Remember: Voluntary assignment 1, to be delivered next monday

Possible structure of answers to assignments:

(1) Identification of theoretical problem
(2) Brief reference to relevant theory
(3) Implications of theory for the question at hand
(4) Possible criticisms
A model including all sectors

Two periods, $t = 1, 2$, two commodities. Agents:

(1) A consumer with $W$ of good 1 and a utility function, the two goods are considered equal.

(2) A producer delivering good 2 at $t = 2$. Input $N_1$ of good 1 must be decided upon in $t = 1$. It can be revised at $t = 2$ to $N_2$ at a cost of $\frac{k}{2}(N_2 - N_1)^2$, production cost is $c$ times output. No need for a loan.

Trade only at $t = 2$, and the consumer must store commodity 1 using one of the two financial agents:
Financial agents

(3) A bank with two investment options:
1. payment of \( I \) at \( t = 1 \): if type \( G \) outcome \( L + I \), if type \( B \) only \( L \). All agents assume that the investment is \( G \) with a probability \( p \).
2. payment of \( x \) and repayment
   \[
   x + f(x) \quad \text{with probability } q
   \]
   \[
   x \quad \text{with probability } 1 - q.
   \]
   This investment must be financed entirely by equity.

(4) A trust fund which can store the commodity – one unit in, one unit out.

What happens?

Consumer puts \( W \) in bank or trustfund, takes it out as \( W_2 \).

Producer adjusts to demand. Price is set to 1, so profit is

\[
(1 - c)W_2 - \frac{k}{2}(W_2 - N_1)^2.
\]

\( W_2 \) is random, so \( N_1 \) is taken as its mean value, expected profit is

\[
(1 - c)E\tilde{W}_2 - \frac{k}{2}E(\tilde{W}_2 - E\tilde{W}_2)^2 = (1 - c)E\tilde{W}_2 - \frac{k}{2}\text{Var}(\tilde{W}_2).
\]
Securitization

Bank offers security based on an investment portfolio:

Repayment: $\mu L$ if portfolio is $B$, $\mu L + \lambda I$ if $G$.

Average return to consumer (and price of security) is $\mu L + p\lambda I$, bank gets rest, used for second investment

Consumer puts remainder $W - \mu L - \lambda pl$ into trust fund and takes it out at $t = 2$, so that $\tilde{W}_2$ has

- mean $p(W + (1 - p)\lambda I) + (1 - p)W - p\lambda I = W$,
- variance $p(1 - p)\lambda^2 l^2$

Regulation

Bank profit is $L + pl - l + qf(\mu L + \lambda pl - l)$.

Increasing in $\mu$ and $\lambda$, without regulation set to 1 (leverage of bank’s investment).

**Regulation**: Assume that regulator can set $\mu$ and $\lambda$ so as to maximize total expected payoff of agents (= bank and producer)

$$L + pl - l + qf(\mu L + \lambda pl - l) + W(1 - c) - \frac{k}{2}p(1 - p)\lambda^2 l^2.$$ 

Optimal value of $\mu$ is 1. First order condition in $\lambda$ is then

$$qf'(\lambda pl + L - l)pl - kp(1 - p)\lambda^2 l^2 = 0$$

The optimal $\lambda^*$ may be set indirectly by a limit on consumer share of investment.
Shadow banking

Now we allow the trust fund to become active: Buy a share in the portfolio (not allowed for consumers).

If price < 1, Bank will sell a share with probability $1 - p + pq$ (otherwise not interested)

The price will be $r = \frac{pq}{1 - p + pq}$ (probability of getting money back)

Bank chooses amount $\lambda'$ maximizing expected profit

$$(p \lambda' + r \lambda')l + L - l + f((p \lambda + r \lambda')l + L - l) + (1 - \lambda - \lambda')l,$$

First order condition

$$rl + f'((p \lambda + r \lambda')l + L - l)r - l = 0,$$

Welfare conditions 1

Insert $r$ and get

$$f'((p \lambda + r \lambda')l + L - l) = \frac{1 - r}{r}$$

If $0 \leq \lambda' \leq 1 - \lambda$ then regulator’s $\lambda$ must lie between

$$\lambda = \varphi\left(\frac{1 - p}{pq}\right) + IL \text{ and } \hat{\lambda} = \varphi\left(\frac{1 - p}{pq}\right) + IL - rl \left(\frac{1}{p - r}\right)$$

($\varphi$ is the inverse of $f'$)

Here $\lambda > \hat{\lambda}$. We conclude:
Welfare conditions 2

(I) If $\lambda \geq \lambda_1$, then no shadow banking at all. If $\lambda \leq \tilde{\lambda}$, then bank sells $(1 - \lambda)l$ to trust fund.

(II) If $\lambda < \lambda_1 < \lambda_2 < \lambda$, then $\lambda_2$ is Pareto-better than $\lambda_1$ (tightening capital regulation is counterproductive).

A dynamic model with credit and collateral

Economy over time, $t = 0, 1, \ldots$:
- one consumption good in each period
- capital good ‘land’ can be used as collateral and for production
- two types of agents:
  (i) entrepreneurs own the land but have no endowment of consumption good
  (ii) lenders with endowments of consumption good
- Production: one unit consumption good plus $k$ units of land to yield $y$ units of a consumption good in the next period.
Dynamics in the model

At time $t$: Price of land is $q_t$.

Entrepreneur can obtain $q_{t+1}(1 + r)^{-1}$ of the consumption good as a loan using land as collateral.

Then units of the consumption good, the entrepreneur needs $kq_{t+1}(1 + r)^{-1}$ units of land.

The remaining $1 - kq_{t+1}(1 + r)^{-1}$ can be rented out at a rate $h_t$, giving income $h_t[1 - kq_{t+1}(1 + r)^{-1}]$.

Assume that rent is determined by $h_t = b - aq_t$.

Fundamental equation

We then have the relation between land price in $t$ and $t + 1$:

$$q_{t+1} + (y - (1 + r))\frac{q_{t+1}}{1 + r} + h_t\left(1 - \frac{kq_{t+1}}{1 + r}\right) = q_t(1 + r).$$

Insert $h_t$, then $q_t$ is a second-degree polynomial in $q_{t+1}$,

$$q_t = \phi(q_{t+1}).$$
A simple model of securities trading

Two periods, 0 and 1; one good, to be consumed or stored

Infinitely many individuals \( i \in [0, 1] \).

Individuals have one unit of the good AND an asset with payoff in the next period:

\[
\begin{align*}
U & \quad i \quad 1 \\
& \quad 1/4 \\
D & \quad i = 0 \\
& \quad i = 1
\end{align*}
\]

Individuals have subjective probability of success, uniformly distributed in \([0, 1]\).
Trade only

Individuals with small values of $i$ sell their asset to individuals with larger $i$.

Equilibrium at price $p$ of asset such that

$$p = i \cdot 1 + (1 - i) \cdot \frac{1}{4} \tag{1}$$

Market clearing occurs when

$$p \cdot i = 1 - i.$$

In our case we get $i = 0.59$, $p = 0.69$.

Trade and loans

Optimistic individuals borrow goods and buy assets

Loans must be fully collateralized (by assets):

Since the asset pays only $1/4$ in the case of a failure, borrowed amount $y$ must satisfy

$$y - 1 = \frac{1 + \frac{1}{4}y}{p},$$

Equilibrium conditions are (1) and:

$$p \cdot i = (1 - i) \cdot 1 + \frac{1}{4}$$

(buyers use their endowment plus the amount borrowed).
Leverage

Solution: \( i = 0.7, \ p = 0.78. \)

Loan to value ratio is \( 0.25/0.78 = 32\% \), and the haircut (percentage subtracted from the market value of the asset) is \( 0.53/0.78 = 70\%. \)

*Leverage*, value of assets divided by own capital invested, is

\[
\frac{0.78}{0.78 - 0.25} = 1.47
\]

One more period

Change the setup to three periods:

Loans are *repo* contract: Lender buys the asset from the borrower, who must buy it back after one period.
Finding equilibrium I

At \( t = 1 \): In the state \( D \) marginal individual is \( i_1 \). At \( t = 0 \): \( i_0 \).

The counterpart of (1) is

\[
p_D = i_1 + (1 - i_1) \frac{1}{4},
\]

and market clearing at state \( D \) is

\[
\frac{i_1}{i_0} p_D = \frac{i_1}{i_0} (i_0 - i_1) + \frac{1}{4}.
\]

To the left: Only the individuals up to \( i \leq i_0 \) in the market, each holds \( \frac{1}{i_0} \).

To the right: Goods for sale with individuals in \([i_1, i_0]\), plus what can be borrowed.

Rewrite market clearing as

\[
i_0 = \frac{4}{5} i_1 (1 + p_D).
\]

Finding equilibrium II

We must also find \( i_0 \) and \( p_0 \). We have

\[
i_0 p_0 = (1 - i_0) + p_D, \tag{2}
\]

(amount of goods transferred to sellers \( t = 0 \) equals amount of goods that the buyers own plus what they can borrow)

Determining the marginal individual \( i_0 \):

An asset gives \( 1 - p_D \) in state \( U \) and 0 in \( D \), against a downpayment of \( p_0 - p_D \), so that

\[
\frac{1 - p_D}{p_0 - p_D}.
\]

Or: \( i_0 \) may keep the good to \( t = 1 \), which with probability \( i_0 \) keeps its value at \( U \) and with probability \( 1 - i_0 \) can be leveraged purchase of asset at \( t = 2 \),

\[
i_0 \frac{1 - 1}{p_D - \frac{1}{4}} = \frac{i_0}{i_1}.
\]
Equilibrium results

Solving the system of 4 equations gives

\[ i_1 = 0.638, \quad p_D = 0.729, \]
\[ i_0 = 0.882, \quad p_0 = 0.959. \]

Leverage at \( t = 0 \):

\[ \frac{0.959}{0.959 - 0.729} = 4.16 \]

At \( t = 1 \) (in D) changed to

\[ \frac{0.729}{0.729 - 0.25} = 1.522 \]