Risk-free rate of return

In this case, r_{t+1} is known and there is only uncertainty about future labor income. Hence, (30.14) reduces to

$$u'(c_t) = \frac{1+r_{t+1}}{1+\rho} E_t [u'(c_{t+1})], \qquad t = 0, 1, 2, ..., T-2. \tag{30.16}$$

It is natural to assume that higher wealth is associated with lower (or at least not higher) absolute risk aversion (i.e., not higher values of $-u''/u'$). In that case, it can be shown that marginal utility u' is a strictly convex function of c, that is, $(u')'' > 0$. But this implies that increased uncertainty in the form of a mean-preserving spread will lead to lower consumption "today" (more saving) than would otherwise be the case. This is what precautionary saving is about.

Fig. 30.1 gives an illustration. We can choose any utility function with $(u')'' > 0$. The often used logarithmic utility function is an example since $u(c) = \ln c$ gives $u'(c) = c^{-1}$, $u''(c) = -c^{-2}$ and $u'''(c) = 2c^{-3} > 0$. In the figure it is understood that $T = 3$ and that we consider the decision problem as seen from period 1. There is uncertainty about labor income in period 2. It can be because the real wage is unknown or because employment is unknown or both. Suppose, for simplicity, that there are only two possible outcomes for labor income y_t ($\equiv w_t n_t$), say y_a and y_b, each with probability $\frac{1}{2}$. That is, given a_2, there are, in view of (30.15), two possible outcomes for c_2:

$$c_2 = \begin{cases} c_a = (1+r_2)a_2 + y_a, & \text{with probability} = \frac{1}{2} \\ c_b = (1+r_2)a_2 + y_b & \text{with probability} = \frac{1}{2}. \end{cases} \tag{30.17}$$

Mean consumption will be $\bar{c} = (1+r_2)a_2 + \bar{y}$, where $\bar{y} = \frac{1}{2}(y_a + y_b)$.

Suppose c_1 is chosen optimally. Then, with $t = 1$ (30.16) is satisfied, and a_2 is given, by (30.10) with $t = 1$. The lower panel of Fig. 30.1 shows graphically, how $E_1 u'(c_2)$ is determined, given this a_2. In case of higher uncertainty in the form of a *mean-preserving spread,* i.e., a higher spread, $|y_b - y_a|$, but the same mean $\bar{y}$, the two possible outcomes for c_2 are c_a^* and c_b^*, if a_2 is unchanged and, hence, $\bar{c}$ unchanged. Then, the expected marginal utility of consumption becomes greater than before, as indicated by $E_1 u'(c_2^*)$ in the figure. In order that (30.16) can still be satisfied, a lower value than before of c_1 must be chosen (since $u'' < 0$), hence, more saving.

True enough, this increases a_2 so that the expected value of c_2 is in fact larger than $\bar{c}$ on the figure. Hereby the new $E_1 u'(c_2)$ ends up somewhere between the old $E_1 u'(c_2)$ and $E_1 u'(c_2^*)$ in the figure. The conclusion is still that the new c_1 has to be lower than the original c_1 in order that the first-order condition (30.16) can be satisfied in the new situation.

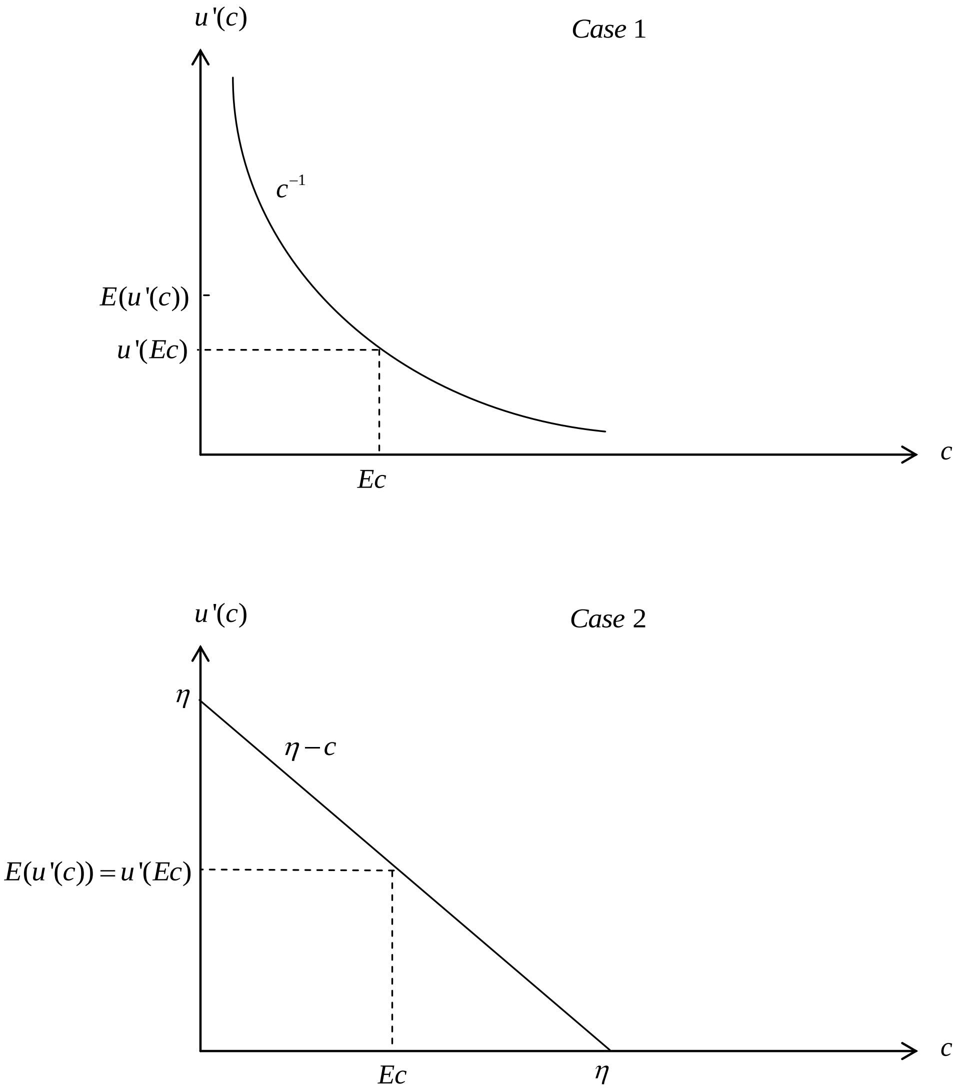

Figure 30.2

This phenomenon is called precautionary saving. To be more precise, we define *precautionary saving* as the increase in saving resulting from increased uncertainty. In the above example, increased uncertainty (a mean-preserving spread) implied lower consumption "today", that is, precautionary saving. Consumption is postponed in order to have a buffer-stock. The intuition is that the household wants to be prepared for meeting bad luck, because it wants to avoid the risk of having to end up starving ("save for the rainy day").

Note that the mathematical background for the phenomenon is the strict convexity of marginal utility, i.e., the assumption that $(u')'' > 0$. This implies $E(u'(c)) > u'(Ec)$, in view of Jensen's inequality (see Appendix). Case 1 in Fig. 30.2 shows the example $u(c) = \ln c$, i.e., $u'(c) = c^{-1}$.

If instead, $(u')'' = 0$, as with a quadratic utility function, then the graph for $u'(c_2)$ is a straight line (cf. case 2 in Fig. 30.2), and then precautionary saving can not occur. Indeed, a quadratic utility function can be written

$$u(c) = \eta c - \frac{1}{2}c^2, \qquad \eta > 0, \eta \text{ "large"}. \tag{30.18}$$

Then $u'(c) = \eta - c$, a straight line. By η "large" is meant "large relative to the likely levels of consumption" so that only the upward-sloping branch of the function becomes relevant in practice (thus avoiding a negative $u'(c)$).

This would be an example of so-called certainty equivalence. We say that *certainty equivalence* is present, if the decision under uncertainty follows the same rule as under certainty, only with actual values of the determining variables replaced by the expected values. The easiest case is to compare a situation where the relevant exogenous variables take on their expected values with a probability one (certainty) and a situation where they do that with a probability *less* than one (uncertainty). If the decision is the same in the two situations, certainty equivalence is present. So, when there is certainty equivalence, the decision under uncertainty is independent of the degree of uncertainty, measured by, say, the variance of the relevant exogenous variable(s). Quadratic utility implies certainty equivalence. Yet, since (30.18) gives $u'' = -1 < 0$, a household with quadratic utility is risk averse. Hence, for precautionary saving to arise, more than risk aversion is needed.

What is needed for precautionary saving to occur is $u''' > 0$, i.e., "prudence". Just as the degree of (absolute) risk aversion is measured by $-u''/u'$ (i.e., the degree of concavity of the utility function), the degree of (absolute) prudence is measured by $-u'''/u''$ (i.e., the degree of convexity of marginal utility). The degree of risk aversion is important for the size of the *required compensation* for uncertainty, whereas the degree of prudence is important for how the household's *saving behavior* is affected by uncertainty.

Uncertain rate of return

We have just argued that strictly convex marginal utility is a necessary condition for precautionary saving. But it is not a sufficient condition. This is so because there may be uncertainty not only about future labor income, but also about the rate of return on saving.

Consider the case where, as seen from period t, r_{t+1} is unknown. Then the relevant first-order condition is (30.14), not (30.16). Now, at least at the theoretical level, the tendency for precautionary saving to arise may be dampened or even turned into its opposite by an offsetting factor. For simplicity, assume first that there is no uncertainty associated with future labor income so that the only uncertainty is about the rate of return, r_{t+1}. In this case it can be shown that there is positive precautionary saving if the *relative* risk aversion, $-cu''/u'$, is larger than 1 ("it is good to have a buffer in case of bad luck") and *negative* precautionary saving if the relative risk aversion is less than 1 ("get while the getting is good").

It is generally believed that the empirically relevant assumption from a macroeconomic point of view is that $-cu''/u' > 1$. Thus, increased uncertainty about the rate of return should lead to more saving. The resulting precautionary saving then *adds* to that arising from increased uncertainty about future labor income.

30.4.2 Precautionary saving in a macroeconomic perspective

Simple calculations as well as empirical investigations (for references, see Romer 2001, p. 357) indicate that precautionary saving is not only a theoretical possibility, but can be quantitatively important. A sudden increase in perceived uncertainty seems capable of creating a sizeable fall in consumption expenditure (in particular expenditure on durable consumption goods) and thereby in aggregate demand. According to a study by Christina Romer (1990) this played a major role for the economic downturn in the US after the crash at the stock market in 1929 (see also Blanchard, 2003, p. 471 ff.).

Note that the conception of precautionary saving as an important business cycle force does not fit equally well in all business cycle theories. In new-classical theories (since the 1980s the RBC theory) a lower propensity to consume is immediately and automatically compensated by higher investment demand and perhaps a larger labor supply and employment in the economy. According to the RBC model from the previous chapter, aggregate demand continues to be sufficient to absorb output at full capacity utilization. Higher uncertainty just leads to a change in the composition of demand, a manifestation of Say's law.

Keynesians consider this story to be contradicted by the data. Less consumption spending seems far form being automatically offset by higher investment spending. Instead, vicious and virtuous circles are emphasized, these phenomena arising from production being in the short term demand-determined rather than supply-determined. An adverse shock, a bursting housing bubble say, will, through precautionary saving, lead to a contraction of demand and therefore a downturn of production.

Also firms' behavior may in an economic crisis have aspects of precautionary financial saving. A deep crisis generates a lot of uncertainty: firms do not understand what has happened and no one knows what actions to choose. The natural thing to do is to pause and wait until the situation becomes clearer. This entails a cutback in the plans for further purchase of investment goods. So on top of households' precautionary saving we have prudent investment behavior by the firms.

30.5 Literature notes

(incomplete)

Paul Krugman's *The Return to Depression Economics* (Krugman 2000) reflects on the need for macroeconomic theory to include depression economics as one of its concerns.

The self-fulfilling prophesy investment theory by Kiyotaki (1988) and the inventory investment theory by Blinder ( ) are examples of business cycle theory emphasizing firms' investment.

A simple model of a dynamic process leading to a liquidity trap and deflationary spirals is presented in Groth (1993).

30.6 Appendix

Jensen's inequality

Jensen's inequality is the proposition that when X is a stochastic variable, and the function f is *convex*, then

$$Ef(X) \geq f(EX)$$

with strict inequality, if f is *strictly* convex (unless X with probability 1 is equal to a constant). It follows that if f is *concave* (i.e., $-f$ is convex), then

$$Ef(X) \leq f(EX)$$

with strict inequality, if f is *strictly* concave (unless X with probability 1 is equal to a constant).

30.7 Exercises