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Market with \( n \) securities with random returns \( \bar{r}_i \) with mean \( r_i \),

covariance matrix \( \Sigma = (\sigma_{ij})_{i=1}^{n} \).

Portfolio \( x = (x_1, \ldots, x_n) \).

Investor with initial wealth \( W_0 \) gets

\[
\tilde{W} = \left( W_0 - \sum_{i=1}^{n} x_i \right) r + \sum_{i=1}^{n} x_i \bar{r}_i
\]

\((r \text{ is the risk-free rate of interest})\), mean and variance are

\[
\mu(x) = E\tilde{W} = \left( W_0 - \sum_{i=1}^{n} x_i \right) r + \sum_{i=1}^{n} x_i r_i = W_0 r + \sum_{i=1}^{n} x_i (r_i - r)
\]

\[
\sigma^2(x) = \text{Var}\tilde{W} = x^t \Sigma x
\]

Assume that \( U \) depends only on \( U(\mu(x), \sigma^2(x)) \). First order conditions are

\[
U'_{\mu} \frac{\partial \mu(x)}{\partial x_i} + U'_{\sigma^2} \frac{\partial \sigma^2(x)}{\partial x_i} = U'_{\mu}(r_i - r) + 2U'_{\sigma^2} \sum_{j=1}^{n} \sigma_{ij} x_j = 0
\]

for \( i = 1, \ldots, n \), in matrix notation,

\[
-\gamma \rho^T + \Sigma x = 0,
\]

where

\[
\gamma = -\frac{U'_{\mu}}{2U'_{\sigma^2}}
\]

is MRS between mean and variance, \( \rho = (r_1 - r, \ldots, r_n - r) \).

\( \Sigma \) has an inverse \( \Sigma^{-1} \), so we get

\[
x^T = \gamma \Sigma^{-1} \rho.
\]
Optimal portfolios of all investors are proportional

Therefore, they are all proportional to the market portfolio

Two-fund separation: Each investor holds a combination of
- money (risk-free asset)
- market portfolio
The proportion of the two depends on the attitude towards risk

Options are rights to buy (or to sell) a security at a given price and date.

Example: European call option on asset $R$ with maturity $T$, exercise price $K$.

The value of this option is

$$V^{BS}(s, R, r, \sigma, K, T) = R\Phi(d_1) - Ke^{-r(T-s)}\Phi(d_2).$$

(the Black-Scholes formula), where

\[d_1 = \frac{\ln(s/R) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}\]

\[d_2 = d_1 - \sigma\sqrt{T-t}\]
The BS formula

\[ \Phi(\cdot) \text{ is standard normal distribution} \]
\[ r \text{ is the riskfree rate of interest} \]
\[ \sigma \text{ the volatility of } R \text{ (follows a Geom. Brownian Motion)}, \]

and

\[ d_1 = \frac{\ln \left( \frac{R}{K} \right) + \left( r + \frac{\sigma^2}{2} \right) (T - s)}{\sigma \sqrt{T - s}}, \quad d_2 = d_1 + \sigma \sqrt{T - s}. \]

Risk assessment

For the risk assessment of an option, we need to identify risk factors:

- Value of security \( R \)
- discount rate \( r \)
- volatility \( \sigma \)

Changes in risk factors indicated by \( \Delta \)
The Greeks

Then linearized loss is

$$L_{t+1}^\Delta = -(V_s^{BS} \Delta + V_R^{BS} R_t \Delta R_{t+1} + V_r^{BS} \Delta r + V_\sigma^{BS} \Delta \sigma).$$

Partial derivatives are “the Greeks”:
- $V_R^{BS}$ (delta) is price risk
- $V_s^{BS}$ (theta) is time decay risk
- $V_\rho^{BS}$ (rho) is discount rate risk
- $V_\sigma^{BS}$ (vega) (!) is volatility risk.
Probability of default

Losses so big that $A$ becomes negative happens with probability

$$P\{A < 0\} = P\left\{ \frac{A - \mu(x)}{\sigma(x)} < -\frac{\mu(x)}{\sigma(x)} \right\} = \Phi\left( -\frac{\mu(x)}{\sigma(x)} \right)$$

Insert $\mu(x) = K + x^t \rho$ and $\sigma(x) = \sqrt{x^t \Sigma x}$:

$$P\{A < 0\} = \Phi\left( -\frac{K + x^t \rho}{\sqrt{x^t \Sigma x}} \right).$$

Define (weighted) capital ratio as

$$\kappa(x) = \frac{K}{\sum_{i=1}^{n} \alpha_i x_i},$$

where $\alpha = (\alpha_1, \ldots, \alpha_n)$ is a weight vector.

Relevance

Probability of ruin, $P\{A < 0\}$, is a decreasing function of $\kappa$:

$$P\{A < 0\} = \Phi\left( -\frac{K}{\sigma(x)} - \hat{x}^t \rho \right) = \Phi\left( -(\alpha^t \hat{x}) \kappa(x) - \hat{x}^t \rho \right), \quad (\ast)$$

where we have used that

$$\kappa(x) = \frac{K}{(\alpha^t \hat{x}) \sigma(x)}.$$ 

Then $(\ast)$ is decreasing in $\kappa$. 
Regulating by capital ratios

If the bank maximizes \( U(\mu(x), \sigma^2(x)) \) under the constraint
\[
\kappa(x) \geq \bar{\kappa}, \text{ or equivalently: } \alpha^T x \bar{\kappa} \leq K,
\]
then first order conditions are
\[
U'_\mu \rho_j + 2U'_\sigma^2 \sum_{j=1}^{n} \sigma_j \bar{x}_j - \lambda \alpha_j, \; i = 1, \ldots, n,
\]
with solution
\[
x^0 = \Sigma^{-1} \left( \gamma \rho + \hat{\lambda} \alpha \right), \quad \gamma = -\frac{U'_\mu}{2U'_\sigma^2}, \quad \hat{\lambda} = \frac{\lambda}{2U'_\sigma^2}.
\]

Two fund separation ceases to hold, market may not be efficient.
Securitization (transformation of loans into securities) has many forms:

- Pass-through
- Asset-backed bonds
- Collateralized mortgage obligations (CMOs)

All versions of the same construction:
Simple model of securitization

Bank finances projects for one period

Initial capital $W$, can finance $kW (= 1)$ projects

Effort of the bank $e_H$ or $e_L$, ($= \text{probability of getting high outcome } y_H$, otherwise $y_L$, with $y_H > 1 > y_L$.

Effort cost $h(e_H) > h(e_L)$.

Effort is desirable for society:

$$e_H y_H + (1 - e_H)y_L - (1 + r)h(e_H) > e_L y_H + (1 - e_L)y_L - (1 + r)h(e_L). \quad (*)$$

Traditional banking or securitization?

Bank can fund the loans at interest rate $r_D$, expected profits as evaluated at $t = 0$ are

$$(1 + r)^{-1}[e y_H + (1 - e)y_L - (1 - W)(1 + r)] - h(e).$$

By $(*)$, bank chooses $e_H$

Securitization: Sell loans in the market at price $p(e^*)$ such that

$$p(e^*) = (1 + r)^{-1}[e^* y_H + (1 - e^*)y_L]$$

In equilibrium, $e^* = e_L$
What is best?

From $r_D \leq r$ we get that

$$(1 + r)^{-1}[e_{HY_H} + (1 - e_H)y_L - (1 + r_D)] - h(e_H) \geq (1 + r)^{-1}[e_{HY_H} + (1 - e_H)y_L - (1 + r)] - h(e_H),$$

and (*) gives

$$e_{HY_H} + (1 - e_H)y_L - (1 + r) - (1 + r)h(e_H) \geq e_Ly_H + (1 - e_L)y_L - (1 + r) - (1 + r)h(e_L),$$

so that

$$(1 + r)^{-1}[e_{HY_H} + (1 - e_H)y_L - (1 + r_D)] - h(e_H) > p(e_L) - 1 - h(e_L),$$

traditional banking is best, securitization used only due to capital constraint.

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Risks from securitization

Economy over three periods $t = 0, 1, 2$.

Investment projects at $t = 0$ or at $t = 1$: 1 invested yields a payoff $Y$ at $t = 2$.

Projects are financed by loans, and the bank demands a fee $f$ when initiating the investment.

Bank has equity $E_0$ at $t = 0$ (in cash),

Loans may be kept by the bank or they may be sold as securities which pay exactly 1 at $t = 2$.

Bank must keep a fraction $d$ of securities.

$P_0 = 1$ and $P_1$ is the security price at $t = 0$ and $t = 1$. Assume $P_1 < 1$ (prices known at $t = 1$).
Role of capital ratio

Banks are subject to capital regulation:

Equity must constitute a fraction $h$ of total assets, or equivalently, of total liabilities, equity plus loans,

$$\frac{E_t}{E_t + L_t} \geq h, \ t = 0, 1,$$

The bank can use funding at $t = 0$ up to $E_0/h$, initiating $N_0 = E_0/dh$ projects.

Bank has a security portfolio $N_0d = \frac{E_0}{h}$ at $t = 0$.

This are the assets of the bank, and it has borrowed $L_0 = E_0(1 - h)/h$.

What happens at $t = 1$

Equity is $E_1 = E_0P_1$ (profits were paid out at $t = 0$ as dividends).

Capital constraint violated, bank must sell assets $S$ at price $P_1$ and pay back loans.

Assets after the sale must be $(N_0d - S)P_1$, $E_1$ is this minus loans $L_0 - P_1S$.

Capital ratio is $\frac{(N_0d - S)P_1 - (L_0 - P_1S)}{(N_0d - S)P_1}$, insert $L_0 = E_0(1 - h)/h$,

$N_0d = E_0/h$, get

$$S = \frac{E_0}{h} \cdot \frac{1 - P_1}{P_1} \cdot \frac{1 - h}{h}$$

Bank has a loss in this period.
Was it profitable after all?

Three possibilities:

(1) Securitization: One unit of money yields:

\[
\frac{f}{dh} - \left[ \frac{1}{h} \cdot \frac{1-P_1}{P_1} \cdot \frac{1-h}{h} \right]
\]

Alternatively, the bank may stay liquid until \( t = 1 \) and then do either:

(2) Traditional banking: One loan may be initiated, profit \( f \),

(3) Buy securities, profit is \( \frac{1-P_1}{P_1} \).

Depending on parameter values, (a) may well be the best option.