$.

Given repayment $R(\cdot)$, the borrower chooses $e^*$ to maximize expected profit

$$\pi(R, e) = \int (y - R(y)) f(y, e) \, dy - C(e)$$

We want $R(\cdot)$ to be chosen optimal for the borrower given that the lender should have $R_L^0$.

$$\max \pi(R, e^*)$$

such that

- $0 \leq R(y) \leq y$, all $y$,
- $\pi(R, e) \leq \pi(R, e^*)$, all $e$,
- $E[R(\tilde{y})|e^*] \geq R_L^0$. 

Lecture 5  
February 2021  
17 / 21
Optimal contract

Proof

Simplify: Replace the IC condition with its 1st order condition

$$\pi'_e(R, y) = \int (y - R(y)) f'_e(y, e) dy - C'(e) = 0.$$ 

For each $y$, the repayment $R(y)$ maximizes the Lagrangian

$$(y - R(y))(f(y, e) + \mu f'_e(y, e)) + \lambda R(y)f(y, e)$$

$$= y(f(y, e) + \mu f'_e(y, e)) + (\lambda - 1)f(y, e)R(y) - \mu f'_e(y, e)R(y),$$

By linearity, in maximum either $R(y) = y$ or $R(y) = 0$. 
The first case arises if

$$(\lambda - 1)f(y, e) \geq \mu f'(y, e)$$

and this can be rewritten as

$$\frac{f'_e(y, e)}{f(y, e)} \leq \frac{\lambda - 1}{\mu}.$$ 

Assume that $\frac{f'_e(y, e)}{f(y, e)}$ is increasing in $y$, then this inequality is satisfied as long as $y$ is $\leq$ some threshold $y^*$. 