Chapter 21

The IS-LM model

After more basic reflections in the previous two chapters about short-run analysis, this chapter revisits what became known as the *IS-LM model*. This model is based on John R. Hicks' summary of the analytical core of Keynes' *General Theory of Employment, Interest and Money* (Hicks, 1937). The distinguishing element of the IS-LM model compared with both the World's Smallest Macroeconomic Model of Chapter 19 and the Blanchard-Kiyotaki model of Chapter 20 is that an interest-bearing asset is added so that money holding is motivated primarily by its liquidity services rather than its role as a store of value.

The version of the IS-LM model presented here is in one respect different from the presentation in many introductory and intermediate textbooks. The tradition is to see the IS-LM model as just one building block of a more involved aggregate supply-aggregate demand (AD-AS) framework where only the wage level is predetermined while the output price is flexible and adjusts in response to shifts in aggregate demand, triggered by changes in the exogenous variables. We interpret the IS-LM model differently, namely as an independent short-run model in its own right, based on the approximation that both wages and prices are set in advance by agents operating in imperfectly competitive markets and being hesitant regarding frequent or large price changes.

The model is quasi-static and deals with mechanisms supposed to be operative within a "short period". We may think of the period length to be a month, a year, or something in between. The focus is on the interaction between the output market and the asset markets. The model conveys the central message of Keynes' theory: the equilibrating role in the output and money markets is taken by output and nominal interest rate changes. We survey the Keynesian tenets known as *spending multipliers*, the *balanced budget multiplier*, the *paradox of thrift*, and the *liquidity trap*. This will serve as an introduction to dynamic versions of the IS-LM model with endogenous forward-looking expectations, presented in subsequent chapters.

We shall in this chapter also take advantage of the suitability of the IS-LM model for a demonstration of the applicability of Cramer's rule in *comparative statics*, given a system of non-linear equations with two endogenous and many exogenous variables.

21.1 The building blocks

We consider a closed economy with a private sector, a government, and a central bank. The produce of the economy consists mainly of manufacturing goods and services, supplied under conditions of imperfect competition, imperfect credit markets, and price stickiness of some sort. The "money supply" in the model is usually interpreted as money in the broad sense and thus includes money created by a commercial bank sector in addition to currency in circulation.

The model starts out directly from presumed aggregate behavioral relationships. These are supposed to characterize the economy-wide behavior of heterogeneous populations of firms and households, respectively, with imperfect information. The aim is to analyze how the economy reacts to changes in the environment and to deliver qualitative answers to questions about the mechanisms and mutual dependencies in the system as a whole in the short run.

21.1.1 The output market

Demand Aggregate output demand is given as

$$\begin{aligned} Y^d &= C(Y^p, Y^e_{+1}, qK, r^e) + I(Y^e_{+1}, K, r^e) + G + \varepsilon_D, && (21.1) \\ C_{Y^p} &> 0, C_{Y^e_{+1}} > 0, I_{Y^e_{+1}} > 0, C_{Y^p} + C_{Y^e_{+1}} + I_{Y^e_{+1}} < 1, && (21.2) \\ C_{(qK)} &> 0, C_{r^e} \leq 0, I_K < 0, I_{r^e} < 0, \end{aligned}$$

where the function $C(\cdot)$ represents private consumption, the function $I(\cdot)$ represents private fixed capital investment, G is public spending on goods and services, and ε_D is a shift parameter summarizing the role of unspecified exogenous variables that suddenly may affect the level of consumption or investment. A rise in the general "state of confidence" may thus be result in a higher level of investment than otherwise and a higher preference for the present relative to the future may result in a higher level of consumption than otherwise. Arguments appearing in the consumption and investment functions include Y^p which is current private disposable income, Y^e_{+1} which is expected output the next period (or period*s*), q and K which are commented on below, and finally r^e which is the *expected* short-term real interest rate. In this first version of the model we assume there are only two assets in the economy, money and a one-period bond with a real interest rate r.

The signs of the partial derivatives of the consumption and investment functions in (21.1) are explained as follows. A general tenet from earlier chapters is that consumption depends positively on household wealth. One component of household wealth is financial wealth, here represented by the market value, $q \cdot K$, of the capital stock K (including the housing stock). Another component is perceived human wealth (the present value of the expected labor earnings stream), which tends to be positively related to both Y^p and Y^e_{+1}. The separate role of disposable income, Y^p, reflects the hypothesis that a substantial fraction of households are credit constrained. The role of the interest rate, r, reflects the hypothesis that the negative substitution and wealth effects on current consumption of a rise in the real interest rate dominate the positive income effect. These hypotheses find support in the empirical literature.

Firms' investment depends positively on Y^e_{+1}. This is because the productive capacity needed next period depends on the expected level of demand next period. In addition, investment in new technologies is more paying when expected sales are high. On the other hand, the more capital firms already have, the less they need to invest, hence $I_K < 0$. Finally, the cost of investing is higher the higher is the real interest rate. These features are consistent with the q-theory of investment when considering an economy where firms' production is demand constrained (cf. Chapter 14).

Disposable income is given by

$$Y^p \equiv Y - \mathbb{T}, \tag{21.3}$$

where Y is aggregate factor income (= GNP) and $\mathbb{T}$ is real net tax revenue in a broad sense, that is, $\mathbb{T}$ equals gross tax revenue minus transfers *and minus* interest service on government debt. We assume a quasi-linear net tax revenue function

$$\mathbb{T} = \tau + T(Y), \qquad 0 \leq T'(Y) < 1,$$

where τ is a constant parameter reflecting "tightness" of discretionary fiscal policy. Fiscal policy is thus described by two variables, G representing government spending on goods and services and τ representing the discretionary element in taxation. A balanced primary budget is the special case $\tau + T(Y) = G$. The endogenous part, $T(Y)$, of the tax revenue is determined by given taxation rules; when $T' > 0$, these rules act as "automatic stabilizers" by softening the effects on disposable income, and thereby on consumption, of changes in output and employment.

With regard to expected output next period, Y^e_{+1}, the model takes a shortcut and assumes Y^e_{+1} is simply an increasing function of current output and nothing else:

$$Y^e_{+1} = \varphi(Y), \qquad 0 < \varphi'(Y) \leq 1. \tag{21.4}$$

We make a couple of simplifications in the specification of aggregate private output demand. First, since we only consider a single period, we treat the amount of installed capital as a given constant, $\bar{K}$, and suppress the explicit reference to $\bar{K}$ in the consumption and investment functions. Second, we ignore the possible influence of q (which may be more problematic). As an implication, we can express aggregate private demand (the sum of C and I) as a function $D(Y, r^e, \tau)$, whereby (21.1) becomes

$$Y^d = D(Y, r^e, \tau) + G + \varepsilon_D, \qquad \text{where} \tag{21.5}$$

$$0 < D_Y = C_{Y^p}(1 - T'(Y)) + (C_{Y^e_{+1}} + I_{Y^e_{+1}})\varphi'(Y) < 1, \tag{21.6}$$

$$D_{r^e} = C_{r^e} + I_{r^e} < 0, \text{ and } D_\tau = -C_{Y^p} \in (-1, 0). \tag{21.7}$$

Behind the scene: production and employment Prices on goods and services have been set in advance by firms operating in markets with monopolistic competition. Owing to either constant marginal costs or the presence of menu costs, when firms face shifts in demand, they change production rather than price. There is scope for maintaining profitability this way because wages are sticky (due to long-term contracts, say) and the preset prices are normally above marginal costs.

Behind the scene there is an aggregate production function, $Y = F(\bar{K}, N)$, where N is *employment.* The conception is that under "normal circumstances" there is abundant capacity. That is, the given capital stock, $\bar{K}$, is large enough so that output demand can be satisfied, i.e.,

$$F(\bar{K}, N) = Y^d, \tag{21.8}$$

without violating the rule of the minimum as defined in Chapter 19. Assuming $F_N > 0$, we can solve the equation (21.8) for firms' desired employment, N^d, and write $N^d = \mathcal{N}(Y^d, \bar{K})$, where $\mathcal{N}_{Y^d} > 0$ and $\mathcal{N}_K < 0$ under the assumption that $F_K > 0$.

Let $\bar{N}$ denote the size of the *labor force,* i.e., those people holding a job or registered as being available for work. The actual employment, N, must satisfy $N \leq \bar{N} - \tilde{U}$, where $\tilde{U}$ is *frictional unemployment.* We use this term in a broad sense comprising people inevitably unemployed in connection with change of job and location in a vibrant economy, people unemployed because of mismatch of skills and job opportunities, and people unemployed because their reservation wage is above the market wage. The remainder of the labor force that are unemployed are said to be *involuntarily unemployed* in the sense of being ready and willing to work at the going wage or even a bit lower wage. The IS-LM model deals with the case where firms' desired employment, $\mathcal{N}(Y^d, K)$, can be realized, that is, the case where $\mathcal{N}(Y^d, \bar{K}) \leq \bar{N} - \tilde{U}$.

With $\hat{U}$ denoting those involuntarily unemployed, total unemployment, U, can be written

$$U = \bar{N} - N = \tilde{U} + \hat{U}.$$

In an alternative decomposition of unemployment one writes

$$U = U^n + U^c,$$

where U^n is the *NAIRU unemployment* level and U^c the remainder unemployment, often called *cyclical* unemployment (a positive number in a recession, a negative number in a boom). So U^n is defined as the level of unemployment prevailing when the unemployment rate, $U/\bar{N}$, equals what is known as the NAIRU, namely that rate of unemployment which generates neither upward nor downward pressure on the inflation rate. The term "NAIRU" (an abbreviation of non-accelerating-inflation-rate-of-unemployment) is in fact a misnomer because the point is not absence of acceleration but merely absence of pressure on the inflation rate in one or the other direction. Nevertheless, we shall stick to this term, because the alternative terms offered in the literature are not better. One is the "natural rate of unemployment"; but there is nothing natural about that unemployment rate – it depends on legal institutions, economic policy, and structural characteristics of the economy. Another – somewhat elusive – name is the "structural rate of unemployment".[1]

In Keynesian theory the *NAIRU unemployment* rate, $U_n/\bar{N}$, is perceived as generally being below $\tilde{U}/\bar{N}$. And business cycle fluctuations in unemployment are perceived as primarily reflecting fluctuations in $\mathcal{N}(Y^d, \bar{K})$ rather than in $\bar{N} - \tilde{U}$. While the size and composition of unemployment generally matter for wage and price changes, the IS-LM model considers such effects as not materializing until at the earliest the next period. Being concerned about only a single short period, the model is therefore often tacit about production and employment aspects and leave them "behind the scene".

21.1.2 Asset markets

In this first version of the IS-LM model we assume that only two financial assets exist, money and an interest-bearing short-term bond. The latter may be issued by the government as well as private agents/firms. Although not directly visible in the model, it is usually understood that there are commercial banks that accept deposits and provide bank loans to households and firms. Bank deposits are then

[1] Our formulations here implitly presuppose that absence of pressure on the inflation rate can be traced to a *single* rate of unemployment. However, there exist empirics as well as theory implying that under certain conditions there is a *range* of unemployment rates within which no pressure on the inflation rate is generated, neither upward nor downward (see Chapter 24).

considered as earning no interest at all.[2] Up to a certain amount bank deposits are nevertheless attractive because for many transactions liquidity is needed. Bank deposits are also a fairly secure store of liquidity, being better protected against theft than cash and being, in modern times, also protected against bank default by government-guaranteed deposit insurance. The interest rate on bank loans allows the banks a revenue over and above the costs associated with banking.

Let M denote the money stock (in the implied broad sense), held by the non-bank public at a given date. That is, in addition to currency in circulation, the bank-created money in the form of liquid deposits in commercial banks is included in M. We may thereby think of M as representing what is in the statistics denoted either M_1 or M_2, cf. Chapter 16. The bank lending rate is assumed equal to the short-term nominal interest rate, i, on government bonds. All interest-bearing assets are considered perfect substitutes from the point of view of the investor and will from now just be called "bonds".

The demand for money is assumed given by

$$M^d = P \cdot (L(Y,i) + \varepsilon_L), \qquad L_Y > 0,\ L_i < 0, \tag{21.9}$$

where P is the output price level (think of the GDP deflator) and ε_L is a shift parameter summarizing the role of unspecified exogenous variables that may affect money demand for any given pair (Y, i). Apart from the shift term, ε_L, real money demand is given by the function $L(Y, i)$, known as the *liquidity preference function*. The first partial derivative of this function is positive reflecting the *transaction motive* for holding money. The output level is an approximate statistic (a "proxy") for the flow of transactions for which money is needed. The negative sign of the second partial derivative reflects that the interest rate, i, is the opportunity cost of holding money instead of interest-bearing assets.

The part of non-human wealth not held in the form of money is held in the form of an interest-bearing asset, a one-period bond. We imagine that also firms' capital investment is financed by issuing such bonds. The bond offers a payoff equal to 1 unit of money at the end of the period. Let the market price of the bond at the beginning of the period be v units of money. The implicit nominal interest rate, i, is then determined by the equation $v(1 + i) = 1$,[3] i.e.,

$$i = (1 - v)/v. \tag{21.10}$$

There is a definitional link between the nominal interest rate and the expected short-term real interest rate, r^e. In continuous time we would have $r^e = i - \pi^e$ with

[2] In practice even checkable deposits in banks may earn a small nominal interest, but this is ignored by the model.

[3] In continuous time with compound interest, $ve^i = 1$ so that $i = -\ln v$.

i as the instantaneous nominal interest rate (with continuous compounding) and π ($\equiv \dot{P}/P$) as the (forward-looking) instantaneous inflation rate, the superscript e indicating expected value. But in discrete time, as we have here, the appropriate way of defining r^e is more involved. The holding of money is motivated by the need, or at least convenience, of ready liquidity to carry out expected as well as unexpected spending in the near future. To perform this role, money must be held in advance, that is, at the beginning of the (short) period in which the purchases are to be made ("cash in advance"). If the price of a good is P euro to be paid at the end of the period and you have to hold this money already from the beginning of the period, you effectively pay $P + iP$ for the good, namely the purchase price, P, plus the opportunity cost, iP. Postponing the purchase one period thus gives savings equal to $P + iP$. The price of the good next period is P_{+1} which, with cash in advance, must be held already from the beginning of that period. So the real gross rate of return obtained by postponing the purchase one period is

$$1 + r = (1 + i)P\frac{1}{P_{+1}} = \frac{1 + i}{1 + \pi_{+1}},$$

where $\pi_{+1} \equiv (P_{+1} - P)/P$ is the inflation rate from the current to the next period. As seen from the current period, P_{+1} and π_{+1} are generally not known. So decisions are based on the expected real interest rate,

$$r^e = \frac{1 + i}{1 + \pi_{+1}^e} - 1 \approx i - \pi_{+1}^e, \tag{21.11}$$

where the approximation is valid for "small" i and π_{+1}^e.

21.2 Keynesian equilibrium

The model assumes that both the output and the money market clear by adjustment of output and nominal interest rate so that supply equals demand:

$$Y = D(Y, i - \pi_{+1}^e, \tau) + G + \varepsilon_D, \quad 0 < D_Y < 1, \; D_{r^e} < 0, \; -1 < D_\tau < 0 \tag{IS}$$

$$\frac{M}{P} = L(Y, i) + \varepsilon_L, \qquad L_Y > 0, \; L_i < 0, \tag{LM}$$

where, for simplicity, we have used the approximation in (21.11), and where M is the available money stock at the beginning of the period. In reality the central bank has direct control only over the monetary base. Yet the traditional understanding of the model is that through this, the central bank has under "normal circumstances" control also over M. With M given by monetary policy, the interpretation of the equations (IS) and (LM) is therefore that output and the nominal interest rate quickly adjust so as to clear the output and money markets.

The equation (IS), known as the *IS equation*, asserts clearing in a *flow* market: so much output *per time unit* matches the effective demand *per time unit* for this output. The name comes from an alternative way of writing it, namely as $I = S$ (investment = saving, where saving $S = Y - C - G - \varepsilon_D$).

In contrast, the equation (LM), known as the *LM equation*, asserts clearing in a *stock* market: so much liquidity demand matches the available money stock, M, at a given point in time. In our discrete time setting we think of asset market openings occurring in a diminutive time interval at the beginning of each period. And we think of changes in the money stock as taking place abruptly from market opening to market opening. Agents' decisions about portfolio composition, consumption, and investment are also thought of as being made at the beginning of each period. Production takes place *during* the period and at the end of the period receipts for work and lending and payment for consumption occur. This interpretation calls for a quite short period length.

At the empirical level we have data for M and i on a daily basis, whereas the period length of data for aggregate output, consumption, and investment, is usually a year or at best a quarter of a year. So, in connection with econometric analyses, instead of linking M and i to a single point in time, one may think of M and i as averages over a year (or a quarter of a year). A possible interpretation would then be that the year still consists of many subperiods with their own asset supplies and demands as well as production and consumption flows. The environment of the system remains unchanged throughout the year, and the system remains in equilibrium with constant stocks and flows.

Having specified the LM equation, should we not also specify a condition for clearing in the market for bonds? Well, we do not have to. The balance sheet constraint of the non-bank private sector guarantees that clearing in the money market implies clearing also in the bond market − and vice versa. To see this, let W denote the nominal financial wealth of the non-bank private sector and let x denote the number of one-period bonds held on net by the non-bank private sector. Each bond offers a payoff of 1 unit of money at the end of the period and is by the market priced $v = 1/(1 + i)$ at the beginning of the period. Then $M + vx \equiv W$. With x^d denoting the on net by the non-bank private sector demanded quantity of bonds, we have $M^d + vx^d = W$. This is an example of a *balance sheet constraint* and implies a "Walras' law for stocks". Subtracting the first from the second of these two equations yields

$$M^d - M + v(x^d - x) = 0. \tag{21.12}$$

Given $v > 0$, it follows that if and only if $M^d = M$, then $x^d = x$. That is, clearing in one of the asset markets implies clearing in the other. Hence it suffices to consider just one of these two markets explicitly. Usually the money market is considered.

The IS and LM equations amount to the traditional IS-LM model in compact form. The exogenous variables are $P, \pi^e_{+1}, \tau, G, \varepsilon_D, \varepsilon_L$, and, in the traditional interpretation, M. Given the values of these variables, a solution, (Y, i), to the equation system consisting of (IS) and (LM) is an example of a *Keynesian equilibrium*. It is an *equilibrium* in the sense that, given the prevailing expectations and preset goods prices, asset markets clear by price adjustment (here adjustment of i) and the traded quantity in the goods market complies with the short-side rule (the rule saying that the short side of the market determines the traded quantity). It is a *Keynesian* equilibrium because it is aggregate demand in the output market which is the binding constraint on output (and implicitly thereby also on employment).

The current price level, P, is seen as predetermined and maintained through the period. But the price level P_{+1} set for the *next* period will presumably not be independent of current events. So expected inflation, π^e_{+1}, *ought* to be endogenous. It is therefore a deficiency of the model that π^e_{+1} is treated as exogenous. Yet this may give an acceptable approximation as long as the sensitivity of expected inflation to current events is small.

21.3 Alternative monetary policy regimes

We shall analyze the functioning of the described economy in three alternative simplistic monetary policy regimes. In the first policy regime the central bank is assumed to maintain the money stock at a certain target level. This is the case of a *money stock rule*. In the second policy regime, trough open market operations the central bank maintains the interest rate at a certain target level for some time. This is the case of a *fixed interest rate rule* (where "fixed" should be interpreted as "fixed but adjustable"). The third policy regime to be considered is a *countercyclical interest rate rule* where both the interest rate and the money stock are endogenous. The static IS-LM model is not suitable for a study of a Taylor-rule regime since that involves dynamics and policy reactions to the rate of inflation.

21.3.1 Money stock rule

Here the central bank maintains the money stock at a certain target level $M > 0$. We assume that given this M, circumstances are such that the generally nonlinear equation system (IS) - (LM) has a solution (Y, i) and, until further notice, that both Y and i are strictly positive.

The IS-LM diagram

For convenience, we repeat our equation system:

$$Y = D(Y, i - \pi^e_{+1}, \tau) + G + \varepsilon_D, \quad 0 < D_Y < 1, \; D_{r^e} < 0, \; -1 < D_\tau < 0, \tag{IS}$$

$$\frac{M}{P} = L(Y, i) + \varepsilon_L, \qquad L_Y > 0, \; L_i < 0, \tag{LM}$$

The determination of Y and i is conveniently illustrated by an IS-LM diagram, cf. Fig. 21.1. First, consider the equation (IS). We *guess* that this equation defines (determines) i as an implicit function of the other variables in the equation, Y, π^e_{+1}, τ, G, and ε_D:

$$i = i_{IS}(Y, \pi^e_{+1}, \tau, G, \varepsilon_D).$$

The partial derivative of this function w.r.t. Y can be found by taking the differential w.r.t. Y and i on both sides of (IS),[4]

$$dY = D_Y dY + D_{r^e} di,$$

and rearranging:

$$\partial i/\partial Y_{|IS} = \frac{di}{dY} = \frac{1 - D_Y}{D_{r^e}} < 0, \tag{21.13}$$

where the first equality is valid by construction, and where the negative sign follows from the information given (IS). The observation that the denominator, D_{r^e}, in (21.13) is not zero confirms our guess that the equation (IS) defines i as an implicit function of the other variables in the equation.

The solution for the derivative in (21.13) tells that higher aggregate demand in equilibrium requires that the interest rate is lower. In Fig. 21.1, this relationship is illustrated by the downward-sloping *IS curve*, which is the locus of combinations of Y and i that are consistent with clearing in the output market. The slope of this locus is given by (21.13).

Next consider the equation (LM). We *guess* that this equation defines i as an implicit function of the other variables in the equation, Y, M/P, and ε_L :

$$i = i_{LM}(Y, \frac{M}{P}, \varepsilon_L).$$

The partial derivative of this function w.r.t. Y can be found by taking the differential w.r.t. Y and i on both sides of (LM),

$$0 = L_Y dY + L_i di,$$

[4] On the concepts of implicit function and differentials, see Math Tools.

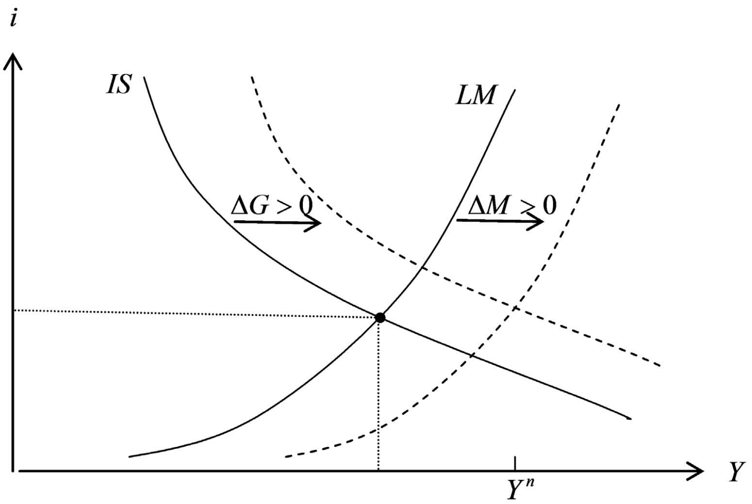

Figure 21.1: The IS-LM cross when M is exogenous; a case with equilibrium output below the NAIRU level, Y^n (π^e_{+1},τ,G,ε_D,M/P, and ε_L given).

and rearranging:

$$\partial i/\partial Y_{|LM} = \frac{di}{dY} = \frac{-L_Y}{L_i} > 0, \tag{21.14}$$

where the first equality is valid by construction, and the positive sign follows from the information given in (LM). The observation that the denominator in (21.14) is *not* zero confirms our guess that the equation (LM) defines i as an implicit function of the other variables in the equation.

The solution for the derivative in (21.14) tells that for the money market to clear, a higher volume of transactions must go hand in hand with a higher interest rate. In Fig. 21.1, this relationship is illustrated by the upward-sloping *LM curve*, which is the locus of combinations of Y and i that are consistent with clearing in the money market.

A solution (Y, i) to the model is unique (the point of intersection in Fig. 21.1). Hence we can write Y and i as (unspecified) functions of all the exogenous variables:

$$Y = f(\frac{M}{P}, \pi^e_{+1}, \tau, G, \varepsilon_D, \varepsilon_L), \tag{21.15}$$

$$i = g(\frac{M}{P}, \pi^e_{+1}, \tau, G, \varepsilon_D, \varepsilon_L). \tag{21.16}$$

Comparative statics

How do Y and i depend on the exogenous variables? A *qualitative* answer can easily be derived by considering in what direction the IS curve and the LM curve shift in response to a change in an exogenous variable. With minimal training, the directions of these shifts can be directly read off the information given in (IS) and (LM) equations. Alternatively one can use the total differentials (21.17) and (21.18) below also for this purpose.

A *quantitative* answer is based on the standard comparative statics method. Starting afresh with the (IS) - (LM) equation system, we *guess* that the system defines (determines) Y and i as implicit functions, f and g, of the other variables, as in (21.15) and (21.16). The aim is to find formulas for the partial derivatives of these implicit functions, evaluated at an equilibrium point (Y, i), a point satisfying (IS) and (LM). We first calculate the *total* differential on both sides of (IS):

$$dY = D_Y dY + D_{r^e}(di - d\pi^e_{+1}) + D_\tau d\tau + dG + d\varepsilon_D. \tag{21.17}$$

Next we calculate the *total* differential on both sides of (LM):

$$d(\frac{M}{P}) = L_Y dY + L_i di + d\varepsilon_L. \tag{21.18}$$

We interpret these two equations as a *new equation system* with two *new endogenous* variables, the differentials dY and di. The changes, $d\pi^e_{+1}$, dG, $d\tau$, $d\varepsilon_D$, $d(M/P)$, and $d\varepsilon_L$, in the exogenous variables are our *new exogenous* variables. The coefficients, D_Y, D_{r^e}, etc., to these endogenous and exogenous variables in the two equations are derivatives evaluated at the equilibrium point (Y, i). Like the original equation system (IS) - (LM), the new system is simultaneous (not recursive).

The key point is that the new system is *linear*. The further procedure is the following. First rearrange (21.17) and (21.18) so that dY and di appear on the left-hand side and the differentials of the exogenous variables on the right-hand side of each equation:

$$(1 - D_Y)dY - D_{r^e}di = -D_{r^e}d\pi^e_{+1} + D_\tau d\tau + dG + d\varepsilon_D, \tag{21.19}$$

$$L_Y dY + L_i di = d\frac{M}{P} - d\varepsilon_L. \tag{21.20}$$

Next, calculate the determinant, Δ, of the coefficient matrix on the left-hand side of the system:

$$\Delta = \begin{vmatrix} 1 - D_Y & -D_{r^e} \\ L_Y & L_i \end{vmatrix} = (1 - D_Y)L_i + D_{r^e}L_Y < 0, \tag{21.21}$$

where the negative sign follows from qualitative information about the functions D and L given in (IS) and (LM), respectively. The observation that the determinant is *not* zero confirms our guess that the (IS) - (LM) system defines Y and i as implicit functions of the other variables.

Now apply Cramer's rule[5] to the linear system (21.19) - (21.20) to determine dY and di:

$$dY = \frac{\begin{vmatrix} -D_{r^e} d\pi^e_{+1} + D_\tau d\tau + dG + d\varepsilon_D & -D_{r^e} \\ d\frac{M}{P} - d\varepsilon_L & L_i \end{vmatrix}}{\Delta} = \frac{L_i(-D_{r^e} d\pi^e_{+1} + D_\tau d\tau + dG + d\varepsilon_D) + D_{r^e}(d\frac{M}{P} - d\varepsilon_L)}{\Delta}, \quad (21.22)$$

and

$$di = \frac{\begin{vmatrix} 1 - D_Y & -D_{r^e} d\pi^e_{+1} + D_\tau d\tau + dG + d\varepsilon_D \\ L_Y & d\frac{M}{P} - d\varepsilon_L \end{vmatrix}}{\Delta} = \frac{(1 - D_Y)(d\frac{M}{P} - d\varepsilon_L) - L_Y(-D_{r^e} d\pi^e_{+1} + D_\tau d\tau + dG + d\varepsilon_D)}{\Delta} \quad (21.23)$$

The partial derivatives of f and g, respectively, w.r.t. the exogenous variables can be directly read off these two formulas.

Suppose we are interested in the effect on Y and i of a change in the real money supply, M/P. By setting $d\pi^e_{+1} = d\tau = dG = d\varepsilon_D = d\varepsilon_L = 0$ in (21.22) and (21.23) and rearranging, we get

$$\frac{\partial Y}{\partial(\frac{M}{P})} = f_{M/P} = \frac{dY}{d(\frac{M}{P})} = \frac{D_r}{(1 - D_Y)L_i + D_{r^e} L_Y} > 0,$$
$$\frac{\partial i}{\partial(\frac{M}{P})} = g_{M/P} = \frac{di}{d(\frac{M}{P})} = \frac{1 - D_Y}{(1 - D_Y)L_i + D_{r^e} L_Y} < 0,$$

where the signs are due to (21.6), 21.7, and (21.21). Such partial derivatives of the endogenous variables w.r.t. an exogenous variable, evaluated at the equilibrium point, are known as *multipliers*. The approximative short-run effect on Y of a given small increase dM in M is calculated as $dY = (\partial Y/\partial(\frac{M}{P}))dM/P$, where we see the role of the partial derivative w.r.t. M/P as a multiplier on the increase in the exogenous variable, M/P.[6]

[5] See Math Tools.

[6] Instead of using Cramer's rule, in the present case we could just substitute di, as determined from (21.18), into (21.17) and then find dY from this equation. In the next step, this solution for dY can be inserted into (21.18), which then gives the solution for di. However, if L_i were a function that *could* take the value nil, this procedure might invite a temptation to rule this out by assumption. That would imply an unnecessary reduction of the domain of $f(\cdot)$ and $g(\cdot)$. The only truly necessary assumption is that $\Delta \neq 0$ and that is automatically satisfied in the present problem.

The intuitive interpretation of the signs of these multipliers is the following. The central bank increases the money supply through an open market purchase of bonds held by the private sector. In practice it is usually short-term government bonds ("treasury bills") that the central bank buys when it wants to increase the money supply (decrease the short-term interest rate). Immediately after the purchase, the supply of money is higher than before and the supply of bonds available to the public is lower. At the initial interest rate there is now excess supply of money and excess demand for bonds. But the attempt of agents to get rid of their excess cash in exchange for more bonds can not succeed in the aggregate because the supplies of bonds and money are given. Instead, what happens is that the price of bonds goes up, that is, the interest rate goes down, cf. (21.10), until the available supplies of money and bonds are willingly held by the agents. Money is therefore *not* neutral.

To find the output multiplier w.r.t. government spending on goods and services, or what is known as the *spending multiplier*, in (21.22) we set $d(M/P) = d\pi^e_{+1} = d\tau = d\varepsilon_D = d\varepsilon_L = 0$ and rearrange to get

$$\frac{\partial Y}{\partial G} = f_G = \frac{dY}{dG} = \frac{L_i}{(1 - D_Y)L_i + D_{r^e} L_Y} = \frac{1}{1 - D_Y + D_{r^e} L_Y / L_i}. \tag{21.24}$$

Under the assumed monetary policy we thus have $0 < \partial Y/\partial G < 1/(1 - D_Y)$. The difference, $1/(1 - D_Y) - \partial Y/\partial G$, is due to the *financial crowding-out effect*, represented by the term $D_{r^e} L_Y / L_i > 0$ in (21.24). Owing to the fixed money stock, the expansionary effect of a rise in G is partly offset by a rise in the interest rate induced by the increased money demand resulting from the "initial rise" in economic activity. If money demand is not sensitive to the interest rate[7] (as the monetarists claimed), the *financial crowding-out* is large and the spending multiplier low in this policy regime.

Another "moderator" comes from the marginal net tax rate, $T'(Y) \in (0, 1)$, which by reducing the private sector's marginal propensity to spend, D_Y in (21.6), acts as an *automatic stabilizer*. When aggregate output (economic activity) rises, disposable income rises less, partly because of higher taxation, partly because of lower aggregate transfers, for example unemployment compensation.[8]

Shifts in the values of the exogenous variables, ε_D and ε_L, may be interpreted as shocks (disturbances) coming from a variety of unspecified events. A positive demand shock, $d\varepsilon_D > 0$, may be due to an upward shift in households' and firms'

[7] This is the case when $|L_i|$ is low, i.e., the LM curve steep.

[8] Outside our static IS-LM model an additional issue is how current consumers repond to the *increased public debt* in the wake of a not fully tax-financed temporary increase in G. Although this takes us outside the static IS-LM model, we shall briefly comment on it towards the end of this chapter.

"confidence". A negative demand shock may come from a "credit crunch" due to a financial crisis. A positive liquidity preference shock may reflect a sudden rise in the perceived risk of default of bond liabilities.

To see how demand shocks and liquidity preference shocks, respectively, affect output under the given monetary policy, in the equation (21.22) we set $d\pi_{+1}^e = d\tau = dG = d\frac{M}{P} = 0$. When in addition we set, first, $d\varepsilon_L = 0$, and next $d\varepsilon_D = 0$, we find the partial derivatives of Y w.r.t. ε_D and ε_L, respectively:

$$\begin{aligned}
\frac{\partial Y}{\partial \varepsilon_D} &= f_{\varepsilon_D} = \frac{dY}{d\varepsilon_D} = \frac{L_i}{(1-D_Y)L_i + D_{r^e}L_Y} = \frac{1}{1 - D_Y + D_{r^e}L_Y/L_i} \\
\frac{\partial Y}{\partial \varepsilon_L} &= f_{\varepsilon_L} = \frac{dY}{d\varepsilon_L} = \frac{-D_{r^e}}{(1-D_Y)L_i + D_{r^e}L_Y} < 0.
\end{aligned}$$

As expected, a positive demand shock is expansionary, while a positive liquidity preference shock is contractionary because it raises the interest rate. Note that $\partial Y/\partial \varepsilon_D = \partial Y/\partial G$ (from (21.24)) in view of the way ε_D enters the IS equation.

As now the method should be clear, we present the further results without detailing. From (21.22) and (21.23), respectively, we calculate the output and interest multipliers w.r.t. fiscal tightness to be

$$\begin{aligned}
\frac{\partial Y}{\partial \tau} &= \frac{L_i D_\tau}{(1-D_Y)L_i + D_{r^e}L_Y} < 0, \\
\frac{\partial i}{\partial \tau} &= \frac{-L_Y D_\tau}{(1-D_Y)L_i + D_{r^e}L_Y} < 0.
\end{aligned}$$

What do (21.22) and (21.23) imply regarding the effect of higher expected inflation on Y, i, and r^e, respectively? We find

$$\begin{aligned}
\frac{\partial Y}{\partial \pi_{+1}^e} &= f_{\pi_{+1}^e} = \frac{-L_i D_{r^e}}{(1-D_Y)L_i + D_{r^e}L_Y} > 0, \\
\frac{\partial i}{\partial \pi_{+1}^e} &= g_{\pi_{+1}^e} = \frac{L_Y D_{r^e}}{(1-D_Y)L_i + D_{r^e}L_Y} \in (0,1), \qquad (21.25) \\
\frac{\partial r^e}{\partial \pi_{+1}^e} &= \frac{\partial (i - \pi_{+1}^e)}{\partial \pi_{+1}^e} = g_{\pi_{+1}^e} - 1 = \frac{-(1-D_Y)L_i}{(1-D_Y)L_i + D_{r^e}L_Y} \in (-1,0).
\end{aligned}$$

A higher expected inflation rate thus leads to a less-than-one-to-one increase in the nominal interest rate and thereby a smaller expected real interest rate. Only if money demand were independent of the nominal interest rate ($L_i = 0$), as in the quantity theory of money, would the nominal interest rate rise one–to-one with π_{+1}^e and the expected real interest rate thereby remain unaffected.

Before proceeding, note that there is a reason that we have set up the IS and LM equations in a general nonlinear form. We want the model to allow

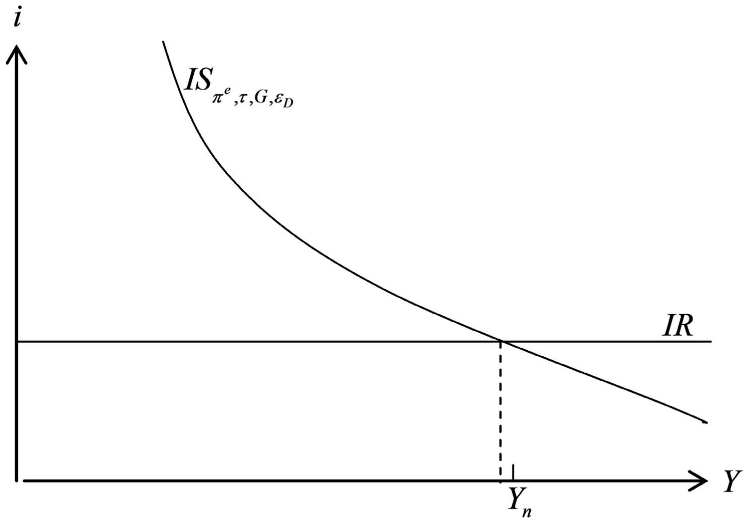

Figure 21.2: A fixed interest rate implying equilibrium output close to the NAIRU level (i,π^e_{+1}, τ, G, and ε_D given).

for the empirical feature that the different multipliers generally depend on the "state of the business cycle". The spending multiplier, for instance, tends to be considerably larger in a slump − with plenty of idle resources − than in a boom. In dynamic extensions of the IS-LM model the length of the time interval associated with the higher G becomes important as does the time profile of the effect on Y. In the present static version of the model it fits intuition best to interpret the rise in G as referring to the "current" period only.

21.3.2 Fixed interest rate rule

Instead of targeting a certain level of the money stock, the central bank now keeps the nominal interest rate at a certain target level $i > 0$. The aim may be to have output unaffected by liquidity preference shocks. This monetary policy seems closer to what most central banks nowadays typically do. They announce a target for the nominal interest rate and then, through open-market operations, adjust the monetary base so that the target rate is realized.

In this regime, i is an exogenous constant > 0, while M and Y are endogenous. Instead of the upward-sloping LM curve we get a horizontal line, the IR line in Fig. 21.2 ("IR" for interest rate). The model is now *recursive*. Since M does not enter the equation (IS), Y is given by this equation independently of the equation (LM). Indeed, in view of $D_Y \neq 0$, the equation (IS) defines Y as an implicit function, h, of the other variables in the equation, i.e.,

$$Y = h(r^e, \tau, G, \varepsilon_D) = h(i - \pi^e_{+1}, \tau, G, \varepsilon_D). \tag{21.26}$$

Comparative statics

The partial derivatives of the function h can be directly read off equation (21.17). We find

$$\begin{aligned}
\frac{\partial Y}{\partial i} &= h_{r^e} = \frac{D_{r^e}}{1 - D_Y} < 0, \\
\frac{\partial Y}{\partial \pi^e_{+1}} &= -h_{r^e} = -\frac{D_{r^e}}{1 - D_Y} > 0, \\
\frac{\partial Y}{\partial \tau} &= h_\tau = \frac{D_\tau}{1 - D_Y} < D_\tau < 0, \\
\frac{\partial Y}{\partial G} &= \frac{\partial Y}{\partial \varepsilon_D} = \frac{1}{1 - D_Y} > 1, \\
\frac{\partial Y}{\partial \varepsilon_L} &= 0.
\end{aligned} \tag{21.27}$$

The observation that the denominator, $1 - D_Y$, is not zero confirms our guess that the equation (IS) defines Y as an implicit function of the other variables in the equation.

The derivative w.r.t. a liquidity preference shock, ε_L, in the last line of (21.27) reflects the principle that a multiplier w.r.t. an exogenous variable not entering the equation(s) determining the endogenous variable directly or indirectly (see below) is nil. In the present case this means that, with a fixed interest rule, a liquidity preference shock has no effect on equilibrium output. The shock is immediately counteracted by a change in the money stock in the same direction so that the interest rate remains unchanged. Thus, the liquidity preference shock is "cushioned" by this monetary policy.

On the other hand, a shock to output demand has a larger effect on output than in the case where the money stock is kept constant (compare (21.27) to (21.24)). This is because keeping the money stock constant allows a dampening rise in the interest rate to take place. But with a constant interest rate this financial crowding-out effect does not occur.

One is tempted to draw the conclusion (from Poole, 1970):

- a money stock rule is preferable (in the sense of implying less volatility) if most shocks are output demand shocks, while
- a fixed interest rate rule is preferable if most shocks are liquidity preference shocks.

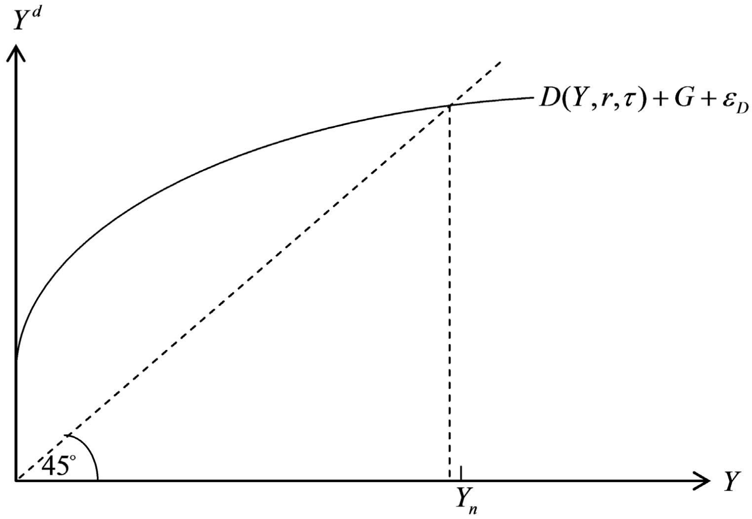

Figure 21.3: Given a fixed interest rate, a "Keynesian cross" diagram is sufficient to display the equilibrium output level (i, π^e_{+1}, τ, G, and ε_D given).

This should be accepted with caution, however, since a static model is not a secure guide for policy rules.

If we are interested also in the required changes in the money stock, we rewrite (LM) as

$$M = P \cdot (L(Y,i) + \varepsilon_L). \tag{LM'}$$

Here, i is exogenous and Y should be seen as already determined from (IS) independently of (LM'), that is, as given by (21.26). In this context we consider (LM') as an equation determining M as an implicit function of the other variables in the equation. To find the partial derivatives of this function, we take the total differential on both sides of (LM'):

$$dM = P(L_Y dY + L_i di + d\varepsilon_L) + (L(Y,i) + \varepsilon_L)dP, \tag{21.28}$$

where dY can be seen as already determined from (21.17) through (21.27), independently of (21.18). For instance, the approximate change in the money stock required for a rise in i of size $di > 0$ to materialize can, by (21.28), be written

$$\Delta M \approx dM = PL_Y dY + PL_i di = PL_Y h_{r^e} di + PL_i di = PL_Y \frac{D_{r^e}}{1 - D_Y} + PL_i di,$$

where the first term after the second equality sign is based on using the chain rule in (LM'). The multiplier of the money stock w.r.t. i is

$$\begin{aligned}\frac{\partial M}{\partial i}_{|(LM')} &= \frac{\partial M}{\partial Y}_{|(LM')} \cdot \frac{\partial Y}{\partial i}_{|(21.26)} + \frac{\partial M}{\partial i}_{|(LM')} = \frac{dM}{dY}_{|21.28} h_{r^e} + \frac{dM}{di}_{|21.28} \\ &= PL_Y \frac{D_{r^e}}{1 - D_Y} + PL_i < 0,\end{aligned}$$

where the first term after the last equality sign represents a negative *indirect effect* on the money stock of the rise in the target and the second term a negative *direct effect*. The direct effect indicates the fall in money stock needed to induce a rise in the interest rate of size di for a fixed output level. But also the output level will be affected by the rise in the interest rate since this rise reduces output demand. Through this indirect channel the transactions-motivated demand for money is reduced, and to match this a further fall in the money stock is required. This is the indirect effect.

The multipliers for the money stock w.r.t. the other exogenous variables are found in a similar way from (21.28) and (21.27), again using the chain rule where appropriate. Let us first consider the multiplier w.r.t. the exogenous variables entering (IS) and thereby (21.26). We get :

$$\begin{aligned}\frac{\partial M}{\partial \pi^e_{+1}} &= PL_Y \frac{\partial Y}{\partial \pi^e_{+1}}_{|(21.26)} = PL_y \cdot (-h_{r^e}) = -PL_Y \frac{D_{r^e}}{1 - D_Y} > 0, \\ \frac{\partial M}{\partial \tau} &= PL_Y \frac{\partial Y}{\partial \tau}_{|(21.26)} = PL_y \cdot h_\tau = PL_Y \frac{D_\tau}{1 - D_Y} < 0, \\ \frac{\partial M}{\partial G} &= \frac{\partial M}{\partial \varepsilon_D} = PL_Y \frac{\partial Y}{\partial G}_{|(21.26)} = PL_y \cdot h_G = PL_Y \frac{1}{1 - D_Y} > 0,\end{aligned}$$

where the inserted partial derives of h come from (21.27).

We see that higher expected inflation implies that the money stock required to maintain a given interest rate is higher. The reason is that for given i, a higher π^e_{+1} means lower expected real interest rate, hence higher output demand and higher output. Hereby the transactions-motivated demand for money is increased. A higher money stock is thus needed to hinder a rise in the nominal interest rate above target.

Finally, as ε_L and P do not enter (IS) and thereby not (21.26), the multipliers of M w.r.t. these two variables are determined directly by (LM'), keeping Y and i constant. We get

$$\frac{\partial M}{\partial \varepsilon_L} = P > 0, \qquad \frac{\partial M}{\partial P} = L(Y, i) + \varepsilon_L = \frac{M}{P} > 0.$$

The "Keynesian cross" Since for a fixed interest rate there is no financial crowding-out, the production outcome can also be illustrated by a standard 45° "Keynesian cross" diagram as in Fig. 21.3.

The spending multiplier under full tax financing

The spending multiplier in the last line of (21.27) is conditional on the fixed interest rate policy *and* constancy of the "fiscal tightness", τ. Although there will be an automatic rise in net tax revenue via $T'(Y) > 0$, unless increased government spending is fully self-financing (which it will be only if $T'(Y) \geq 1 - D_Y$, as we will see in a moment), the result is $d\mathbb{T} < dG$. This amounts to a larger budget deficit than otherwise and thereby increased public debt and higher taxes in the future. In Section 21.4 below we assess the possible feedback effects of this on the spending multiplier (effects that are ignored by the static IS-LM model).

In the present section we will consider the alternative case, a useful benchmark, where the increase in G is *accompanied* by an adjustment of the fiscal tightness parameter, τ, so as to ensure $d\mathbb{T} = dG$, thereby leaving the budget balance unchanged, it be negative, positive, or nil. The net tax revenue is

$$\mathbb{T} = \tau + T(Y), \qquad 0 \leq T' < 1, \tag{21.29}$$

cf. Section 21.1. We impose the requirement that the primary budget deficit, $G - \mathbb{T}$, remains equal to some constant k in spite of the change in G.This gives the equation

$$G - \tau - T(Y) = k, \tag{*}$$

where both τ and Y are endogenous. We have a second equation where these two variables enter, namely the (IS) equation with i exogenous:

$$Y = D(Y, i - \pi^e_{+1}, \tau) + G, \tag{**}$$

ignoring the shift term ε_D. The equation system (*) - (**) thus determines the pair τ and Y as implicit functions of the remaining variables, all of which are exogenous.

Taking differentials w.r.t. Y, τ, and G on both sides of (*) and (**) gives, after ordering, the linear equation system in $d\tau$ and dY :

$$\begin{aligned} d\tau + T'(Y)dY &= dG \\ -D_\tau d\tau + (1 - D_Y)dY &= dG. \end{aligned}$$

The determinant of the coefficient matrix on the left-hand side of this system is

$$\begin{aligned} \bar{\Delta} &= 1 - D_Y + D_\tau T'(Y) = 1 - D_Y - C_{Y^p} T'(Y) \\ &= 1 - \left[C_{Y^p}(1 - T'(Y)) + (C_{Y^e_{+1}} + I_{Y^e_{+1}})\varphi'(Y) \right] - C_{Y^p} T'(Y) \\ &= 1 - C_{Y^p} - (C_{Y^e_{+1}} + I_{Y^e_{+1}})\varphi'(Y) \in (0, 1), \end{aligned} \tag{21.30}$$

where the second equality sign comes from (21.7) and the third from (21.6). The stated inclusion follows from (21.2) and (21.4). By Cramer's rule

$$\begin{aligned} d\tau &= \frac{(1-D_Y)dG - T'(Y)dG}{\bar{\Delta}}, \\ dY &= \frac{dG + D_\tau dG}{\bar{\Delta}} = \frac{(1-C_{Y^p})dG}{\bar{\Delta}}, \end{aligned}$$

where the last equality sign follows from (21.7). Substituting $\bar{\Delta}$ from (21.30), the first line gives

$$\frac{\partial \tau}{\partial G}_{|(*),(**)} = \frac{1 - D_Y - T'(Y)}{1 - C_{Y^p} - (C_{Y^e_{+1}} + I_{Y^e_{+1}})\varphi'(Y)} \gtreqless \text{ for } T'(Y) \lesseqgtr 1 - D_Y, \quad (21.31)$$

The second line gives the derivative of Y w.r.t. G, conditional on full tax financing:

$$\frac{\partial Y}{\partial G}_{|(*),(**)} = \frac{1 - C_{Y^p}}{1 - C_{Y^p} - (C_{Y^e_{+1}} + I_{Y^e_{+1}})\varphi'(Y)} \geq 1. \quad (21.32)$$

Although valid (within the fixed interest rate regime) for any unchanged budget balance, this result is known as the *balanced budget multiplier* in the sense of spending multiplier under a balanced budget. In case $C_{Y^e_{+1}} + I_{Y^e_{+1}} = 0$, the multiplier is exactly 1, which is the original *Haavelmo result* (Haavelmo, 1945).

Let us underline two important results within the IS-LM model:

Result 1: *Even fully tax-financed government spending is expansionary.* Given a constant interest rate, under the unchanged-budget-balance policy (*), $dY \geq dG = d\mathbb{T} > 0$, in view of the spending multiplier being at least 1. Thereby, the change in disposable income is $dY - d\mathbb{T} \geq 0$. Thereby private consumption, C, tends to *rise*, if anything. The rise in G therefore does not crowd out private consumption. It rather crowds it *in*. As Y is raised and monetary policy keeps the interest rate unchanged, according to the model also private investment is "crowded in" rather than "crowded out" (this follows from the assumptions (21.2) and (21.4)).

Result 2: *The timing of (lump-sum) taxes generally matter.* To disentangle the role of timing, we compare the unchanged-budget-balance policy (*) with the case where the rise in G is not accompanied by a change in the fiscal tightness parameter, τ. Only the automatic stabilizer, $T'(Y)dY$, is operative. This will generally result in $dG - T'(Y)dY \neq 0$. If $T'(Y) < 1 - D_Y$ (by many considered the "normal case"[9]), from (21.31) follows that $dG - T'(Y)dY > 0$, that is, the budget balance is allowed to worsen. With fixed interest rate the spending multiplier will

[9] But in a large downturn it may be otherwise, cf. e.g. DeLong and Summers (2012).

be $1/(1-D_Y)$, cf. (21.27), and exceed that under an unchanged budget balance, given in (21.32).[10] So in the considered case, postponing the taxation needed to provide the ultimate financing of the rise in G makes this rise more expansionary. The timing of taxes matter.

In Section 21.4 we briefly discuss what happens to these two results if we imagine that the household sector consists of a fixed number of utility-maximizing infinitely-lived households.

The paradox of thrift

Another proposition of Keynesian theory is known as the *paradox of thrift*. Consider the following special case of the IS equation:

$$Y = C + I + G = c_0 + c_1(Y - \mathbb{T}) + \tilde{C}(i - \pi^e_{+1}) + c_2 Y + \tilde{I}(i - \pi^e_{+1}) + G, \quad (21.33)$$

where c_0, c_1, and c_2 are given constants satisfying

$$c_0 > 0, 0 < c_1 \leq c_1 + c_2 < 1, \quad (21.34)$$

and $\tilde{C}(\cdot)$ and $\tilde{I}(\cdot)$ are decreasing functions of the expected real interest rate, $i - \pi^e_{+1}$. We have excluded the demand shift parameter ε_D and linearized the income-dependent parts of the consumption and investment functions. We take G, π^e_{+1}, and i as exogenous (fixed interest rate rule).

The paradox of thrift comes out most clear-cut if we ignore the public sector.

No public sector: $G = \mathbb{T} = 0$. In this case equilibrium output is

$$Y = \frac{c_0 + \tilde{C}(i - \pi^e_{+1}) + \tilde{I}(i - \pi^e_{+1})}{1 - c_1 - c_2}.$$

Suppose that all households for some reason decide to save more at any level of income so that c_0 is decreased. What happens to aggregate private saving S^p? We have

$$S^p = Y - C = I = c_2 Y + \tilde{I}(i - \pi^e_{+1}), \quad (21.35)$$

by (21.33) with $G = \mathbb{T} = 0$. Hence,

$$\frac{\partial S^p}{\partial c_0} = c_2 \frac{\partial Y}{\partial c_0} = \frac{c_2}{1 - c_1 - c_2} \geq 0,$$

from (21.27). Considering a reduction of c_0, i.e., $\Delta c_0 < 0$, the resulting change in S^p is thus

$$\Delta S^p = \frac{\partial S^p}{\partial c_0} \Delta c_0 = \frac{c_2}{1 - c_1 - c_2} \Delta c_0 \leq 0.$$

[10] Indeed, $1/(1-D_Y) \gtreqless (1 - C_{Y^P})/(1 - D_Y - C_{Y^P} T'(Y))$ if $T'(Y) < 1 - D_Y$, respectively.

The attempt to save more thus defeats itself. What happens is that income decreases by an amount such that saving is either unchanged or even reduced. More precisely, if the income coefficient in the investment function, c_2, is nil, we get $\Delta S^p = 0$ because aggregate investment remains unchanged and income is reduced exactly as much as consumption, leaving saving unchanged. If $c_2 > 0$, we get $\Delta S^p < 0$ because income is reduced *more* than consumption since also investment is reduced when income is reduced. In this case the attempt to save more is directly counterproductive and leads to *less* aggregate saving.

The background to these results is that when aggregate output and income is demand-determined, the decreased propensity to consume lowers aggregate demand, thereby reducing production and income. The resulting lower income brings aggregate consumption further down through the *Kahn-Keynes multiplier process* (see below). While consumption is reduced, there is nothing in the situation to stimulate aggregate investment (at least not as long as the central bank maintains an unchanged interest rate). Thereby aggregate saving can not rise, since in a closed economy aggregate saving and aggregate investment are in equilibrium just two sides of the same thing as testified by national income accounting, cf. (21.35).

This story is known as the *paradox of thrift*. It is an example of a *fallacy of composition*, a term used by philosophers to denote the error of concluding from what is *locally* valid to what is *globally* valid. Such inference overlooks the possibility that when many agents act at the same time, the conditions framing each agent's actions are affected. As Keynes put it:

> ... although the amount of his own saving is unlikely to have any significant influence on his own income, the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums. Every such attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself (Keynes 1936, p. 84).

With public sector We return to (21.33) with $G > 0$ and $\mathbb{T} > 0$. The essence of the paradox of thrift remains but it may be partly blurred by the tendency of the government budget deficit to rise when private consumption, and therefore aggregate income, is reduced.

Consider first the case where public dissaving does *not* emerge. This is the case where the government budget is always balanced. Then, net tax revenue is $\mathbb{T} = G$, and private saving is

$$S^p \equiv Y - \mathbb{T} - C = Y - G - C = I = c_2 Y + \tilde{I}(i - \pi^e_{+1}),$$

by (21.33). So in this case the paradox of thrift comes out in the same strong form as above.

Consider instead the more realistic case where alternating budget deficits and surpluses are allowed to arise as a result of the net tax revenue following the rule

$$\mathbb{T} = \tau + \tau_1 Y, \qquad 0 < \tau_1 < 1. \tag{21.36}$$

Equilibrium output now is

$$Y = \frac{c_0 - c_1\tau + \tilde{C}(i - \pi^e_{+1}) + \tilde{I}(i - \pi^e_{+1}) + G}{1 - c_1(1 - \tau_1) - c_2}, \tag{21.37}$$

so that

$$\frac{\partial Y}{\partial c_0} = \frac{1}{1 - c_1(1 - \tau_1) - c_2} > 1, \tag{21.38}$$

the inequality being due to (21.34) and (21.36). Private saving is

$$\begin{aligned} S^p &= Y - \mathbb{T} - C = I - (\mathbb{T} - G) = I - S^g = I + G - (\tau + \tau_1 Y) \\ &= c_2 Y + \tilde{I}(i - \pi^e_{+1}) + G - (\tau + \tau_1 Y), \end{aligned}$$

where the second equality comes from (21.33) and the fourth from the taxation rule (21.36). We see that.....(continuation not yet available)

Adjustment: the Kahn-Keynes multiplier process (no text available)

21.3.3 Counter-cyclical interest rate rule

Assuming a fixed interest rate rule may fit the very short run well. If we think of a time interval of a year's length or more, we may imagine a counter-cyclical interest rate rule aiming at dampening fluctuations in aggregate economic activity. Such a policy may take the form

$$i = i_0 + i_1 Y, \qquad i_1 > 0, \tag{21.39}$$

where i_0 and i_1 are policy parameters. The present version of the IS-LM model does not rule out that the parameter i_0 can be negative. But in case $i_0 < 0$, at least i_0 is not so small that even under "normal circumstances", the zero lower bound for i can become operative. The term "counter-cyclical" refers to the attempt to stabilize output by raising i when output goes up and reducing i when output goes down.[11]

[11] The label "counter-cyclical" should not be confused with what is in the terminology of business cycle econometrics named "counter-cyclical" behavior. In this terminology a variable is characterized as "pro-" or "counter-cyclical" depending on whether its correlation with aggregate output is positive or negative, respectively. So (21.39) would in this language exemplify "pro-cyclical" behavior.

If the LM curve in Fig. 21.1 is made linear and its label changed into IRR (for Interest Rate Rule), that figure covers the counter-cyclical interest rate rule (21.39). Instead of a LM curve (which requires a fixed M), we have an upward sloping IRR curve. Both i and M are here endogenous. The fixed interest rate rule from the previous section is a limiting case of this rule, namely the case $i_1 = 0$. By having $i_1 > 0$, the counter-cyclical interest rate rule yields qualitative effects more in line with those of a money stock rule. If $i_1 > \partial i/\partial Y_{|LM}$ from (21.14), the stabilizing response of i to a decrease in Y is *stronger* than under the money stock rule.

Comparative statics

Inserting (21.39) into (IS) gives

$$Y = D(Y, i_0 + i_1 Y - \pi^e_{+1}, \tau) + G + \varepsilon_D.$$

By taking the total differential on both sides we find

$$\begin{aligned}
\frac{\partial Y}{\partial G} &= \frac{\partial Y}{\partial \varepsilon_D} = \frac{1}{1 - D_Y - D_{r^e} i_1} \in (0, \frac{1}{1 - D_Y}), \\
\frac{\partial Y}{\partial i_1} &= \frac{D_{r^e} Y}{1 - D_Y - D_{r^e} i_1} < 0, \\
\frac{\partial Y}{\partial \pi^e_{+1}} &= -\frac{D_{r^e}}{1 - D_Y - D_{r^e} i_1} > 0, \\
\frac{\partial Y}{\partial \varepsilon_L} &= 0.
\end{aligned}$$

We see that all multipliers become become close to 0, if the reaction coefficient i_1 is large enough. In particular, undesired fluctuations due to demand shocks are damped this way.

The corresponding changes in i are given as $\partial i/\partial x = i_1 \partial Y/\partial x$ for $x = G, \varepsilon_D, i_1, \pi^e_{+1}$, and ε_L, respectively. From (21.28) we find the corresponding changes in M as $\partial M/\partial x = P(L_Y + i_1 L_i)\partial Y/\partial x$ for $x = G, \varepsilon_D, i_1$, and π^e_{+1}; finally, from (21.28) we have again $\partial M/\partial \varepsilon_L = P > 0$.

21.3.4 Further aspects

The loanable funds theory of the interest rate

As we have seen, two Keynesian tenets are that involuntary unemployment can be a state of rest and that an increased propensity to save makes things worse. Several of Keynes' contemporaries (for instance NAME, YEAR) objected that the interest rate would adjust so as to bring the demand for new loans by users (primarily home and business investors).in line with an increased supply of new loans by financial savers. This is known as the "loanable funds theory of the interest rate" according to which the interest rate is determined by "the supply and demand for saving". The pre-Keynesian version of this theory does not take into account that aggregate saving depends not only on the interest rate, but also on aggregate income (the same could be said about investment but this is of no help for the pre-Keynesian version).

To clarify the issue, we consider the simple case where $C = C(Y, r^e)$ and $I = I(r^e)$, $0 < C_Y < 1$, $C_{r^e} < 0$, $I_{r^e} < 0$, $r^e = i - \pi^e_{+1}$ and where government spending and taxation are ignored. Let S denote aggregate saving. Then in our closed economy, $S = Y - C = Y - C(Y, r^e) \equiv S(Y, r^e)$, $S_Y = 1 - C_Y > 0$, $S_{r^e} = -C_{r^e} > 0$. Equilibrium in the output market requires $Y = C(Y, r^e) + I(r^e)$. By subtracting $C(Y, r^e)$ on both sides and inserting $r^e = i - \pi^e_{+1}$, we get

$$S(Y, i - \pi^e_{+1}) = I(i - \pi^e_{+1}), \tag{21.40}$$

which may be interpreted as supply of saving being equilibrated with demand for saving. Conditional on a given income level, Y, we could draw an upward-sloping supply curve and a downward-sloping demand curve in the (S, i) plane for given π^e_{+1}. But this would not determine i since the position of the supply curve will depend on the endogenous variable, Y. An extra equation is needed. This is what the money market equilibrium condition, $M/P = L(Y, i)$ delivers, combined with exogeneity of M, P and π^e. In the Keynesian version of the loanable funds theory of the interest rate there are thus two endogenous variables, i and Y, and two equations, (21.40) and $M/P = L(Y, i)$.

If we want to illustrate the solution graphically, we can use the standard IS-LM diagram from Fig. 21.1. This is because the equation (21.40) in the (Y, i) plane is nothing but the standard IS curve. Indeed, by adding consumption on both sides of the equation, we get $Y = C(Y, i - \pi^e_{+1}) + I(i - \pi^e_{+1})$, the standard IS equation. And whether we combine the LM equation with this or with (21.40), the solution for the pair (Y, i) will be the same.

A liquidity trap

We return to the general IS-LM model,

$$Y = D(Y, i - \pi^e_{+1}, \tau) + G + \varepsilon_D, \tag{IS}$$

$$\frac{M}{P} = L(Y, i) + \varepsilon_L, \tag{LM}$$

where M is again exogenous and Y and i endogenous. Suppose a large adverse demand shock $\varepsilon_D < 0$ takes place. This shock could be due to a bursting housing

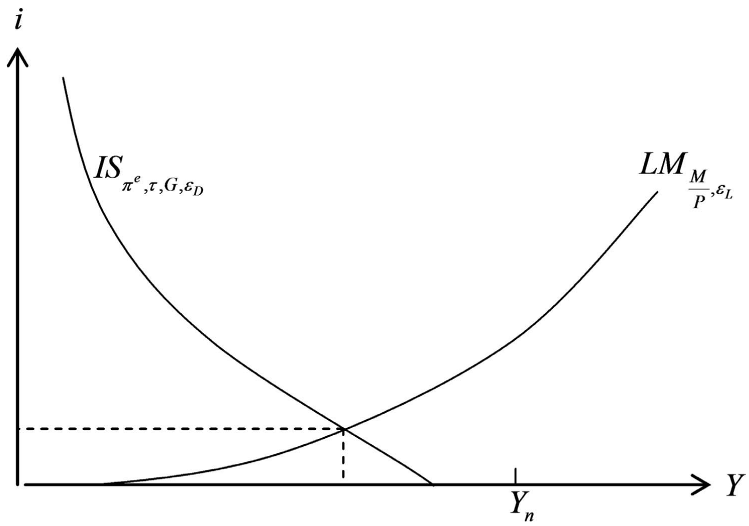

Figure 21.4: A situation where the given IS curve is such that no non-negative nominal interest rate can generate full employment (π_{+1}^e, τ, G, ε_D, M/P, and ε_L given). The value of Y where the IS curve crosses the abscissas axis is dented Y_0.

price bubble making creditors worried and demanding that debtors deleverage. This amounts to decreased consumption and investment and as a consequence, the IS curve may be moved so much leftward in the IS-LM diagram that whatever the money stock, output will end up smaller than the full-employment level, Y^n. Then the economy is in a *liquidity trap*: "conventional" monetary policy is not able to move output back to full employment. By "conventional" monetary policy is meant a policy where the central bank buys bonds in the open market with the aim of reducing the short-term nominal interest rate and thereby stimulate aggregate demand . The situation resembles a "trap" in the sense that when the central bank strives to stimulate aggregate demand by lowering the interest rate through open market operations, it is like attempting to fill a leaking bucket with water. The phenomenon is illustrated in Fig. 21.4.

The crux of the matter is that the nominal interest rate has the lower bound, 0, known as the *zero lower bound*. An increase in M can not bring i below 0. Agents would prefer holding cash at zero interest rather than short-term bonds at negative interest. That is, equilibrium in the asset markets is then consistent with the "=" in the LM equation being replaced by "$\geq$".

Suppose that expected inflation is very low, say nil. Then the (expected) real interest rate can not be brought below zero. The real interest rate *required* for full employment is *negative*, however, given the IS curve in Fig. 21.4. For the given π^e, to solve the demand problem expansionary fiscal policy moving the IS curve rightward is called for. Coordinated fiscal and monetary policy with the aim of raising π^e may also be a way out.

When an economy is at the zero lower bound, the government spending multiplier tends to be relatively large for two reasons. The first reason is the more trivial one that being in a liquidity trap is a *symptom* of a serious deficient aggregate demand problem and low capacity utilization so that there is no hindrance for fast expansion of production. The second reason is that there will be no financial crowding-out effect of a fiscal stimulus as long as the central bank aims at an interest rate as low as possible. (REFER to lit.)

Note that the economy can be in a liquidity trap, as we have defined it, before the zero lower bound on the nominal interest rate has been reached. Fig. 21.4 illustrates such a case. In spite of the current nominal interest rate being above zero, conventional monetary policy is not able to move output back to full employment. Conventional monetary policy can move the LM curve to the right, but the point of intersection with the IS curve can not be moved to the right of Y_0. An alternative − and more common − definition is simply to identify a liquidity trap with a situation in which the short-term nominal interest rate is zero.

Keynes (1936, p. 207) was the first to consider the possibility of a liquidity trap. After the second world war the issue appeared in textbooks, but not in practice, and so it gradually was given less and less attention. Almost at the same time as the textbooks had stopped mentioning it, it turned up in reality, first in Japan from the middle of the 1990, then in several countries, including USA, in the wake of the Great Recession. It became a problem of urgent practical importance and lead to suggestions for non-conventional monetary policies as well as more emphasis on expansionary fiscal policy, aspects to which we return later in this book.

A proviso concerning the exact character of the zero lower bound on the interest rate should be added. The zero bound should only be interpreted as exactly 0.0 if storage, administration, and safety cost are negligible, and − in an open economy − if there is no chance of a sudden appreciation of the currency in which the government debt is denominated. In the wake of the European debt crisis 2010-14, government bonds of some European countries (e.g., Germany, Finland, Switzerland, Denmark, and the Netherlands) were sold at slightly negative yields.

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Unfinished:

Some empirics about spending multipliers and their dependence on the state of the economy.

21.4 Some robustness checks

21.4.1 Presence of an interest rate spread (banks' lending rate $= i + \omega > i$).

(currently no text)

21.4.2 What if households are infinitely-lived?

Here we shall reconsider Result 1 and Result 2 from Section 21.3.2. They were:

Result 1: *Even fully tax-financed government spending is expansionary.*

Result 2: *The timing of (lump-sum) taxes generally matter.*

We ask whether these two results are likely to still hold in some form if we imagine that the household sector consists of a fixed number of utility-maximizing infinitely-lived households. The assumption that involuntary unemployment and abundant capacity are present is maintained.

Concerning Result 1 the answer is *yes* in the sense that the spending multiplier under a balanced budget will remain positive, albeit not necessarily ≥ 1. The reason is that although under a balanced budget the households face a temporary rise in taxes, $d\mathbb{T}$, equal to the temporary rise in spending, dG, they will reduce their *current* consumption by less than $d\mathbb{T}$, if at all. This is because they want to smooth consumption. If they at all have to reduce their total consumption, they will spread this reduction out over all future periods so that the present value of the total reduction is sufficient to cover the rise in taxes. Thereby, $-dC < dG$ so that there is necessarily an "initial" stimulus to aggregate demand equal to $dG - dC > 0$. Owing to unemployment and abundant production capacity, there need not be any crowding out of investment and so aggregate demand, output, and employment will be higher in this "first round" than without the rise in G. This means that current *before*-tax incomes increase and this stimulates private consumption and, therefore, production in the "second round". And so on through the "multiplier process". In the end private consumption in the current period need not at all fall and may even rise. So even with infinitely-lived households, the rise in G is expansionary under the stated circumstances.

Concerning Result 2 the answer depends on whether the credit market is perfect or not. With a perfect credit market current consumption of the infinitely-lived households will *not* depend on the timing of the extra taxes that are needed to finance dG. Whether the tax rise occurs now or later is irrelevant, as long as the present value of the tax rise is the same for the individual household. So the spending multiplier will be the *same* in the two situations. In this case, in spite of the rise in G being expansionary, there is *Ricardian equivalence* in the sense that for a given time path of government expenditures, the time path of (lump-sum) taxes does not matter for aggregate private consumption (whether the taxes are lump-sum or distortionary is in fact not so important in the present context where production and employment are demand-determined rather than supply-determined).

If the credit market is imperfect, however, in a heterogeneous population some of the infinitely-lived households, the less patient, say, may be currently credit constrained. The timing of the extra taxes then *does* matter and Ricardian equivalence is absent. Indeed, the lower current taxes associated with a budget deficit loosens the limit to current consumption of the credit-constrained households. Their consumption demand is thereby stimulated. Aggregate demand and therefore output and employment are thus raised. Through the automatic stabilizers the budget deficit hereby becomes smaller than otherwise. This means that the future extra tax burden becomes lower for everybody, including the households that are not currently credit-constrained. So also *their* current consumption is stimulated, and aggregate demand is raised further. We conclude that in spite of households being infinitely-lived, when credit markets are imperfect, for a given rise in government spending, the spending multiplier is likely to be larger under deficit financing than under balanced budget financing. So even Result 2 seems relatively robust.

21.5 Concluding remarks

The distinguishing feature of the IS-LM model compared with classical and new-classical theory is the treatment of the general price level for goods and services as given in the short run, that is, as a *state variable* of the system, hence very different from an asset price. The IS-LM model is not about why it is so (the two previous chapters suggested some answers to that question), but about the consequences for how the interaction between goods and asset markets works out. There are two different assets, money and an interest-bearing asset in the form of bonds, where money is held because of its liquidity services while as a store of value money is generally dominated by bonds.

Traditionally, the IS-LM model has been seen as only one building block of a more involved aggregate supply-aggregate demand (AS-AD) framework of many macroeconomic textbooks. In that framework the IS-LM model describes just the demand side of a model where the level of nominal wages is an exogenous constant in the short run, but the price level adjusts in response to shifts in aggregate demand for fixed money stock.

In this chapter we have interpreted the IS-LM model another way, namely as an independent short-run model in its own right, based on the approximation that both nominal wages and prices are set "in advance" by agents operating in imperfectly competitive markets and being hesitant regarding frequent or large price changes. The traditional AS-AD version of the Keynesian framework blurs the distinction between short-run equilibrium and a sequence of such equilibria. In a sequence of short-run equilibria some kind of Phillips curve, a dynamic relation, is operative rather than an upward-sloping AS curve in the (Y, P) plane (a static relation).[12]

Given the pre-set wages and prices, in every short period output is demand-determined. Likewise, but behind the scene, also employment is demand-determined. Not prices on goods and services, but quantities are the equilibrating factors. This is the polar opposite of Walrasian microeconomics and neoclassical long-run theory, cf. Part II-IV of this book, where output and employment are treated as supply-determined − with absolute and relative prices as the equilibrating factors.

A striking implication of this role switch is the *paradox of thrift* which is Keynes' favorite example of a *fallacy of composition.* As Keynes put it:

> ... although the amount of his own saving is unlikely to have any significant influence on his own income, the reactions of the amount of his consumption on the incomes of others makes it impossible for all individuals simultaneously to save any given sums. Every such attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself (Keynes 1936, p. 84).

Empirically the IS-LM model, in the interpretation given here but extended with an expectation-augmented Phillips curve, does a quite good job (see Gali, 1992, and Rudebusch and Svensson, 1998). And for instance the surveys in the *Handbook of Macroeconomics* (1999) and *Handbook of Monetary Economics* (201?) support the view that under "normal circumstances", the empirics say that the level of production and employment is significantly sensitive to fiscal and monetary policy.

21.6 Literature notes

The IS-LM model as presented here is essentially based on the attempt by Hicks (1937) to summarize the analytical content of Keynes' *General Theory of Em-*

[12] If one insists on something related to AS-AD, one could interpret this chapter's model as imposing a horizontal AS curve in the output-price plane. But that's it. No AD curve in this plane appears in the model. The only place an AD curve appears is in the output-interest plane in the form of an IS curve. When it comes to the study of sequences of short-run equilibria (Chapter 22), a medium-term AD curve in the output-*inflation* plane will arise.

ployment, Interest and Money. Keynes (WHERE?) mainly approved the interpretation. Of course Keynes' book contained many additional ideas and there has subsequently been controversies about "what Keynes really meant" (see, e.g., Leijonhufvud 1968). Yet the IS-LM framework has remained a cornerstone of mainstream short-run macroeconomics. The demand side of the large macroeconometric models which governments, financial institutions, and trade unions apply to forecast macroeconomic evolution in the near future is essentially built on the IS-LM model. At the theoretical level the IS-LM model has been criticized for being *ad hoc,* i.e., not derived from "primitives" (optimizing firms and households, given specified technology, preferences, budget constraints, and market structures combined with an intertemporal perspective with forward-looking expectations) and not ensuring mutual compatibility of agents budget constraints. In recent years, however, more elaborate micro-founded versions of the IS-LM model have been suggested (Goodfriend and King 1997, McCallum and Nelson 1999, Sims 2000, Dubey and Geanakoplos 2003, Walsh 2003, Woodford 2003, Casares and McCallum, 2006). Some of these "modernizations" and consistency checks are considered in Part VII.

To be added:

Barro's and others' critique of the traditional AS-AD interpretation of the IS-LM model.

The case with investment goods industries with monopolistic competition: Kiyotaki, QJE 1987.

Keynes 1937. Comparison between Keynes (1936) and Keynes (1939).

Balanced budget multiplier: Haavelmo (1945).

The natural range of unemployment: McDonald. See also Dixon and Rankin, eds., p. 56.

Keynes and DeLong: Say's law vs. the treasury view.

21.7 Appendix

21.8 Exercises