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Remaining discussion of loan contracts:

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What is collateral?

- Borrower with outcome $y$ has pledged a collateral of size $C$.
- If borrower reports $y = 0$, then gain is only $R - C$.
- No gain at all if $C > R$ — typically the case.

Consequence: No problems of asymmetric information if contract with collateral

Collateral may change in value

Suppose that value of collateral is random, $f(C)$. The probability of default

$$p = P\left\{ \tilde{C} < R \right\} = \int_{-\infty}^{R} f(C) \, dC,$$

is in $R$, and $\frac{dp}{dR} = f(R)$. Given default, expected value of collateral is

$$\frac{1}{p} \int_{-\infty}^{R} Cf(C) \, dC < R,$$

so expected repayment is

$$\int_{-\infty}^{R} Cf(C) \, dC < pR.$$

Thus, collateral induces moral hazard: incentive to strategic default.

Consequence: over-collateralization.
The BTU model

Investment project: Outcome $y$ with probability $p_\theta(e)$, otherwise 0, where

$$\theta = \begin{cases} B \\ G \end{cases}$$

is investor type and

$$e = \begin{cases} e_H \\ e_L \end{cases}$$

is the effort of the borrower-investor with cost $V(e_L) < V(e_H)$.

Intuitively, the model should be such that

$$p_B(e_H) - p_B(e_L) > p_G(e_H) - p_G(e_L)$$

(Effort matters more for the bad than for the good borrower)

Optimum for society

Choose $e \in \{e_L, e_H\}$ maximize

$$p_\theta(e)y - V(e) - \rho,$$

with $\rho$ is the repayment for society.

For $\theta = G$, $e_L$ is optimal if

$$p_G(e_L)y - V(e_L) - \rho \geq p_G(e_H)y - V(e_H) - \rho$$

or

$$p_G(e_H) - p_G(e_L) \leq \frac{V(e_H) - V(e_L)}{y},$$

and for $\theta = B$, $e_H$ is optimal if

$$p_B(e_H) - p_B(e_L) \geq \frac{V(e_H) - V(e_L)}{y},$$
Optimum for society II

If we assume:

\[
[p_B(e_H) - p_B(e_L)] y \geq V(e_H) - V(e_L) \geq [p_G(e_H) - p_G(e_L)] y,
\]

(ranking of expected gain from more effort and cost of more effort)

then first-best optimum is where \( G \) uses \( e_L \) and \( B \) uses \( e_H \).

Can this optimum be sustained by financial intermediation?

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Moral hazard

Suppose that \( B \) is offered an unsecured loan with repayment \( \frac{\rho}{p_B(e_H)} \).

If failure, \( B \) pays nothing.

But then \( e_L \) is better for \( B \) when when

\[
p_B(e_L) \left( y - \frac{\rho}{p_B(e_H)} \right) - V(e_L) \geq p_B(e_H) \left( y - \frac{\rho}{p_B(e_H)} \right) - V(e_H),
\]

or equivalently, when

\[
[p_B(e_H) - p_B(e_L)] \left( y - \frac{\rho}{p_B(e_H)} \right)y \leq (V(e_H) - V(e_L)).
\]

(net expected gain from extra effort not big enough to cover increase in cost)
Introducing collateral

We look for a second-best equilibrium:

Bank proposes a contracts \((R_\theta, C_\theta)\) with collateral depending on types.

Expected borrower payoff is

\[
p_\theta(e^*)[y - R_\theta] - (1 - p_\theta(e^*))C_\theta - V(e^*)
\]

subject to the constraints

\[
p_\theta(e^*)R_\theta + (1 - p_\theta(e^*))C_\theta \geq \rho,
\]

\[
e^* \in \text{argmax}_{e \in \{e_L, e_H\}} p_\theta(e)[y - R_\theta] - (1 - p_\theta(e))C_\theta - V(e)
\]

The second-best contract

\textit{In the second best equilibrium, G-investors get an unsecured loan with repayment}

\[
\rho(G) = \frac{\rho}{p_G(e_L)}, \quad C_G = 0.
\]

If \((p_B(e_H) - p_B(e_L))y - (V(e_H) - V(e_L)) \geq 0\), then B-investors get the contract

\[
R_B = \frac{\rho}{p_B(e_H)} - (1 - p_B(e_H)) \frac{C_B}{p_B(e_H)},
\]

\[
C_B = -p_B(e_H)y + \rho + \frac{p_B(e_H)[V(e_H) - V(e_L)]}{p_B(e_H) - p_B(e_L)}.
\]
Sketch of proof:

We are maximizing expected payoff of $B$ under the given constraints, so repayment $R_B$ and collateral $C_B$ should be as small as possible under these constraints.

From the participation constraint

$$p_B(e_H) R_B + (1 - p_B(e_H))C_B = \rho$$

which gives $R_B$ for given $C_B$. From incentive compatibility,

$$p_B(e_H)(y - R_B) - V(e_H) - (1 - p_B(e_H))C_B$$

$$\geq p_B(e_L)(y - R_B) - V(e_L) - (1 - p_B(e_L))C_B$$

we get that $(p_B(e_H) - p_B(e_L))[y - R_B + C_B] = V(e_H) - V(e_L)$ or

$$C_B = -y + R_B + \frac{V(e_H) - V(e_L)}{p_B(e_H) - p_B(e_L)}.$$

Inserting $R_B$ and solving gives the solution.

Microfinance

A formal model of group lending

Borrowers can choose one of two projects, with outcome (if success)

$$y_G(L) \text{ or } y_B(L) \quad (\text{depending on } L!)$$

The probabilities of $\pi_G$ and $\pi_B$ are such that $\pi_G > \pi_B$ and

$$\pi_G y_G(L) > \pi_B y_B(L), \text{ all } L$$

Initial fixed cost $\bar{L}_j$ such that $y_j(L) = 0$ for $L \leq \bar{L}_j$,

$$\bar{L}_G < \bar{L}_B \quad \text{but} \quad \frac{dy_G}{dL} < \frac{dy_B}{dL} \quad \text{for } L \geq \bar{L}_B.$$

Risky project has a higher fixed cost and marginal product. There is an effort cost $\nu(L)$ with $\nu' > 0$. 
Consider pairs \((L, R)\) where 

\[ U_G(L, R) = U_B(L, R). \]

Then 

\[ \frac{\partial U_G}{\partial L} = \pi_G u'_G(y_G(L) - RL) \left[ \frac{dy_G}{dL} - R \right] - \nu'(L) \]
\[ < \pi_B u'_B(y_B(L) - RL) \left[ \frac{dy_B}{dL} - R \right] - \nu'(L) = \frac{\partial U_B}{\partial L}. \]

Increasing the loan and project size slightly \(\Rightarrow\) borrowers choose \(B\).
Equilibrium 2

Zero profit contracts satisfy

$$\pi_G R = r \text{ or } R = \frac{r}{\pi_G}, \quad R = \frac{r}{\pi_B}$$

where $r$ is the funding rate.

The equilibrium contract $(L^*, R^*)$ must be on the boundary of the $G$-region.

Equilibrium 3
Improvement 1

What happens if we introduce joint liability?

- Repayment rates can be lowered since probability of default decreases, but:
- Each individual must pay also if other individuals default

The two effects cancel out each other, BUT:

The boundary between $G$ and $B$ moves outward!

Improvement 2

![Diagram showing $G$ and $B$ chosen with points $(L^*, R^*)$, $r/p_B$, and $r/p_G$.]
Credit Rationing

Demand and supply for credits

- The price mechanism doesn’t work for the credit market:
- Individuals may agree to pay arbitrary high interest rates but cannot get loans

A possible explanation: Backward-bended supply?

Backward-bending supply

![Graph showing demand and supply curves for credits.](image-url)
Credit Rationing

But why?

So far, so good, but:

Why should the supply of credits be backward-bending?

One rather obvious possibility: Relationship between nominal and expected repayment

We shall be interested in explanations of this phenomenon in 3 different ways:
- Adverse selection
- Costly monitoring
- Moral hazard

Credit Rationing

Nominal and expected repayment

\[ E = ER \]

\[ R \]