Economics of Banking
Lecture 9

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A model including all sectors

Two periods, $t = 1, 2$, two commodities. Agents:

(1) A *consumer* with W of good 1 and a utility function, the two goods are considered equal.

(2) A *producer* delivering good 2 at $t = 2$. Input N_1 of good 1 must be decided upon in $t = 1$. It can be revised at $t = 2$ to N_2 at a cost of $\frac{k}{2}(N_2 - N_1)^2$, production cost is c times output. No need for a loan.

Trade only at $t = 2$, and the consumer must store commodity 1 using one of the two financial agents:

Financial agents

(3) A *bank* with two investment options:

1. payment of I at $t = 1$: if type G outcome $L + I$, if type B only L. All agents assume that the investment is G with a probability p.
2. payment of x and repayment
$$\begin{array}{ll} x + f(x) & \text{with probability } q \\ x & \text{with probability } 1 - q. \end{array}$$
This investment must be financed entirely by equity.

(4) A *trust fond* which can store the commodity – one unit in, one unit out.

What happens?

Consumer puts W in bank or trustfund, takes it out as W_2.

Producer adjusts to demand. Price is set to 1, so profit is

$$(1-c)W_2 - \frac{k}{2}(W_2 - N_1)^2.$$

W_2 is random, so N_1 is taken as its mean value, expected profit is

$$(1-c)\mathrm{E}\widetilde{W_2} - \frac{k}{2}\mathrm{E}(\widetilde{W_2} - \mathrm{E}\widetilde{W_2})^2 = (1-c)\mathrm{E}\widetilde{W_2} - \frac{k}{2}\mathrm{Var}(\widetilde{W_2}).$$

Securitization

Bank offers security based on an investment portfolio:

Repayment: μL if portfolio is B, $\mu L + \lambda l$ if G.

Average return to consumer (and price of security) is $\mu L + p\lambda l$, bank gets rest, uses the cash payment for second investment

Consumer puts remainder $W - \mu L - \lambda p l$ into trust fund and takes it out at $t = 2$, so that $\widetilde{W}_2$ has

- mean $p(W + (1-p)\lambda l) + (1-p)W - p\lambda l = W$,
- variance $p(1-p)\lambda^2 l^2$

Regulation

Bank profit is

$$L + pI - I + qf(\mu L + \lambda pI - I).$$

increasing in μ and λ, without regulation set to 1 (leverage of bank's investment).

Regulation: Assume that regulator can set μ and λ so as to maximize total expected payoff of agents (= bank and producer)

$$L + pI - I + qf(\mu L + \lambda pI - I) + W(1 - c) - \frac{k}{2}p(1 - p)\lambda^2 I^2.$$

Optimal value of μ is 1. First order condition in λ is then

$$qf'(\lambda pI + L - I)pI - kp(1 - p)\lambda I^2 = 0$$

The optimal λ^* may be set indirectly by a limit on consumer share of investment.

Shadow banking

Now we allow the trust fund to become active: Buy a share in the portfolio (not allowed for consumers).

If price < 1, Bank will sell a share with probability $1 - p + pq$ (otherwise not interested)

The price will be $r = \frac{pq}{1-p+pq}$ (probability of getting money back)

Bank chooses λ' maximizing expected profit

$$(p\lambda + r\lambda')I + L - I + f((p\lambda + r\lambda')I + L - I) + (1 - \lambda - \lambda')I,$$

First order condition

$$rI + f'((p\lambda + r\lambda')I + L - I)rI - I = 0,$$

Welfare conditions 1

Insert r and get

$$\boxed{f'((p\lambda + r\lambda')I + L - I) = \frac{1-r}{r}}$$

If $0 \leq \lambda' \leq 1 - \lambda$ then regulator's λ must lie between

$$\overline{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L}{pI} \quad \text{and} \quad \widehat{\lambda} = \frac{\varphi\left(\frac{1-p}{pq}\right) + I - L - rI}{(p-r)I}$$

(φ is the inverse of f')

Here $\overline{\lambda} > \widehat{\lambda}$. We conclude:

Welfare conditions 2

(I) If $\lambda \geq \overline{\lambda}$, then no shadow banking at all. If $\lambda \leq \widehat{\lambda}$, then bank sells $(1-\lambda)I$ to trust fund.

(II) If $\widehat{\lambda} < \lambda_1 < \lambda_2 < \overline{\lambda}$, then λ_2 is Pareto-better than λ_1 (tightening capital regulation is counterproductive).

A dynamic model with credit and collateral

Economy over time, $t = 0, 1, \ldots$:

- one consumption good in each period
- capital good 'land' can be used as collateral and for production
- two types of agents:
  - (i) *entrepreneurs* own the land but have no endowment of consumption good
  - (ii) *lenders* with endowments of consumption good
- Production: one unit consumption good plus k units of land to yield y units of the consumption good in the next period.

Dynamics in the model

At time t: Price of land is q_t.

Entrepreneur can obtain $q_{t+1}(1+r)^{-1}$ of the consumption good as a loan using land as collateral.

Then units of the consumption good, the entrepreneur needs $kq_{t+1}(1+r)^{-1}$ units of land.

The remaining $1 - kq_{t+1}(1+r)^{-1}$ can be rented out at a rate h_t, giving income $h_t[1 - kq_{t+1}(1+r)^{-1}]$.

Assume that rent is determined by $h_t = b - al_t$.

Fundamental equation

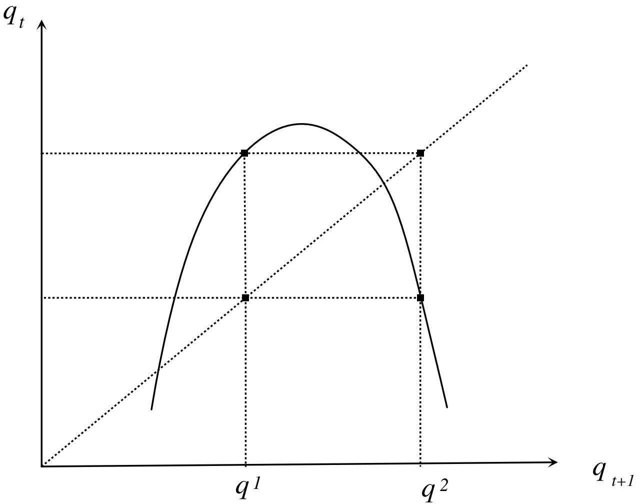
We then have the relation between land price in t and $t+1$:

$$q_{t+1} + (y - (1+r))\frac{q_{t+1}}{1+r} + h_t\left(1 - \frac{kq_{t+1}}{1+r}\right) = q_t(1+r).$$

Insert h_t, then q_t is a second-degree polynomial in q_{t+1},

$$q_t = \phi(q_{t+1}).$$

Backward dynamics

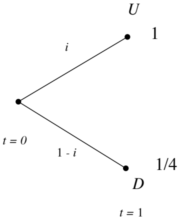


A simple model of securities trading

Two periods, 0 and 1; one good, to be consumed or stored

Infinitely many individuals $i \in [0, 1]$.

Individuals have one unit of the good AND an asset with payoff in the next period:



Individuals have subjective probability of success, uniformly distributed in $[0, 1]$.

Trade only

Individuals with small values of i sell their asset to individuals with larger i.

Equilibrium at price p of asset such that

$$p = i \cdot 1 + (1 - i)\,\frac{1}{4} \tag{1}$$

Market clearing occurs when

$$p \cdot i = 1 - i.$$

In our case we get $i = 0.59$, $p = 0.69$.

Trade and loans

Optimistic individuals borrow goods and buy assets

Loans must be fully collateralized (by assets):

Since the asset pays only 1/4 in the case of a failure, borrowed amount y must satisfy

$$y - 1 = \frac{1 + \frac{1}{4}y}{p},$$

Equilibrium conditions are (**??**) and:

$$p \cdot i = (1 - i) \cdot 1 + \frac{1}{4}$$

(buyers use their endowment plus the amount borrowed).

Leverage

Solution: $i = 0.7$, $p = 0.78$.

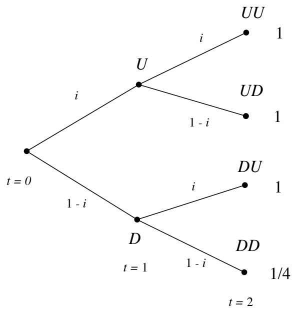
Loan to value ratio is $0.25/0.78 = 32\%$, and the haircut (percentage subtracted from the market value of the asset) is $0.53/0.78 = 70\%$.

Leverage, value of assets divided by own capital invested, is

$$\frac{0.78}{0.78 - 0.25} = 1.47$$

One more period

Change the setup to three periods:



Loans are *repo* contract: Lender buys the asset from the borrower, who must buy it back after one period.

Finding equilibrium I

At $t = 1$: In the state D marginal individual is i_1. At $t = 0$: i_0.
The counterpart of (**??**) is

$$p_D = i_1 1 + (1 - i_1)\frac{1}{4},$$

and market clearing at state D is

$$\frac{i_1}{i_0} p_D = \frac{i_1}{i_0}(i_0 - i_1) + \frac{1}{4}.$$

To the left: Only the individuals up to $i \leq i_0$ in the market, each holds $\frac{1}{i_0}$
To the right: Goods for sale with individuals in $[i_1, i_0]$, plus what can be borrowed.
Rewrite market clearing as

$$i_0 = \frac{4}{5} i_1 (1 + p_D).$$

Finding equilibrium II

We must also find i_0 and p_0. We have

$$i_0 p_0 = (1 - i_0) + p_D, \qquad (2)$$

(amount of goods transferred to sellers $t = 0$ equals amount of goods that the buyers own plus what they can borrow)

Determining the marginal individual i_0:

An asset gives $1 - p_D$ in state U and 0 in D, against a downpayment of $p_0 - p_D$, so that

$$i_0 \frac{1 - p_D}{p_0 - p_D}.$$

Or: i_0 may keep the good to $t = 1$, which with probability i_0 keeps its value at U and with probability $(1 - i_0)$ can be leveraged purchase of asset at $t = 2$,

$$i_0 \frac{1 - \frac{1}{4}}{p_D - \frac{1}{4}} = \frac{i_0}{i_1},$$

Equilibrium results

Solving the system of 4 equations gives

$$i_1 = 0.638, \quad p_D = 0.729,$$
$$i_0 = 0.882, \quad p_0 = 0.959.$$

Leverage at $t = 0$:

$$\frac{0.959}{0.959 - 0.729} = 4.16$$

At $t = 1$ (in D) changed to

$$\frac{0.729}{0.729 - 0.25} = 1.522$$