Economics of Banking
Lecture 5

February 2023

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We shall be concerned with several aspects of the loan contract:

- What is the loan contract?
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What is in the loan contract?

Simple view: a loan contract specifies when and how much to repay

Less simple view: Contract specifies:

- Repayment
- What happens if borrower cannot repay this amount

Loan contracts with full information

Borrower gets $\tilde{y}$, is **observable** and **contractable**

But $\tilde{y}$ is random, so:

Borrower has utility u, lender has v, and expected outcomes are

$$\mathrm{E}[u(\tilde{y}-R(\tilde{y}))] = \int u(y-R(y))f(y)\,dy, \quad \mathrm{E}[v(R(\tilde{y}))] = \int v(R(y))f(y)\,dy$$

Contracts should be *Pareto optimal:* none of the parties can be made better off without the other party becoming worse off.

Pareto optimal contracts

We want to characterize such contracts:

PO contracts maximize a weighted sum of the expected utilities of borrowers and lender:

$$\lambda_B \mathrm{E}[u(\tilde{y} - R(\tilde{y}))] + \lambda_L \mathrm{E}[v(R(\tilde{y}))]$$

is maximized for some positive numbers λ_B, λ_L.

First order conditions are

$$\lambda_B u'(y - R(y)) - \lambda_L v'(R(y)) = 0.$$

for each value y of the random variable $\tilde{y}$.

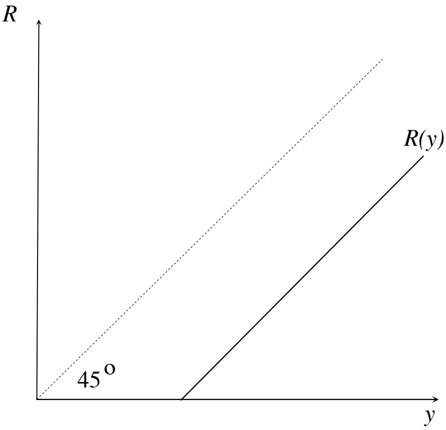
Implicit function

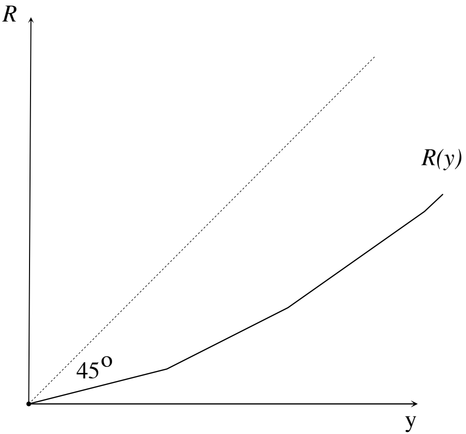
This equation gives us R as a function of y.

We now use the implicit function theorem to get

$$\frac{dR}{dy} = \frac{\lambda_B u''}{\lambda_B u'' + \lambda_L v''}.$$

Using the result:





Interpreting the result

If the bank is risk neutral, so that $v'' = 0$, we get

$$\frac{dR}{dy} = 1$$

If both are risk averse, then $R'(y) < 1$ *(risk-sharing)*

Truthful reporting

If the outcome of $\tilde{y}$ is observed only by borrower, there is an incentive problem

Assume that true y can be inspected at a cost (not specified here)

We want truth-telling to be optimal for the borrower, but to use as little inspection as possible.

Let A be the reports from the borrower which will be audited.

Properties of such a repayment function

(1) If $y_1 < y_2$ both are not audited, then we cannot have $R(y_1) < R(y_2)$

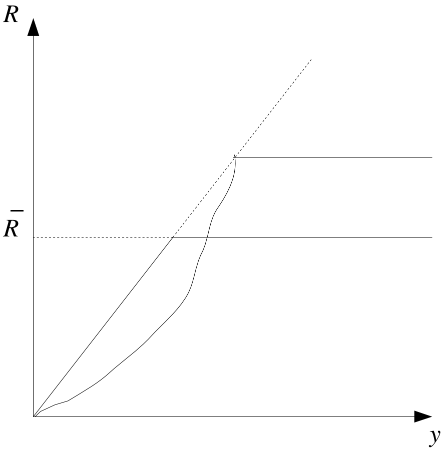
Thus, repayment is constant, say $R(y) = \overline{R}$, in the no-auditing region.

(2) If y_1 is audited, and $R(y_2) < R(y_1)$, then also y_2 must be audited. In particular,

$$\begin{aligned} R(y) &= \overline{R}, \; y \notin A, \\ R(y) &\leq \overline{R}, \; y \in A. \end{aligned}$$

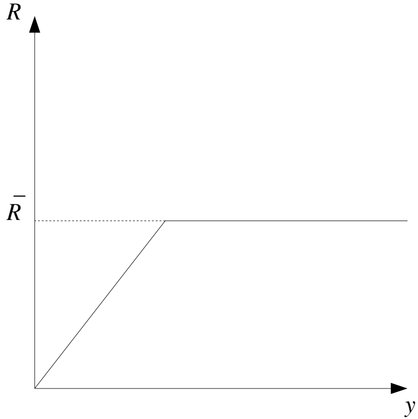
Incentive compatible contract

A repayment function with these properties may look as this:



Minimizing cost

We now add a condition of efficiency: contract maximizes expected repayment for given probability of audit.
Then we get the *standard* contract



Threat of no renewal

Model of repeated engagements: 2 periods, in each period outcome y_H with probability p, otherwise y_L. No discounting.

We assume $y_L < 1$.

At $t = 2$, borrower reports y_L.

Rule: New engagement at $t = 1$ only if reported outcome is y_H.

Conditions for feasibility

Incentive compatibility condition for the borrower (at $t = 1$)

$$-R + p(y_H - y_L) \geq -y_L$$

Present value for the bank is nonegative if

$$-1 + (1 - p)y_L + p(R - 1 + y_L) = p(R - 1) - 1 + y_L \geq 0$$

Combine them to get

$$\boxed{1 - y_L \leq p\left(\mathrm{E}\tilde{y} - 1\right)}$$

Special case: sovereign debt

Simple (Solow) model of a country:

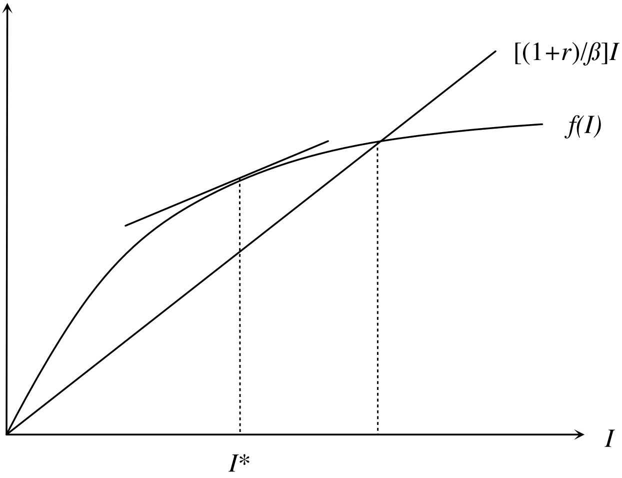
Country borrows I, invests one-period production with output $f(I)$.

Repayment after one period $(1+r)I$.

Optimal level of investment I^* maximizes $f(I)-(1+r)I$, first order condition

$$f'(I^*) = 1 + r.$$

Sovereign debt



Repudiating debt

What if debt is not paid back? Lenders' reply: no new debt any more

Future loss (at discount rate β) is

$$\sum_{t=1}^{\infty} \beta^t \left[f(I) - (1+r)I\right] = \frac{\beta}{1-\beta}[f(I) - (1+r)I].$$

Debt is repaid if loss $\geq$ gain from repudiating debt:

$$(1+r)I \leq \beta f(I).$$

The model

Outcome $\tilde{y}$ has density function $f(y, e)$ which depends on **effort** e.

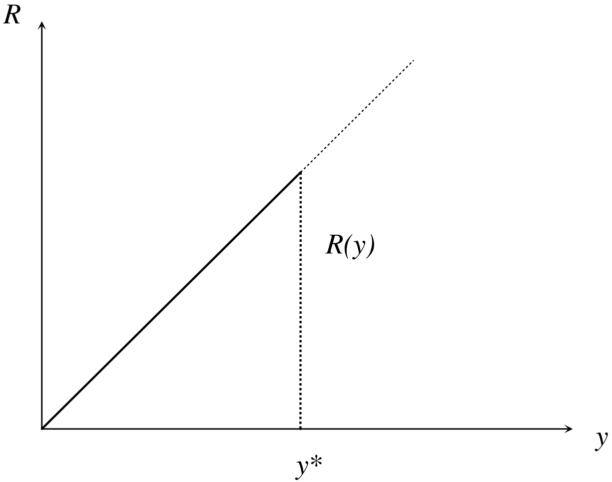
Given repayment $R(\cdot)$, the *borrower* chooses e^* to maximize expected profit

$$\pi(R, e) = \int (y - R(y)) f(y, e)\, dy - C(e)$$

We want $R(\cdot)$ to be chosen optimal for the borrower given that the lender should have R_L^0:

$$\begin{aligned}
&\max \ \pi(R, e^*) \\
&\text{such that} \\
&0 \leq R(y) \leq y, \ \text{all } y, \\
&\pi(R, e) \leq \pi(R, e^*), \ \text{all } e, \\
&\mathrm{E}[R(\tilde{y})|e^*] \geq R_L^0.
\end{aligned}$$

Optimal contract



Proof

Simplify: Replace the IC condition with its 1st order condition

$$\pi'_e(R, y) = \int (y - R(y)) f'_e(y, e)\, dy - C'(e) = 0.$$

For each y, the repayment $R(y)$ maximizes the Lagrangian

$$\begin{aligned}(y - R(y))(f(y, e) + \mu f'_e(y, e)) + \lambda R(y) f(y, e)\\ = y(f(y, e) + \mu f'_e(y, e)) + (\lambda - 1) f(y, e) R(y) - \mu f'_e(y, e) R(y),\end{aligned}$$

By linearity, in maximum either $R(y) = y$ or $R(y) = 0$.

Proof, end

The first case arises if

$$(\lambda - 1)f(y,e) \geq \mu f'_e(y,e)$$

and this can be rewritten as

$$\frac{f'_e(y,e)}{f(y,e)} \leq \frac{\lambda - 1}{\mu}.$$

Assume that $\frac{f'_e(y,e)}{f(y,e)}$ is increasing in y, then this inequality is satisfied as long as y is $\leq$ some threshold y^*.