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Discussion of credit rationing:

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Market risk: An introduction to CAPM
Adverse selection 1

Potential entrepreneurs:

Project $\tau$ has random outcome $\tilde{y}(\tau) = \mu + \tilde{z}(\tau)$.

Here $\tilde{z}(\tau)$ random with mean 0, distribution $F(z|\tau)$, s.t.

$$\tau' > \tau \iff F(z|\tau) \text{ 2nd order stochastically dominates } F(z|\tau')$$

(project $\tau'$ is more risky than $\tau$)

Stochastic dominance 1

Stochastic dominance:

Comparing probability distributions (lotteries) independent of decision maker.

1st order: $F$ 1st order dominates $F'$ if expected utility is higher for any person with increasing utility.

It can be shown that in this case $F'(x) \geq F(x)$ for each $x$. 
Stochastic dominance 2

Comparing lotteries with the same mean (say, $\mu = 0$)

2nd order: $F$ 2nd order dominates $F'$ if expected utility is higher for any person with concave utility.

It can be shown that in this case $\int_0^y F'(x) \, dx \leq \int_0^y F'(x) \, dx$ for each $y$.

Adverse selection 2

Bank uses standard contract with repayment $R$.
Entrepreneur’s profit at $z$ is

$$\pi(z, R) = \max\{\mu + z - R, 0\},$$

Let $\Pi(\tau, R) = \int_{-\infty}^{+\infty} \pi(z, R) f(z|\tau) \, dz$ be the expected outcome for type $\tau$.

This can be written as

$$\Pi(\tau, R) = (\mu - R)(1 - F(R - \mu|\tau)) + \int_{R-\mu}^{\infty} zf(z|\tau) \, dz.$$
Adverse selection 3

Expected profit can be written (after some manipulations) as

$$\Pi(\tau, R) = \mu - R + \int_{-\infty}^{R-\mu} F(z | \tau) \, dz.$$  

$\pi(\cdot, R)$ is convex, $-\pi(\cdot, R)$ concave, so: $\Pi(\tau, R)$ is increasing in $\tau$!

Now we come to adverse selection: Let $\theta(R)$ be smallest type such that expected profit $\geq 0$, so that

$$\Pi(\theta(R), R) = 0$$

Adverse selection 4

Use implicit function theorem to get

$$\frac{\partial \Pi}{\partial \tau} = \theta'(R) = -\frac{1 - F(R - \mu | \theta(R))}{\int_{-\infty}^{R-\mu} F_z(z | \theta(R)) \, dz} > 0,$$  

(1)

so that increasing $R$ forces the low-risk types out of the market!

Average (over borrowers) profit is

$$\Pi(R) = \frac{1}{1 - G(\theta(R))} \int_{\theta(R)}^{1} \Pi(\tau, R) g(\tau) \, d\tau$$

$G$ is distribution of borrower types
Adverse selection 5

Define \( \rho(R) = \mu - \bar{I}(R) \) – expected repayment to bank

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Adverse selection 6

The curve has a (local) maximum at \( R^0 \) with \( \rho(R^0) = r^0 \)

— expected repayment first increases, then decreases with nominal repayment.

(At \( R^{\text{max}} \) – the highest repayment rate at which there can be borrowers – repayment equals expected outcome

— showing that \( \rho \) eventually increases in our case)
An alternative view: Oversupply of credits

Investment projects: \( y_H \) if success, \( y_L \) if failure

Investors differ in probability \( \pi \) of success (investors distributed with density \( f(\pi) \))

Borrowers have initial wealth \( W \), borrow \( B = 1 - W \)

Investment project profitable for society as a whole if

\[
\pi y_H + (1 - \pi) y_L \geq 1 + r
\]

Oversupply 2

Given repayment \( R \), borrower will accept loan if

\[
\pi(y_H - RB) \geq (1 + r)W,
\]

Smallest probability at \( R \) (with \( = \)) is \( \pi(R) \).

Bank supplies credits as long as

\[
\pi R + (1 - \pi)(1 - r)B \geq (1 - r)W
\]

Assume free entry of banks, so that we have \( = \) in this equation.
Oversupply 3

Let \( f_R(\pi) \) be conditional density given that \( \pi \geq \pi(R) \) (investor gets credits).

Average project in the market \( \bar{\pi}(R) = \int_{\pi(R)}^{1} \pi f_R(\pi) \, d\pi \).

Then bank profits = 0 means that
\[
\bar{\pi}(R)RB + (1 - \bar{\pi}(R))y_L = \bar{\pi}(R)(RB - y_L) + y_L = (1 + r)B,
\]
and \( \bar{\pi}(R) > \pi(R) \) implies
\[
\pi(R)RB + (1 - \pi(R))y_L < \bar{\pi}(R)RB + (1 - \pi(R))y_L = (1 + r)B.
\]
Add the condition for the investor with \( \pi = \pi(R) \),
\[
\pi(R)(y_H - RB) = (1 + r)W,
\]
to get
\[
\bar{\pi}(R)y_H + (1 - \bar{\pi}(R))y_L < 1 + r.
\]

Costly monitoring

Assume standard contract, monitoring cost is \( c_m \).

Monitoring when \( y \) is \( 0 < R \).

Outcome \( y \) has distribution function \( F(y) \) and density \( f(y) \).

Then
\[
\rho(R) = \int_{0}^{R} (y - c_m)f(y) \, dy + \int_{R}^{\infty} Rf(y) \, dy.
\]
Take the derivative w.r.t. \( R \),
\[
\rho'(R) = (R - c_m)f(R) - Rf(R) + \int_{R}^{\infty} f(y) \, dy = [1 - F(R)] - c_m f(R).
\]
For \( R \) large, first member goes to 0, so the derivative becomes negative.
Moral hazard 1

Recall the simple model of moral hazard:

Borrower may choose between projects $I_B$ and $I_G$.

We have $B > G$ but $\pi_G G > 1 > \pi_B B$.

The choice of project cannot be observed by the bank.

Invester chooses $I_G$ if $R \leq R^* = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B}$.

Expected payment is $\pi_G R$ if $R \leq R^*$, $\pi_B R$ otherwise.

Moral hazard 2

\[ R^* = \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B} \]
Use of collateral

**An adverse selection model with collateral**

Investment project: Invest 1, get \( y \) with some probability, otherwise 0.

Two types of borrowers \( G \) and \( B \) with \( \pi_G > \pi_B \). Repayment \( R \), collateral \( C \) may be used.

Investor with utility \( u \) has expected utility of a loan contract with repayment \( R \) and collateral \( C \)

\[
U(R, C; \pi_j) = \pi_j u(W + y - R) + (1 - \pi_j) u(W - C), \quad j = B, G,
\]

where \( W \) is initial wealth.

Collateral 2

The value of the collateral \( C \) is \( v(C) \), \( v \) concave, expected profit is

\[
V(R, C; \pi_j) = \pi_j R + (1 - \pi_j) v(C) - (1 + r),
\]

where \( r \) is the risk-free rate of interest (funding the bank).

Draw the zero-expected-profit curves in a \((C, R)\) diagram. Contracts are points in the diagram.

We look for a *separating equilibrium*: Each type chooses the contract which is designated for this type.
Equilibrium conditions in this market:

1. contracts are *incentive compatible*
2. bank has zero expected profit on each contract,
3. no bank can propose a new contract on which it can earn positive profit
The crucial equilibrium condition: No competitor can earn money on a pooling contract.

The \( G \)-indifference curve through the point \( G \) must not intersect the all-types-together zero-profit curve!

If it does, there is no separating equilibrium, but

The pooling contract is \textit{not} an equilibrium, so there is no equilibrium in this model....
Market risk

And now to something completely different...