Economics of Banking
Lecture 7

March 2025

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Adverse selection 1

Potential entrepreneurs:

Project τ has random outcome $\tilde{y}(\tau) = \mu + \tilde{z}(\tau)$,

Here $\tilde{z}(\tau)$ random with mean 0, distribution $F(z|\tau)$, s.t.

$$\tau' > \tau \quad \Leftrightarrow \quad F(z|\tau) \text{ 2nd order stochastically dominates } F(z|\tau')$$

(project τ' is more risky than τ)

Stochastic dominance 1

Stochastic dominance:

Comparing probability distributions (lotteries) **independent** of decision maker.

1st order: F 1st order dominates F' if expected utility is higher for any person with **increasing** utility.

It can be shown that in this case $F'(x) \geq F(x)$ for each x.

Stochastic dominance 2

Comparing lotteries with the same mean (say, $= 0$)

2nd order: F 2nd order dominates F' if expected utility is higher for any person with **concave** utility.

It can be shown that in this case $\int_{\infty}^{y} F(x)\,\mathrm{d}x \leq \int_{\infty}^{y} F'(x)\,\mathrm{d}x$ for each y.

Adverse selection 2

Bank uses standard contract with repayment R.
Entrepreneur's profit at z is

$$\pi(z, R) = \max\{\mu + z - R, 0\},$$

Let $\Pi(\tau, R) = \int_{-\infty}^{+\infty} \pi(z, R) f(z|\tau) \, dz$ be the expected outcome for type τ.

This can be written as

$$\Pi(\tau, R) = (\mu - R)(1 - F(R - \mu|\tau)) + \int_{R-\mu}^{\infty} z f(z|\tau) \, dz.$$

Adverse selection 3

Expected profit can be written (after some manipulations) as

$$\Pi(\tau, R) = \mu - R + \int_{-\infty}^{R-\mu} F(z|\tau)\, dz.$$

$\pi(\cdot, R)$ is convex, $-\pi(\cdot, R)$ concace, so: $\Pi(\tau, R)$ is increasing in τ !

Now we come to **adverse selection:** Let $\theta(R)$ be smallest type such that expected profit ≥ 0, so that

$$\Pi(\theta(R), R) = 0$$

Adverse selection 4

Use implicit function theorem to get

$$\theta'(R) = -\frac{1 - F(R-\mu|\theta(R))}{\int_{-\infty}^{R-\mu} F'_{\tau}(z|\theta(R))\, dz} > 0, \tag{1}$$

so that *increasing R forces the low-risk types out of the market!*

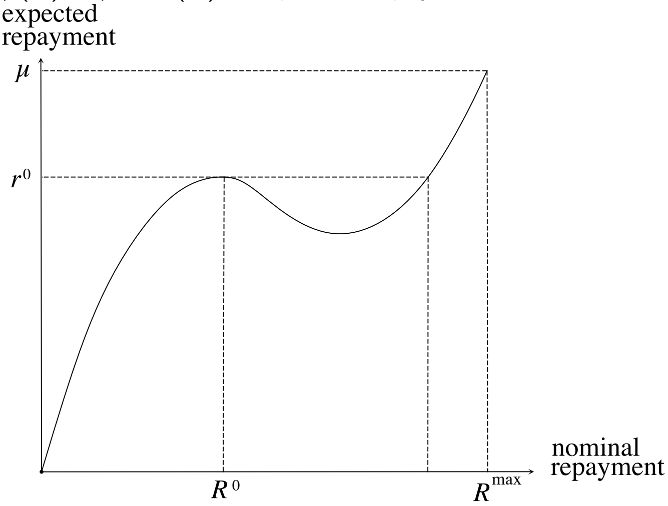
Average (over borrowers) profit is

$$\overline{\Pi}(R) = \frac{1}{1 - G(\theta(R))} \int_{\theta(R)}^{1} \Pi(\tau, R) g(\tau)\, \mathrm{d}\tau$$

G is distribution of borrower types

Adverse selection 5

Define $\rho(R) = \mu - \overline{\Pi}(R)$ – expected repayment to bank



Adverse selection 6

The curve has a (local) maximum at R^0 with $\rho(R^0) = r^0$

— expected repayment first increases, then decreases with nominal repayment.

(At R^{max} – the highest repayment rate at which there can be borrowers – repayment equals expected outcome

– showing that ρ eventually increases in our case)

An alternative view: Oversupply of credits

Investment projects: y_H if success, y_L if failure

Investors differ in probability π of success (investors distributed with density $f(\pi)$)

Borrowers have initial wealth W, borrow $B = 1 - W$

Investment project profitable for society as a whole if

$$\boxed{\pi y_H + (1 - \pi) y_L \geq 1 + r}$$

Oversupply 2

Given repayment R, borrower will accept loan if

$$\pi(y_H - RB) \geq (1 + r)W,$$

Smallest probability at R (with $=$) is $\pi(R)$.

Bank supplies credits as long as

$$\pi RB + (1 - \pi)y_L \geq (1 + r)B,$$

Assume free entry of banks, so that we have $=$ in this equation.

Oversupply 3

Let $f_R(\pi)$ be conditional density given that $\pi \geq \pi(R)$ (investor gets credits),

Average project in the market $\overline{\pi}(R) = \int_{\pi(R)}^{1} \pi f_R(\pi)\, d\pi$.

Then bank profits $= 0$ means that

$$\overline{\pi}(R)RB + (1 - \overline{\pi}(R))y_L = \overline{\pi}(R)(RB - y_L) + y_L = (1 + r)B,$$

and $\overline{\pi}(R) > \pi(R)$ implies

$$\pi(R)RB + (1 - \pi(R))y_L < \overline{\pi}(R)RB + (1 - \overline{\pi}(R))y_L = (1 + r)B.$$

Add the condition for the investor with $\pi = \pi(R)$,

$$\pi(R)(y_H - RB) = (1 + r)W,$$

to get

$$\boxed{\pi(R)y_H + (1 - \pi(R))y_L < 1 + r}$$

Costly monitoring

Assume standard contract, monitoring cost is c_m,

Monitoring when y is $< R$.

Outcome $\tilde{y}$ has distribution function $F(y)$ and density $f(y)$.

Then

$$\rho(R) = \int_0^R (y - c_m) f(y) dy + \int_R^\infty R f(y) dy.$$

Take the derivative w.r.t. R,

$$\rho'(R) = (R - c_m) f(R) - R f(R) + \int_R^\infty f(y) dy = [1 - F(R)] - c_m f(R).$$

For R large, first member goes to 0, so the derivative becomes negative

Moral hazard 1

Recall the simple model of moral hazard:

Borrower may choose between projects I_B and I_G,

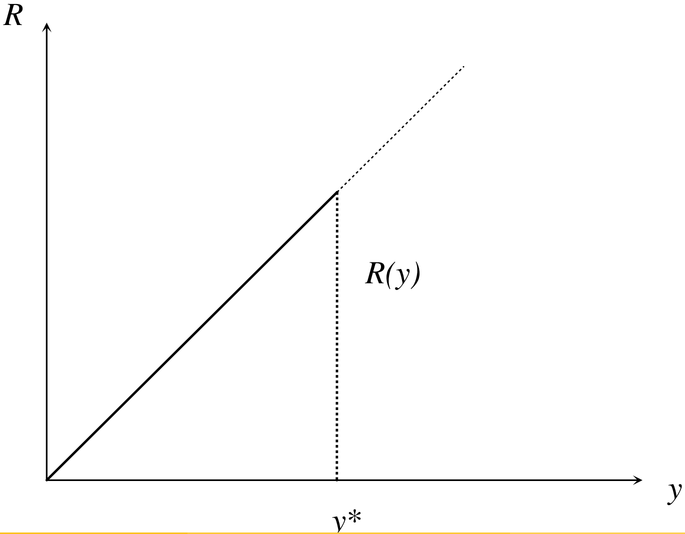
We have $B > G$ but $\pi_G G > 1 > \pi_B B$.

The choice of project cannot be observed by the bank.

Invester chooses I_G if $R \leq R^* = \dfrac{\pi_G G - \pi_B B}{\pi_G - \pi_B}$.

Expected payment is $\pi_G R$ if $R \leq R^*$, $\pi_B R$ otherwise.

Moral hazard 2



Use of collateral 1

An adverse selection model with collateral Investment project: Invest 1, get y with some probability, otherwise 0.

Two types of borrowers G and B with $\pi_G > \pi_B$. Repayment R, collateral C may be used.

Investor with utility u has expected utility of a loan contract with repayment R and collateral C

$$U(R, C; \pi_j) = \pi_j u(W + y - R) + (1 - \pi_j) u(W - C), \; j = B, G,$$

where W is initial wealth.

Collateral 2

The value of the collateral C is $v(C)$, v concave, expected profit is

$$V(R, C; \pi_j) = \pi_j R + (1 - \pi_j)v(C) - (1 + r),$$

where r is the risk-free rate of interest (funding the bank).

Draw the zero-expected-profit curves in a (C, R) diagram. Contracts are points in the diagram.

We look for a *separating equilibrium:* Each type chooses the contract which is designated for this tyoe.

Collateral 3

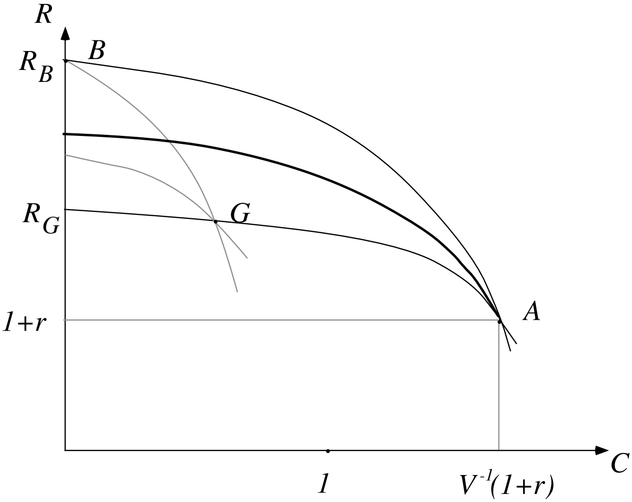
Equilibrium conditions in this market:

(1) contracts are *incentive compatible*

(2) bank has zero expected profit on each contract,

(3) no bank can propose a new contract on which it can earn positive profit

Collateral 4



Collateral 6

The crucial equilibrium condition: No competitor can earn money on a pooling contract

The G-indifference curve through the point G must not intersect the all-types-together zero-profit curve!

If it does, there is no separating equilibrium, but

The pooling contract is *not* an equilibrium, so there is no equilibrium in this model....

Collateral 7