Opgave III.1

$$N = 4440 \qquad N_1 = 609 \qquad N_2 = 962 \qquad N_3 = 412 \qquad N_4 = 2457$$

giver

$$W_1 = 0.137 \qquad W_2 = 0.217 \qquad W_3 = 0.093 \qquad W_4 = 0.553$$

a) Allokeringen:

$$w_1 = \frac{42}{223} = 0.188 \qquad w_2 = \frac{43}{223} = 0.193 \qquad w_3 = \frac{23}{223} = 0.103 \qquad w_4 = \frac{115}{223} = 0.516$$

$$\bar{y}_{STRAT} = \sum_{k=1}^{K} w_k \bar{y}_k$$

Pålægschokolade:

$$\bar{y}_{STRAT} = 0.137 \cdot 43572 + 0.217 \cdot 19685 + 0.093 \cdot 42264 + 0.553 \cdot 24612 = 27782$$

Chips:

$$\bar{y}_{STRAT} = 0.137 \cdot 177 + 0.217 \cdot 117 + 0.093 \cdot 216 + 0.553 \cdot 112 = 132$$

Shampoo:

$$\bar{y}_{STRAT} = 0.137 \cdot 250 + 0.217 \cdot 105 + 0.093 \cdot 265 + 0.553 \cdot 129 = 153$$

Instant kaffe: (droppes p.g.a. trykfejl)

b)

$$\text{prop:}\ n_k = W_k \cdot n \qquad\qquad \text{opt:}\ n_k = \frac{W_k s_k}{\sum_{k=1}^{K} W_k s_k} \cdot n$$

Pålægschokolade:

Stratum	n_{PROP}	n_{PROP} (afrundet)	n_{OPT}	n_{OPT} (afrundet)
1	0.137 · 500 = 68.5	69	10822/61795.5 · 500 = 87.6	88
2	0.217 · 500 = 108.5	108	. 54.3	54
3	0.093 · 500 = 46.5	47	. 68.6	69
4	0.553 · 500 = 276.5	276	35780.2/61795.5 · 500 = 289.5	289
	500	500	500	500

$$\bar{s} = \sum_{k=1}^{K} W_k s_k = 61795.5$$

$$\mathrm{var}[\bar{y}_{OPT}] = \frac{1}{n}\left(\sum_{k=1}^{K} W_k s_k\right)^2 - \frac{1}{N}\sum_{k=1}^{K} W_k s_k^2$$

$$\mathrm{var}[\bar{y}_{OPT}] = \frac{1}{500}(61795.5)^2 - \frac{1}{4440} \cdot 4150744813 = 2589^2$$

$$\mathrm{var}[\bar{y}_{PROP}] = \frac{N - n}{Nn}\sum_{k=1}^{K} W_k s_k^2$$

$$\mathrm{var}[\bar{y}_{PROP}]$$

$$= \frac{4440 - 500}{4440 \cdot 500}[0.137 \cdot 78993^2 + 0.217 \cdot 30940^2 + 0.093 \cdot 91175^2 + 0.553 \cdot 64702^2$$

$$= \frac{3940}{4440 \cdot 500} \cdot 4150744813 = 2714^2$$

Chips:

Stratum	n_{OPT}		n_{OPT} (afrundet)
1	$55.76/323.6 \cdot 500 =$	86.15	86
2	.	83.15	83
3	.	65.81	66
4	$171.43/323.6 \cdot 500 =$	264.88	265
		499.99	500

$$\bar{s} = \sum_{k=1}^{K} w_k s_k = 323.6$$

$$\mathrm{var}[\bar{y}_{OPT}] = \frac{1}{500}(323.6)^2 - \frac{1}{4440} \cdot 108691.6 = 13.6^2$$

Shampoo:

Stratum	n_{OPT}		n_{OPT} (afrundet)
1	$69.32/357.7 \cdot 500 =$	97.7	98
2	.	43.4	43
3	.	80.5	81
4	$197.4/357.7 \cdot 500 =$	278.3	278
		499.99	500

$$\bar{s} = \sum_{k=1}^{K} w_k s_k = 354.7$$

$$\mathrm{var}[\bar{y}_{PROP}] = \frac{4440 - 500}{4440 \cdot 500}[0.137 \cdot 506^2 + 0.217 \cdot 142^2 + 0.093 \cdot 614^2 + 0.553 \cdot 357^2]$$

$$= \frac{3940}{4440 \cdot 500} \cdot 144992.4 = 16^2$$

$$\mathrm{var}[\bar{y}_{OPT}] = \frac{1}{500}(354.7)^2 - \frac{1}{4440} \cdot 144992.4 = 14.8^2$$

c)

$$L_0 = 50 = 2 \cdot 1.96\sqrt{V_0} \Leftrightarrow V_0 = \left(\frac{50}{2 \cdot 1.96}\right)^2 = 162.7$$

$$n_0 = \frac{\bar{s}^2}{V_0 + \frac{1}{N}\sum W_k s_k^2}$$

ved optimal allokering.

Pålægschokolade:

$$n_0 = \frac{(61795.5)^2}{162.7 + \frac{1}{4440} \cdot 4150744813} = 4084$$

Chips:

$$n_0 = \frac{(323.6)^2}{162.7 + \frac{1}{4440} \cdot 108691.6} = 560$$

Shampoo:

$$n_0 = \frac{(354.7)^2}{162.7 + \frac{1}{4440} \cdot 144992.4} = 645$$

d)

$$L_0 = 50 \Rightarrow V_0 = 162.7$$

$$n_0 = \frac{\sum w_k s_k^2}{V_0 + \frac{1}{N}\sum w_k s_k^2}$$

ved proportional allokering.

Pålægschokolade:

$$n_0 = \frac{4150744813}{162.7 + \frac{1}{4440} \cdot 4150744813} = 4439.2 \sim 4440$$

Chips:

$$n_0 = \frac{108691.6}{162.7 + \frac{1}{4440} \cdot 108691.6} = 581$$

Shampoo:

$$n_0 = \frac{144992.4}{162.7 + \frac{1}{4440} \cdot 144992.4} = 743$$

n_0: mindste (hel)tal for hvilket L_0 er mindre end eller lig 50.

Opgave III.2

$N = 1385000$ $W_K = 0.52$ $W_M = 0.48$ $n = 1000$

a)

	n_{PROP}
Mænd	$n_M = 1000 \cdot 0.48 = 480$
Kvinder	$n_K = 1000 \cdot 0.52 = 520$

Nok søvn	n_{OPT}	n_{OPT} (afrundet)
Mænd	$1000 \frac{0.48\sqrt{0.72(1-0.72)}}{0.48\sqrt{0.72(1-0.72)} + 0.52\sqrt{0.67(1-0.67)}} = 468.49$	468
Kvinder	$1000 \frac{0.52\sqrt{0.67(1-0.67)}}{0.48\sqrt{0.72(1-0.72)} + 0.52\sqrt{0.67(1-0.67)}} = 531.51$	532
	1000	1000

Piberygning	n_{OPT}	n_{OPT} (afrundet)
Mænd	$1000\dfrac{0.48\sqrt{0.23(1-0.23)}}{0.48\sqrt{0.23(1-0.23)}+0.52\sqrt{0.23(1-0.23)}} = 796.09$	796
Kvinder	$1000\dfrac{0.52\sqrt{0.01(1-0.01)}}{0.48\sqrt{0.01\cdot 0.99}+0.52\sqrt{0.01\cdot 0.99}} = 203.91$	204
	1000	1000

Nok søvn:

$$\mathrm{var}[p_{PROP}] = \frac{1385000-1000}{1385000-1000}\cdot[0.48\cdot 0.72(1-0.72)+0.52\cdot 0.67(1-0.67)] = 0.0145^2$$

$$\mathrm{var}[p_{OPT}]$$

$$= \frac{1}{1000}\cdot(0.48\sqrt{0.72\cdot 0.28}+0.52\sqrt{0.67\cdot 0.33}^2$$

$$-\frac{1}{1385000}[0.48\cdot 0.72\cdot 0.28+0.52\cdot 0.67\cdot 0.33]$$

$$= 0.0145^2$$

$$\mathrm{var}[p_{SI}]$$

$$= \mathrm{var}[\bar{y}_{PROP}] + \frac{1385000 - 1000}{1385000 \cdot 1000}\left[0.48(0.72 - 0.694)^2 + 0.52(0.67 - 0.694)^2\right]$$

$$- 7.21 \cdot 10^{-10} \cdot \left[\left(1 - \frac{520}{1385000}\right) \cdot 0.67 \cdot 0.33 + \left(1 - \frac{480}{1385000}\right) \cdot 0.72 \cdot 0.28\right]$$

$$= 0.0145^2 + 0.000001 - 3.04 \cdot 10^{-10} = 0.0145^2$$

(sætning III.4.2)

$$p = w_K\bar{y}_K + w_M\bar{y}_M = 0.52 \cdot 0.67 + 0.48 \cdot 0.72 = 0.694 .$$

Piberygning:

$$\mathrm{var}[p_{OPT}]$$

$$= \frac{1}{1000} \cdot (0.48\sqrt{0.23 \cdot 0.77} + 0.52\sqrt{0.01 \cdot 0.99}^2$$

$$- \frac{1}{1385000}[0.48 \cdot 0.23 \cdot 0.77 + 0.52 \cdot 0.01 \cdot 0.99]$$

$$= 0.0080^2$$

$$\mathrm{var}[p_{PROP}] = \frac{1385000 - 1000}{1385000 - 1000} \cdot [0.48 \cdot 0.23 \cdot 0.77 + 0.52 \cdot 0.01 \cdot 0.99] = 0.0095^2$$

b)

$$L_0 = 0.05 \Rightarrow V_0 = \left(\frac{0.05}{2 \cdot 1.96}\right)^2 = 0.0128^2$$

Søvn nok: $p = 0.48 \cdot 0.72 + 0.52 \cdot 0.67 = 0.694$

SI:

$$n_0 = \frac{0.694(1 - 0.694)}{0.0128^2 + \dfrac{0.694(1 - 0.694)}{1384999}} = 1295$$

PROP:

$$n_0 = \frac{0.21174}{0.0128^2 + 0.21174/1385000} = 1292$$

OPT:

$$n_0 = \frac{0.211628}{0.0128^2 + 0.21174/1385000} = 1291$$

Piberygning: $p = 0.48 \cdot 0.23 + 0.52 \cdot 0.01 = 0.1156$

SI:

$$n_0 = \frac{0.1156(1 - 0.1156)}{0.0128^2 + \dfrac{0.1156(1 - 0.1156)}{1384999}} = 624$$

PROP:

$$n_0 = \frac{0.090156}{0.0128^2 + 0.090156/1385000} = 551$$

OPT:

$$n_0 = \frac{0.064383}{0.0128^2 + 0.090156/1385000} = 393$$

n_0 er mindste (hel)tal for hvilket L_0 er mindre end eller lig med 0.05. I alle beregninger er stratumvarianserne estimeret ved $s_k^2 = p_k(1 - p_k)$, d.v.s. at faktoren n/(n - 1) er udeladt.

Opgave III.3

N = 1385000 n = 749 $W_k = w_k$ f = 0.0005

a)

$$\bar{y}_{PROP} = \frac{72}{749} \cdot 60 + \frac{82}{749} \cdot 47 + \ldots + \frac{21}{749} \cdot 55 = \frac{30708}{749} = 41$$

$$\mathrm{var}[\bar{y}_{PROP}] = \frac{1385000 - 749}{1385000 \cdot 749} \left[\frac{72}{749} \cdot 46^2 + \frac{82}{749} \cdot 46^2 + \ldots + \frac{21}{749} \cdot 73^2\right]$$

$$= \frac{1}{749} \cdot (1 - 0.0005) \cdot 46.15^2 = 1.69^2$$

95% konfidensinterval: 41 ± 1.96 · 1.69 = [37.7 min. ; 44.3 min.]
95% konf.int. for $N\bar{y}_{PROP}$: [52197325 min. ; 61372674 min.] .

b)

$$S^2 = \frac{1}{N-1}\sum_{k=1}^{K}(N_k - 1)S_k^2 + \frac{1}{N-1}\sum_{k=1}^{K} N_k(\bar{Y}_{k.} - \bar{Y})^2$$

$$\simeq \sum_{k=1}^{K} W_k S_k^2 + \sum_{k=1}^{K} W_k(\bar{Y}_{k.} - \bar{Y})^2$$

Indsættes de estimerede værdier s_k^2, $\bar{y}_k$ og $\bar{y}$ heri, fås:

$$s^2 = 46.15^2 = \left[\frac{72}{749}(60 - 41)^2 + \frac{82}{749}(47 - 41)\ 2 + \ldots + \frac{21}{749}(55 - 41)^2\right]$$

$$= 46.15^2 + \frac{1}{749} \cdot 59847 = 47.01^2 \simeq 46.6^2$$

c)

$$\mathrm{var}[\bar{y}_{SI}] = \frac{1}{749}(1 - 0.0005) \cdot 46.6^2 = 1.70^2$$

$$L_0 = 2 \cdot 1.96 \cdot 1.70 = 6.67$$

d)

$$n_k = n \cdot \frac{W_k s_k}{\sum_{k=1}^{K} W_k s_k}$$

$$\sum_{k=1}^{13} W_k s_k = \frac{33842}{749} = 45.18$$

$$n = 749$$

Stratum	n_{OPT}	n_{OPT} (afrundet)
1	$749 \cdot \dfrac{\frac{72}{749} \cdot 46}{45.18} = 73.30$	73
2		83
3	83.48	157
4	157.07	18
5	17.93	7
6	6.82	58
7	57.54	30
8	30.10	63
9	62.61	113
10	112.79	63
11	62.74	22
12	22.44	28
	28.24	
13	$749 \cdot \dfrac{\frac{21}{749} \cdot 73}{45.18} = 33.93$	34
	748.99	749

$\Sigma W_k S_k^2 = 46.15^2$

$\Sigma W_k S_k = 45.18^2$

$var(\bar{y}_{OPT}) = 45.18^2/749 - 46.15^2/138500 = 1.65^2$

e)

$var(\bar{y}_{SI}) = 1.70^2$

$var(\bar{y}_{PROP}) = 1.69^2$

$var(\bar{y}_{OPT}) = 1.65^2$

Opgave III.4

$$N = 2527020 + 2601258 = 5128278 \qquad n = 1000$$

Stratum	$\bar{Y}_k$	S_k^2	W_k	W_kS_k	$W_kS_k^2$
Mænd	161800	107900^2	0.4928	53173.1	5737379648
Kvinder	107600	79400^2	0.5072	40271.7	3197571392
				$\bar{S}$ = 93444.8	8934951040

Stratum	n_{PROP}	n_{PROP}(afr.)	n_{OPT}
Mænd	1000·0.4928=492.8	493	$1000 \cdot \frac{53173.1}{93444.8} = 569.0$
Kvinder	1000·0.5072=507.2	507	$1000 \cdot \frac{40217.7}{93444.8} = 431.0$

a)

$$var[\bar{y}_{PROP}] = \frac{5128278 - 1000}{5128278 \cdot 1000} \cdot 8934951040 = 2988.8^2$$

$$L_0 = 2 \cdot 1.96 \cdot 2988.8 = 11716.3$$

$$\mathrm{var}[\bar{y}_{OPT}] = \frac{1}{1000}(93444.8)^2 + \frac{1}{5128278} \cdot 8934951040 = 2955.3^2$$

$$L_0 = 2 \cdot 1.96 \cdot 2955.3 = 11584.7$$

$$s^2 = \frac{1}{n-1}\sum_{k=1}^{K}(N_k - 1)s_k^2 + \frac{1}{N-1}\sum_{k=1}^{K} N_k(\bar{Y}_{k.} - \bar{Y})^2 \approx \sum_{k=1}^{K} w_k s_k^2 + \sum_{k=1}^{K} N_k(\bar{Y}_{k.} - \bar{Y})^2$$

$$\bar{y} = 0.9428 \cdot 161800 + 0.5072 \cdot 107600 = 134310$$

Heraf estimeres populationsvariansen til

$$S^2 = 8934951040 + [04928(161800 - 134310)^2 + 0.5072(107600 - 134310)^2] = 98332^2$$

$$\mathrm{var}[\bar{y}_{SI}] = \frac{1}{1000}\left(1 - \frac{1000}{5128278}\right) \cdot 98332^2 = 3109.2^2$$

$$L_0 = 2 \cdot 1.96 \cdot 3109.2 = 12188.2$$

Konklusion: Der er ikke særlig stor forskel på størrelsen af L_0 ved de 3 indsamlingsmetoder. Skyldes sammenlignelige stratagennemsnit og stratavarianser.

b)

$$L_0 = 5000 \Leftrightarrow V_0 = \left(\frac{5000}{2 \cdot 1.96}\right)^2 = 1275.5^2$$

Prop. allokering:

$$n_0 = \frac{8934951040}{1275.5^2 + \frac{8934951040}{5128278}} = 5487$$

Opt. allokering:

$$n_0 = \frac{(93444.8)^2}{1275.5^2 + \dfrac{8934951040}{5128278}} = 5362$$