Chapter 6

Long-run aspects of fiscal policy and public debt

We consider an economy with a government providing public goods and services. It finances its spending by taxation and borrowing. The term *fiscal policy* refers to the government's decisions about spending and the financing of this spending, be it by taxes or debt issue. The government's choice concerning the level and composition of its spending and how to finance it, may aim at:

1 affecting resource allocation (provide public goods that would otherwise not be supplied in a sufficient amount, correct externalities and other markets failures, prevent monopoly inefficiencies, provide social insurance);

2 affecting income distribution, be it a) within generations or b) between generations;

3 contribute to macroeconomic stabilization (dampening of business cycle fluctuations through aggregate demand policies).

The design of fiscal policy with regard to the aims 1 and 2 at a disaggregate level is a major theme within the field of public economics. Macroeconomics studies ways of dealing with aim 3 as well as big-picture aspects of 1 and 2, like overall policies to maintain and promote sustainable prosperity.

In this chapter we address fiscal sustainability and long-run implications of debt finance. This relates to one of the conditions that constrain public financing instruments. To see the issue of fiscal sustainability in a broader context, Section 6.1 provides an overview of conditions and factors that constrain public financing instruments. Section 6.2 introduces the basics of government budgeting and Section 6.3 defines the concepts of *government solvency* and *fiscal sustainability.* In Section 6.4 the analytics of debt dynamics is presented. As an example, the

Stability and Growth Pact of the EMU (the Economic and Monetary Union of the European Union) is discussed. Section 6.5 looks more closely at the link between government solvency and the government's *No-Ponzi-Game condition* and *intertemporal budget constraint.* In Section 6.6 we widen public sector accounting by introducing separate operating and capital budgets so as to allow for proper accounting of public investment. A theoretical claim, known as the *Ricardian equivalence* proposition, is studied in Section 6.7. The question whether Ricardian equivalence is likely to be a good approximation to reality, is addressed, applying the Diamond OLG framework extended with a public sector.

6.1 An overview of government spending and financing issues

Before entering the more specialized sections, it is useful to have a general idea about circumstances that condition public spending and financing. These circumstances include:

(i) financing by debt issue is constrained by the need to remain solvent and avoid catastrophic debt dynamics;

(ii) financing by taxes is limited by problems arising from:

 (a) distortionary supply-side effects of many kinds of taxes;

 (b) tax evasion (cf. the rise of the shadow economy, tax havens used by multinationals, etc.).

(iii) time lags in spending as well as taxing may interfere with attempts to stabilize the economy (recognition lag, decision lag, implementation lag, and effect lag);

(iv) credibility problems due to time-inconsistency;

(v) conditions imposed by political processes, bureaucratic self-interest, lobbying, and rent seeking.

Point (i) is the main focus of sections 6.2-6.6. Point (ii) is briefly considered in Section 6.4.1 in connection with the *Laffer curve.* In Section 6.6 point (iii) is briefly commented on. The remaining points, (iv) - (v), are not addressed specifically in this chapter. They should always be kept in mind, however, when discussing fiscal policy in practice. Hence some remarks at the end of the chapter.

Now to the specifics of government budget accounting and debt financing.

6.2 The government budget

We generally perceive the *public sector* (or the *nation state*) as consisting of the *national government* and a *central bank.* In economics the term "government" does not generally refer to the particular administration in office at a point in time. The term is rather used in a broad sense, encompassing both legislation and central and local administration. The aspects of legislation and administration in focus in macroeconomics are the rules and decisions concerning spending on public consumption, public investment, transfers, and subsidies on the expenditure side and on levying taxes and incurring debts on the financing side. Within certain limits the national government has usually delegated the management of the nation's currency to the central bank, a separate governmental institution, often called the monetary authority. Yet, from an overall macroeconomic point of view it is useful to treat "government budgeting" as covering the public sector as a whole: the consolidated government and central bank. Government bonds held by the central bank are thus excluded from what we call "government debt".

The basics of government budget accounting cannot be described without including money, nominal prices, and inflation. Elementary aspects of money and inflation will therefore be included in this section. We shall not, however, consider money and inflation in any systematic way until later chapters. Whether the economy considered is a closed or open economy will generally not be important in this chapter. We use the terms *government debt* and *public debt* synonymously.

Table 6.1 lists key variables of government budgeting.

Table 6.1. List of main variable symbols

Symbol	*Meaning*
Y_t	real GDP (= real GNP if the economy is closed)
C_t^g	public consumption
I_t^g	public fixed capital investment
G_t	$\equiv C_t^g + I_t^g$ real public purchases (spending on goods and services)
X_t	real transfer payments
$\tilde{T}_t$	real gross tax revenue
T_t	$\equiv \tilde{T}_t - X_t$ real net tax revenue
M_t	the monetary base (currency and bank reserves in the central bank)
P_t	price level (in money) for goods and services (the GDP deflator)
D_t	nominal net public debt (including possible debt of local government)
B_t	$\equiv \frac{D_t}{P_{t-1}}$ real net public debt
b_t	$\equiv \frac{B_t}{Y_t}$ government debt-to-income ratio
i_t	nominal short-term interest rate
Δx_t	$\equiv x_t - x_{t-1}$ (where x is some arbitrary variable)
π_t	$\equiv \frac{\Delta P_t}{P_{t-1}} \equiv \frac{P_t - P_{t-1}}{P_{t-1}}$ inflation rate
$1 + r_t$	$\equiv \frac{P_{t-1}(1+i_t)}{P_t} \equiv \frac{1+i_t}{1+\pi_t}$ real short-term interest rate

Note that Y_t, G_t, and T_t are quantities defined *per period*, or more generally, *per time unit,* and are thus flow variables. On the other hand, M_t, D_t, and B_t are stock variables, that is, quantities defined at a given point in time, here at the *beginning* of period t. We measure D_t and B_t *net* of financial claims held by the government. Almost all countries have positive government net debt, but in principle $D_t < 0$ is possible.[1] The monetary base, M_t, is currency plus fully liquid deposits in the central bank held by the private sector at the beginning of period t; M_t is by definition nonnegative.

Until further notice, we shall in this chapter ignore uncertainty and default risk. We shall also ignore the fact that government bonds are usually more liquid (easier to quickly convert into cash) than other financial assets. Under these circumstances the market interest rate on government bonds must be the same as that on other interest-bearing assets. There is thus only one interest rate, i_t, in the economy. For ease of exposition we imagine that all government bonds are *one-period bonds.* That is, each government bond promises a payout equal to one unit of account at the end of the period and then the bond expires. Given the interest rate, i_t, the market value of a bond at the start of period t is $v_t = 1/(1 + i_t)$. If the number of outstanding bonds (the quantity of bonds) in

[1] If $D_t < 0$, the government has positive net financial claims on the private sector and earns interest on these claims − which is then an additional source of government revenue besides taxation.

period t is q_t, the government debt has face value (value at maturity) equal to q_t. The market value at the start of period t of this quantity of bonds will be $D_t \equiv q_t v_t = q_t/(1+i_t)$. The nominal expenditure to be made at the end of the period to redeem the outstanding debt can then be written

$$q_t = D_t(1+i_t). \tag{6.1}$$

This is the usual way of writing the expenditure to be made, namely as *if* the government debt were like a given bank loan of size D_t with a variable rate of interest. We should not forget, however, that given the quantity, q_t, of the bonds, the value, D_t, of the government debt at the issue date depends negatively on i_t.

Anyway, the total nominal government expenditure in period t can be written

$$P_t(G_t + X_t) + D_t(1+i_t).$$

It is common to refer to this expression as expenditure "in period t". Yet, in a discrete time model (with a period length of a year or a quarter corresponding to typical macroeconomic data) one has to imagine that the *payment* for goods and services delivered in the period occurs either at the beginning or the end of the period. We follow the latter interpretation and so the nominal price level P_t for period-t goods and services refers to payment occurring at the *end* of period t. As an implication, the real value, B_t, of government debt at the beginning of period t (= end of period $t-1$) is D_t/P_{t-1}. This may look a little awkward but is nevertheless meaningful. Indeed, D_t is a *stock* of liabilities at the beginning of period t while P_{t-1} is a price referring to a flow *paid for* at the *end* of period $t-1$ which is essentially the same point in time as the beginning of period t. Anyway, whatever timing convention is chosen, some kind of awkwardness will always arise in discrete time analysis. This is because the discrete time approach artificially treats the continuous flow of time as a sequence of discrete points in time.[2]

The government's expenditure is financed, effectively, by a combination of taxes, bonds issue, and increase in the monetary base:

$$P_t\tilde{T}_t + D_{t+1} + \Delta M_{t+1} = P_t(G_t + X_t) + D_t(1+i_t). \tag{6.2}$$

By rearranging we have

$$\Delta D_{t+1} + \Delta M_{t+1} = P_t(G_t + X_t - \tilde{T}_t) + i_t D_t. \tag{6.3}$$

Although in many developed countries the central bank is prohibited from buying government bonds directly from the government, it may buy them from private

[2] In a theoretical model this kind of problems is avoided when government budgeting is formulated in continuous time, cf. Chapter 13.

entities shortly after these have bought them from the government. Over the year the newly issued government debt may thus be more or less "monetarized".

In customary government budget accounting the nominal *government budget deficit,* GBD, is defined as the excess of total government spending over government revenue, $P\tilde{T}$. That is, according to this definition the right-hand side of (6.3) is the nominal budget deficit in period t, GBD_t. The first term on the right-hand side, $P_t(G_t + X_t - \tilde{T}_t)$, is named the nominal *primary budget deficit* (non-interest spending less taxes). The second term, $i_t D_t$, is called the nominal *debt service*. Similarly, $P_t(\tilde{T}_t - X_t - G_t)$ is called the nominal *primary budget surplus*. A negative value of a "deficit" thus amounts to a positive value of a corresponding "surplus", and a negative value of a "surplus" amounts to a positive value of a corresponding "deficit".

We immediately see that this accounting deviates from "normal" principles. Business companies typically have sharply separated capital and operating budgets. In contrast, the budget deficit defined above treats that part of G which represents government *net investment* as parallel to government consumption. Government net investment is attributed as an *expense* in a single year's account. According to "normal" principles it is only the *depreciation* on the public capital that should figure as an expense. Likewise, the above accounting does not consider that a part of D, or perhaps more than D, may be backed by the value of public physical capital. And if the government sells a physical asset to the private sector, the sale will appear as a reduction of the government budget deficit while in reality it is merely a conversion of an asset from a physical form to a financial form. The expense and asset aspects of government net investment are thus not properly dealt with in the standard public accounting.[3]

With the exception of Section 6.6 we will nevertheless stick to the traditional vocabulary. Where this might create logical difficulties, it helps to imagine that:

(a) all of G *is* public consumption, i.e., $G_t = C_t^g$ for all t;

(b) there is *no* public physical capital.

Now, from (6.2) and the definition $T_t \equiv \tilde{T}_t - X_t$ (net tax revenue) follows that

[3] Another anomaly is related to the fact that some countries, for instance Denmark, have large implicit government assets due to deferred taxes on the part of personal income invested in pension funds. If the government then decides to reverse the deferred taxation (as the Danish government did 2012 and 2014 to comply better with the 3%-deficit rule of the Stability and Growth Pact of the EMU), the official budget deficit is reduced. But essentially, all that has happened is that one government asset has been replaced by another.

real government debt at the beginning of period $t+1$ is:

$$\begin{aligned} B_{t+1} &\equiv \frac{D_{t+1}}{P_t} = G_t + X_t - \tilde{T}_t + (1+i_t)\frac{D_t}{P_t} - \frac{\Delta M_{t+1}}{P_t} \\ &= G_t - T_t + (1+i_t)\frac{D_t/P_{t-1}}{P_t/P_{t-1}} - \frac{\Delta M_{t+1}}{P_t} = G_t - T_t + \frac{1+i_t}{1+\pi_t}B_t - \frac{\Delta M_{t+1}}{P_t} \\ &\equiv (1+r_t)B_t + G_t - T_t - \frac{\Delta M_{t+1}}{P_t}. \end{aligned} \tag{6.4}$$

This is the law of motion of real government debt.

The last term, $\Delta M_{t+1}/P_t$, in (6.4) is *seigniorage,* i.e., public sector revenue obtained by issuing base money (ignoring the diminutive cost of printing money). To get a sense of this variable, suppose real output grows at the constant rate g_Y so that $Y_{t+1} = (1+g_Y)Y_t$. Then the public debt-to-income ratio can be written

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{1+r_t}{1+g_Y}b_t + \frac{G_t - T_t}{(1+g_Y)Y_t} - \frac{\Delta M_{t+1}}{P_t(1+g_Y)Y_t}. \tag{6.5}$$

Apart from the growth-correcting factor, $(1+g_Y)^{-1}$, the last term is the seigniorage-income ratio,

$$\frac{\Delta M_{t+1}}{P_t Y_t} = \frac{\Delta M_{t+1}}{M_t}\frac{M_t}{P_t Y_t}.$$

If in the long run the base money growth rate, $\Delta M_{t+1}/M_t$, as well as the nominal interest rate (i.e., the opportunity cost of holding money) are constant, then the velocity of money and its inverse, the money-nominal income ratio, $M_t/(P_t Y_t)$, are also likely to be roughly constant. So is, therefore, the seigniorage-income ratio.[4] For the more developed countries this ratio tends to be a fairly small number although not immaterial. For emerging economies with poor institutions for collecting taxes seigniorage matters more.[5]

The U.S. has a single monetary authority, the central bank, and a single fiscal authority, the treasury. The seigniorage created is immediately transferred from the first to the latter. The Eurozone has a single monetary authority but

[4] A reasonable money demand function is $M_t^d = P_t Y_t e^{-\alpha i}$, $\alpha > 0$, where i is the nominal interest rate. With clearing in the money market, we thus have $M_t/(P_t Y_t) = e^{-\alpha i}$. In view of $1+i \equiv (1+r)(1+\pi)$, when r and π are constant, so is i and, thereby, $M_t/(P_t Y_t)$.

[5] In the U.S. over the period 1909-1950s seigniorage fluctuated a lot and peaked 4 % of GDP in the 1930s and 3 % of GDP at the end of WW II. But over the period from the late 1960s to 1986 seigniorage fluctuated less around an average close to 0.5 %.of GDP (Walsh, 2003, p. 177). In Denmark seigniorage was around 0.2 % of GDP during the 1990s (*Kvartalsoversigt 4. kvartal 2000*, Danmarks Nationalbank). In Bolivia, up to the event of hyperinflation 1984-85, seigniorage reached 5 % of GDP and more than 50 % of government revenue (Sachs and Larrain, 1993).

multiple fiscal authorities, namely the treasuries of the member countries. The seigniorage created by the ECB is every year shared by the national central banks of the Eurozone countries in proportion to their equity share in the ECB. And the national central banks then transfer their share to the national treasuries. This makes up a ΔM_{t+1} term for the consolidated public sector of the individual Eurozone countries.

In monetary unions and countries with their own currency, government budget deficits are thus, from a macroeconomic point of view, generally financed both by debt creation and money creation, as envisioned by the above equations. Nonetheless, from now on, for simplicity, in this chapter we will predominantly ignore the seigniorage term in (6.5) and only occasionally refer to the modifications implied by taking it into account.

We thus proceed with the simple government accounting equation:

$$B_{t+1} - B_t = r_t B_t + G_t - T_t, \tag{DGBC}$$

where the right-hand side is the *real budget deficit.* This equation is often called the *dynamic government budget constraint* (or DGBC for short). It is in fact just an accounting identity conditional on $\Delta M = 0$. It says that if the real budget deficit is positive and there is essentially no financing by money creation, then the real public debt grows. We come closer to a *constraint* when combining (DGBC) with the requirement that the government stays *solvent.*

A terminological remark before proceeding: One is tempted to call the right-hand side of (DGBC) the *real* budget deficit. And there is nothing wrong with that as long as one keeps in mind that right-hand side of (DGBC) is *not* the same as the nominal budget deficit deflated by P_t. Indeed,

$$r_t B_t + G_t - T_t = \left(\frac{1+i_t}{1+\pi_t} - 1\right)\frac{D_t}{P_{t-1}} + G_t - T_t = \frac{i_t - \pi_t}{1+\pi_t}\frac{D_t}{P_{t-1}} + G_t - T_t = \frac{GBD_t - \pi_t D_t}{P_t},$$

by definition of the nominal budget deficit GBD_t. The reason that the term $\pi_t D_t$ is subtracted is that *inflation* curtails the increase in *real* debt, given the nominal interest rate

6.3 Government solvency and fiscal sustainability

To be *solvent* means being able to meet the financial commitments as they fall due. In practice this concept is closely related to the government's No-Ponzi-Game condition and intertemporal budget constraint (to which we return in Section 6.5), but at the theoretical level it is more fundamental.

We may view the public sector as an infinitely-lived agent in the sense that there is no last date where all public debt has to be repaid. Nevertheless, as we shall see, there tends to be stringent constraints on government debt creation in the long run.

6.3.1 The critical role of the growth-corrected interest factor

Very much depends on whether the real interest rate in the long-run is higher than the growth rate of GDP or not.

To see this, suppose the country considered has positive government debt at time 0 and that the government levies taxes equal to its non-interest spending:

$$\tilde{T}_t = G_t + X_t \quad \text{or} \quad T_t \equiv \tilde{T}_t - X_t = G_t \text{ for all } t \geq 0. \tag{6.6}$$

So taxes cover only the primary expenses while interest payments (and debt repayments when necessary) are financed by issuing new debt. That is, the government attempts a permanent *roll-over* of the debt including the interest due for payment. In view of (DGBC), this implies that $B_{t+1} = (1 + r_t)B_t$, saying that the debt grows at the rate r_t. Assuming, for simplicity, that $r_t = r$ (a given constant), the law of motion for the public debt-to-income ratio is

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{1+r}{1+g_Y}\frac{B_t}{Y_t} \equiv \frac{1+r}{1+g_Y}b_t, \qquad b_0 > 0,$$

where we have maintained the assumption of a constant output growth rate, g_Y. The solution to this linear difference equation then becomes

$$b_t = b_0(\frac{1+r}{1+g_Y})^t,$$

where we consider both r and g_Y as exogenous. We see that the growth-corrected interest rate, $\frac{1+r}{1+g_Y} - 1 \approx r - g_Y$ (for g_Y and r "small") plays a key role. There are contrasting cases to discuss.

Case 1: $r > g_Y$. In this case, $b_t \to \infty$ for $t \to \infty$. Owing to compound interest, the debt grows so large in the long run that the government will be unable to find buyers for the newly issued debt. Permanent debt roll-over is thus not feasible. Imagine for example an economy described by the Diamond OLG model. Here the buyers of the debt are the young who place part of their saving in government bonds. But if the stock of these bonds grows at a higher rate than income, the saving of the young cannot in the long run keep track with the fast-growing government debt. In this situation the private sector will understand that bankruptcy is threatening and nobody will buy government bonds except at a low price, which means a high interest rate. The high interest rate only aggravates the problem. That is, the fiscal policy (6.6) breaks down. Either the government defaults on the debt or T must be increased or G decreased (or both) until the growth rate of the debt is no longer higher than g_Y.

If the debt is denominated in the country's own currency, an alternative way out is of course a shift to money financing of the budget deficit, that is, seigniorage. When capacity utilization is high, this leads to rising inflation and thus the real value of the debt is eroded. Bond holders will then demand a higher nominal interest rate, thus aggravating the fiscal difficulties. The economic and social chaos of *hyperinflation* threatens.[6] The hyperinflation in Germany 1922-23 peaked in Nov. 1923 at 29,525% per month; it eroded the real value of the huge government debt of Germany after WW I by 95 percent.

Case 2: $r = g_Y$. If $r = g_Y$, we get $b_t = b_0$ for all $t \geq 0$. Since the debt, increasing at the rate r, does not increase faster than national income, the government has no problem finding buyers of its newly issued bonds − the government stays solvent. Thereby the government is able to finance its interest payments simply by issuing new debt. The growing debt is passed on to ever new generations with higher income and saving and the debt roll-over implied by (6.6) can continue forever.

Case 3: $r < g_Y$. Here we get $b_t \to 0$ for $t \to \infty$, and the same conclusion holds *a fortiori*.

In Case 2 as well as Case 3, where the interest rate is not higher than the growth rate of the economy, the government can thus pursue a permanent debt roll-over policy as implied by (6.6) and still remain solvent. But in Case 1, permanent debt roll-over is impossible and sooner or later the interest payments must be tax financed.

Which of the cases is relevant in real life? Fig. 6.1 shows for Denmark (upper panel) and the US (lower panel) the time paths of the real short-term interest rate and the GDP growth rate, both on an annual basis. Overall, the levels of the two are more or less the same, although on average the interest rate is in Denmark slightly higher but in the US somewhat lower than the growth rate. (Note that the interest rates referred to are not the average rate of return in the economy but a proxy for the lower interest rate on government bonds.)

Nevertheless, many macroeconomists believe there is good reason for paying attention to the case $r > g_Y$, also for a country like the US. This is because we live

[6] In economists' standard terminology "hyperinflation" is present when the inflation rate exceeds 50 percent *per month*. As we shall see in Chapter 18, the monetary financing route comes to a dead end if the needed seigniorage reaches the backward-bending part of the "seigniorage Laffer curve".

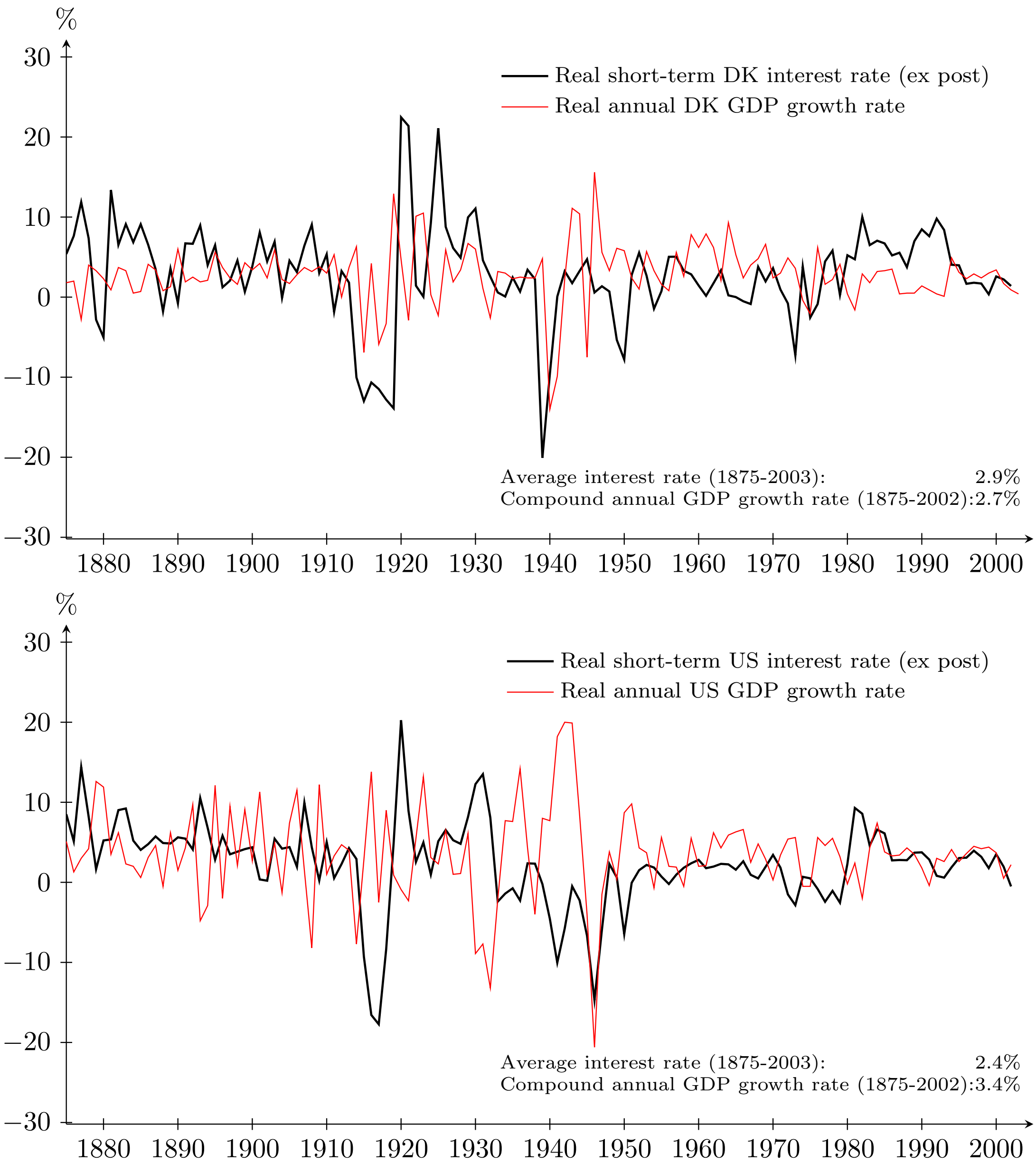

Figure 6.1: Real short-term interest rate and annual growth rate of real GDP in Denmark and the US since 1875. The real short-term interest rate is calculated as the money market rate minus the contemporaneous rate of consumer price inflation. Source: Abildgren (2005) and Maddison (2003).

in a world of *uncertainty,* with many different interest rates, and imperfect credit markets, aspects the above line of reasoning has not incorporated. The prudent debt policy needed whenever, under certainty, $r > g_Y$ can be shown to apply to a larger range of circumstances when uncertainty is present (see Literature notes). To give a flavor we may say that a prudent debt policy is needed when the average interest rate on the public debt exceeds $g_Y - \varepsilon$ for some "small" but positive ε.[7] On the other hand there is a different feature which draws the matter in the opposite direction. This is the possibility that a tax, $\tau \in (0, 1)$, on interest income is in force so that the net interest rate on the government debt is $(1 - \tau)r$ rather than r.

6.3.2 Sustainable fiscal policy

The concept of sustainable fiscal policy is closely related to the concept of government solvency. As already noted, to be *solvent* means being able to meet the financial commitments as they fall due. A given fiscal policy is called *sustainable* if by applying its spending and tax rules forever, the government stays solvent. "Sustainable" conveys the intuitive meaning. The issue is: can the current tax and spending rules continue forever?

To be more specific, suppose G_t and T_t are determined by fiscal policy rules represented by the functions

$$G_t = \mathcal{G}(x_{1t}, ..., x_{nt}, t), \quad \text{and} \quad T_t = \mathcal{T}(x_{1t}, ..., x_{nt}, t),$$

where $t = 0, 1, 2, \ldots$, and x_{1t},..., x_{nt} are key macroeconomic and demographic variables (like national income, old-age dependency ratio, rate of unemployment, extraction of natural resources, say oil from the North Sea, etc.). In this way a given fiscal policy is characterized by the rules $\mathcal{G}(\cdot)$ and $\mathcal{T}(\cdot)$. Suppose further that we have an economic model, $\mathcal{M}$, of how the economy functions.

DEFINITION Let the current period be period 0 and let the public debt at the beginning of period 0 be given. Then, given a forecast of the evolution of the demographic and foreign economic environment in the future and given the economic model $\mathcal{M}$, the fiscal policy $(\mathcal{G}(\cdot), \mathcal{T}(\cdot))$ is said to be *sustainable* relative to this model if the forecast calculated on the basis of $\mathcal{M}$ is that the government stays solvent under this policy. The fiscal policy $(\mathcal{G}(\cdot), \mathcal{T}(\cdot))$ is called *unsustainable*, if it is not sustainable.

This definition of fiscal sustainability is silent about the presence of uncertainty. Without going into detail about this difficult issue, suppose the model $\mathcal{M}$ is stochastic and let ε be a "small" positive number. Then we may say that

[7] This is only a "rough" characterization, see, e.g., Blanchard and Weil (2001).

the fiscal policy $(\mathcal{G}(\cdot), \mathcal{T}(\cdot))$ with 100-ε percent probability is *sustainable* relative to the model $\mathcal{M}$ if the forecast calculated on the basis of $\mathcal{M}$ is that with 100-ε percent probability the government stays solvent under this policy.

Governments, rating agencies, and other institutions evaluate sustainability of fiscal policy on the basis of simulations of giant macroeconometric models. Essentially, the operational criterion for sustainability is whether the fiscal policy can be deemed compatible with upward boundedness of the public debt-to-income ratio. Normally, the income measure applied here is GDP. Other measures are conceivable such as GNP, taxable income, or after-tax income. Moreover, even if a debt spiral is not (yet) underway in a given country, a high *level* of the debt-income ratio may in itself be worrisome. This is because a high level of debt under certain conditions may trigger a spiral of self-fulfilling expectations of default. We come back to this in the section to follow.

Owing to the increasing pressure on public finances caused by factors such as reduced birth rates, increased life expectancy, and a fast-growing demand for medical care, many industrialized countries have for a long time been assessed to be in a situation where their fiscal policy is not sustainable (Elmendorf and Mankiw 1999). The implication is that sooner or later one or more expenditure rules and/or tax rules (in a broad sense) will probably have to be changed.

Two major kinds of strategies have been suggested. One kind of strategy is the *pre-funding strategy*. The idea is to prevent sharp future tax increases by ensuring a fiscal consolidation prior to the expected future demographic changes. Another strategy (alternative or complementary to the former) is to attempt a gradual increase in the labor force by letting the age limits for retirement and pension increase along with expected lifetime – this is the *indexed retirement strategy*. The first strategy implies that current generations bear a large part of the adjustment cost. In the second strategy the costs are shared by current and future generations in a way more similar to the way the benefits in the form of increasing life expectancy are shared. We shall not go into detail about these matters here, but refer the reader to a large literature about securing fiscal sustainability in the ageing society, see Literature notes.

6.4 Debt arithmetic

A key tool for evaluating fiscal sustainability is *debt arithmetic*, i.e., the analytics of debt dynamics. The previous section described the important role of the growth-corrected interest rate. The next subsection considers the minimum primary budget surplus required for fiscal sustainability in different situations.

6.4.1 The required primary budget surplus

Ignoring the seigniorage term $\Delta M_{t+1}/P_t$ in the dynamic government budget identity (6.4) and assuming a constant interest rate r, we have:

$$B_{t+1} = (1+r)B_t - (T_t - G_t), \tag{DGBC}$$

where $T_t - G_t \equiv \tilde{T}_t - X_t - G_t$ is the primary budget surplus in real terms. Suppose aggregate income, Y_t, grows at a given constant rate g_Y. Let the spending-to-income ratio, G_t/Y_t, and the (net) tax revenue-to-income ratio, T_t/Y_t, be constants, γ and τ, respectively. We assume that interest income on government bonds is not taxed. It follows that the public debt-to-income ratio $b_t \equiv B_t/Y_t$ (from now just denoted debt-income ratio) changes over time according to

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{1+r}{1+g_Y}b_t - \frac{\tau-\gamma}{1+g_Y}, \tag{6.7}$$

where we have assumed a constant interest rate, r. There are (again) three cases to consider.

Case 1: $r > g_Y$. As emphasized above this case is generally considered the one of most practical relevance. And it is in this case that *latent debt instability* is present and the government has to pay attention to the danger of runaway debt dynamics. To see this, note that the solution of the linear difference equation (6.7) is

$$\begin{aligned} b_t &= (b_0 - b^*)\left(\frac{1+r}{1+g_Y}\right)^t + b^*, \quad \text{where} && (6.8) \\ b^* &= -\frac{\tau-\gamma}{1+g_Y}\left(1-\frac{1+r}{1+g_Y}\right)^{-1} = \frac{\tau-\gamma}{r-g_Y} \equiv \frac{s}{r-g_Y}, && (6.9) \end{aligned}$$

where s is the *primary surplus as a share of GDP*. Here b_0 is historically given. But the steady-state debt-income ratio, b^*, depends on fiscal policy. The important feature is that the growth-corrected interest factor is in this case higher than 1 and has the exponent t. Therefore, if fiscal policy is such that $b^* < b_0$, the debt-income ratio exhibits geometric growth. The solid curve in the topmost panel in Fig. 6.2 shows a case where fiscal policy is such that $\tau - \gamma < (r - g_Y)b_0$ whereby we get $b^* < b_0$ when $r > g_Y$, so that the debt-income ratio, b_t, grows without bound. This reflects that with $r > g_Y$, compound interest is stronger than compound growth. The sequence of discrete points implied by our discrete-time model is in the figure smoothed out as a continuous curve.

The American economist and Nobel Prize laureate George Akerlof (2004, p. 6) came up with this analogy:

> "It takes some time after running off the cliff before you begin to fall. But the law of gravity works, and that fall is a certainty".

Somewhat surprisingly, perhaps, when $r > g_Y$, there can be debt explosion in the long run even if $\tau > \gamma$, namely if $0 < \tau - \gamma < (r - g_Y)b_0$. Debt explosion can also arise if $b_0 < 0$, namely if $\tau - \gamma < (r - g_Y)b_0 < 0$.

The only way to avoid the snowball effects of compound interest when the growth-corrected interest rate is positive is to ensure a primary budget surplus as a share of GDP, $\tau - \gamma$, high enough such that $b^* \geq b_0$. So the *minimum* primary surplus as a share of GDP, $\hat{s}$, required for fiscal sustainability is the one implying $b^* = b_0$, i.e., by (6.9),

$$\hat{s} = (r - g_Y)b_0. \tag{6.10}$$

If by adjusting τ and/or γ, the government obtains $\tau - \gamma = \hat{s}$, then $b^* = b_0$ whereby $b_t = b_0$ for all $t \geq 0$ according to (6.8), cf. the second from the top panel in Fig. 6.2. The difference between $\hat{s}$ and the actual primary surplus as a share of GDP is named the *primary surplus gap* or the *sustainability gap*.

Note that $\hat{s}$ will be larger:

- the higher is the initial level of debt, b_0; and,
- when $b_0 > 0$, the higher is the growth-corrected interest rate, $r - g_Y$.

Delaying the adjustment increases the size of the needed policy action, since the debt-income ratio, and thereby $\hat{s}$, will become higher in the meantime.

For fixed spending-income ratio γ, the minimum tax-to-income ratio needed for fiscal sustainability is

$$\hat{\tau} = \gamma + (r - g_Y)b_0. \tag{6.11}$$

Given b_0 and γ, this tax-to-income ratio is sometimes called the *sustainable tax rate*. The difference between this rate and the actual tax rate, τ, indicates the size of the needed tax adjustment, were it to take place at time 0, assuming a given γ.

Suppose that the debt build-up can be − and is − prevented already at time 0 by ensuring that the primary surplus as a share of income, $\tau - \gamma$, at least equals $\hat{s}$ so that $b^* \geq b_0$. The solid curve in the midmost panel in Fig. 6.2 illustrates the resulting evolution of the debt-income ratio if b^* is at the level corresponding to the hatched horizontal line while b_0 is unchanged compared with the top panel. Presumably, the government would in such a state of affairs relax its fiscal policy after a while in order not to accumulate large government financial net wealth. Yet, the pre-funding strategy vis-a-vis the fiscal challenge of population ageing (referred to above) is in fact based on accumulating some positive public financial net wealth as a buffer before the substantial effects of population ageing set in. In

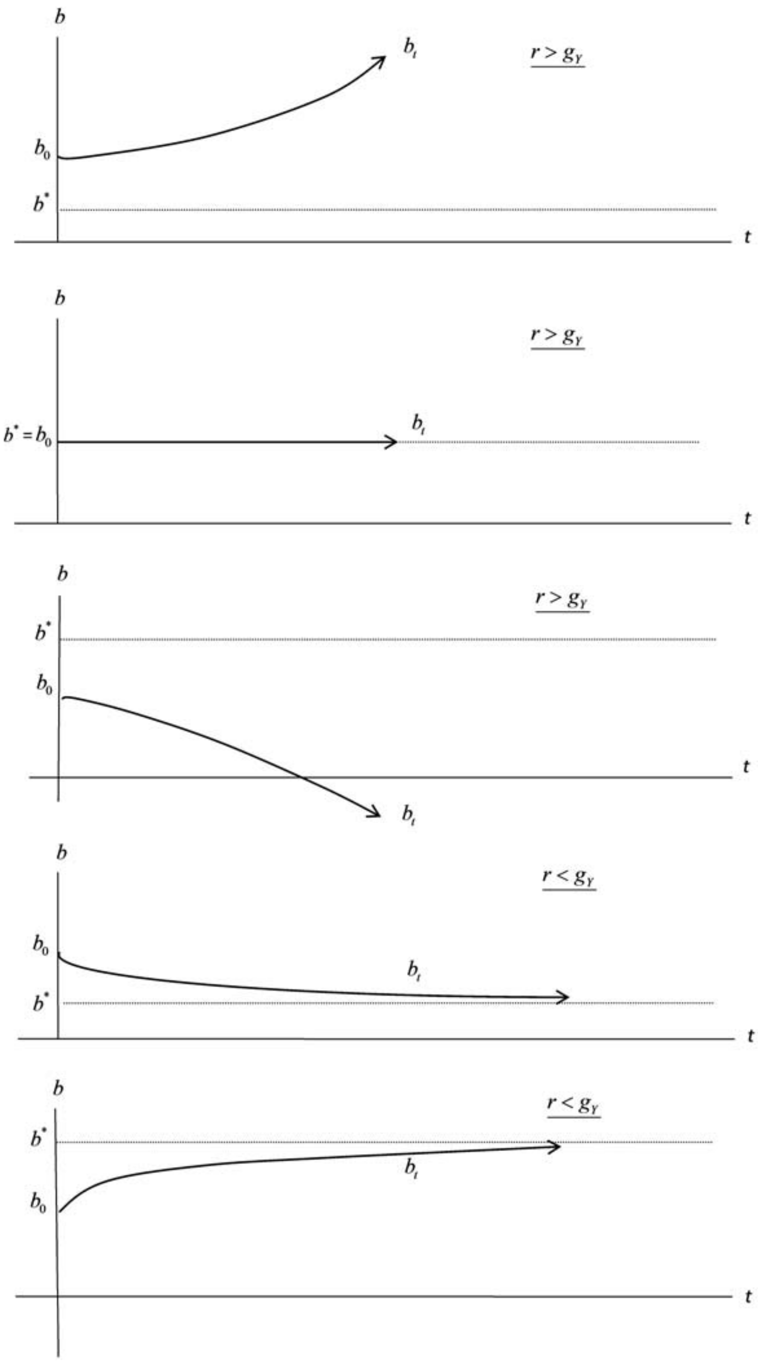

Figure 6.2: Evolution of the debt-income ratio, depending on the sign of $b_0 - b^*$, in the cases $r > g_Y$ (the three upper panels) and $r < g_Y$ (the two lower panels), respectively.

this context, the higher the growth-corrected interest rate, the shorter the time needed to reach a given positive net wealth position.

Case 2: $r = g_Y$. In this knife-edge case there is still a danger of runaway dynamics, but of a less explosive form. The formula (6.8) is no longer valid. Instead the solution of (6.7) is $b_t = b_0 + [(\gamma - \tau)/(1 + g_Y)]\, t = b_0 - [(\tau - \gamma)/(1 + g_Y)]\, t$. Here, a non-negative primary surplus is both necessary and sufficient to avoid $b_t \to \infty$ for $t \to \infty$.

Case 3: $r < g_Y$. This is the case of stable debt dynamics. The formula (6.8) is again valid, but now implying that the debt-income ratio is non-explosive. Indeed, $b_t \to b^*$ for $t \to \infty$, whatever the level of the initial debt-income ratio and whatever the sign of the budget surplus. Moreover, when $r < g_Y$,

$$b^* = \frac{\tau - \gamma}{r - g_Y} \lesseqqgtr 0 \text{ for } \tau - \gamma \gtreqqless 0. \tag{*}$$

So, if there is a forever positive primary surplus, the result is a negative long-run debt, i.e., a positive government financial net wealth in the long run. And if there is a forever negative primary surplus, the result is not debt explosion but just convergence toward some positive long-run debt-income ratio. The second from bottom panel in Fig. 6.2 illustrates this case for a situation where $b_0 > b^*$ and $b^* > 0$, i.e., $\tau - \gamma < 0$, by (*). When the GDP growth rate continues to exceed the interest rate on government debt, a large debt-income ratio can be brought down quite fast, as witnessed by the evolution of both UK and US government debt in the first three decades after the second world war. Indeed, if the growth-corrected interest rate remains negative, permanent debt roll-over can handle the financing, and taxes need never be levied.[8]

Finally, the bottom panel in Fig. 6.2 shows the case where, with a *large* primary deficit ($\tau - \gamma < 0$ but large in absolute value), excess of output growth over the interest rate still implies convergence towards a constant debt-income ratio, albeit a high one.

In this discussion we have treated r as exogenous. But r may to some extent be dependent on prolonged budget deficits. Indeed, in Chapter 13 we shall see that with prolonged budget deficits, r tends to become higher than otherwise. Everything else equal, this reduces the likelihood of Case 2 and Case 3.

Laffer curve*

We return to Case 1 because we have ignored supply-side effects of taxation, and such effects could be important in Case 1.

[8] On the other hand, we should not forget that this analysis presupposes absence of uncertainty. As touched on in Section 6.3.1, in the presence of uncertainty and therefore existence of many interest rates, the issue becomes more complicated.

A *Laffer curve* (so named after the American economist Arthur Laffer, 1940-) refers to a hump-shaped relationship between the income tax rate and the tax revenue. For simplicity, suppose the (gross) tax revenue equals taxable income times a given average tax rate. A 0% tax rate and likely also a 100% tax rate generate no tax revenue. As the tax rate increases from a low initial level, a rising tax revenue is obtained. But after a certain point some people may begin to work less (in the legal economy), stop reporting all their income, and stop investing. So it is reasonable to think of a tax rate above which the tax revenue begins to decline.

While Laffer was wrong about where USA was "on the curve" (see, e.g., Fullerton 2008), and while, strictly speaking, there is no such thing as *the* Laffer curve and *the* tax rate,[9] Laffer's intuition is hardly controversial. Ignoring, for simplicity, transfers, we therefore now assume that for a *given* tax *system* there is a gross tax-income ratio, τ_L, above which the tax revenue declines. Then, if the presumed sustainable tax-income ratio, $\hat{\tau}$, in (6.11) exceeds τ_L, the tax revenue aimed at can not be realized.

To see what the value of τ_L could be, suppose aggregate taxable income before tax is a function, φ, of the net-of-tax share $1-\tau$. Then tax revenue is

$$\tilde{T} = \tau \cdot \varphi(1-\tau) \equiv R(\tau) \ ,$$

which we assume is a hump-shaped function of τ in the interval $[0,1]$. Taking logs and differentiating w.r.t. τ gives the first-order condition $R'(\tau)/R(\tau) = 1/\tau - \varphi'(1-\tau)/\varphi(1-\tau) = 0$, which holds for $\tau = \tau_L$, the tax-income ratio that maximizes R. It follows that $1/\tau_L = \varphi'(1-\tau_L)/\varphi(1-\tau_L)$, hence

$$\frac{1-\tau_L}{\tau_L} = \frac{1-\tau_L}{\varphi(1-\tau_L)}\varphi'(1-\tau_L) \equiv \mathrm{E}\ell_{1-\tau}\varphi(1-\tau_L).$$

Rearranging gives

$$\tau_L = \frac{1}{1+\mathrm{E}\ell_{1-\tau}\varphi(1-\tau_L)}.$$

If the elasticity of income w.r.t. $1-\tau$ is given as 0.4,[10] we get $\tau_L \simeq 0.7$. Thus, if the required tax-income ratio, $\hat{\tau}$, calculated on the basis of (6.11) (under the simplifying assumption of no transfers), exceeds 0.7, fiscal sustainability can not be obtained by just raising taxation.

[9] A lot of contingencies are involved: income taxes are typically progressive (i.e., average tax rates rise with income); it matters whether a part of tax revenue is spent to reduce tax evasion, etc.

[10] As suggested for the U.S. by Gruber and Saez (2002).

The level of the debt-income ratio and self-fulfilling expectations of default

We again consider Case 1: $r > g_Y$. As incumbent chief economist at the IMF, Olivier Blanchard remarked in the midst of the 2010-2012 debt crisis in the Eurozone:

> "The higher the level of debt, the smaller is the distance between solvency and default".[11]

The background for this remark is the following. There is likely to be an upper bound for the tax-income ratio deemed politically or economically feasible by the government as well as the market participants. Similarly, a lower bound for the spending-income ratio is likely to exist, be it for economic or political reasons. In the present framework we therefore let the government face the constraints $\tau \leq \bar{\tau}$ and $\gamma \geq \bar{\gamma}$, where $\bar{\tau}$ is the least upper bound for the tax-income ratio and $\bar{\gamma}$ is the greatest lower bound for the spending-income ratio. We assume that $\bar{\tau} > \bar{\gamma}$. Then the actual primary surplus, s, can at most equal $\bar{s} \equiv \bar{\tau} - \bar{\gamma}$.

Suppose that at first the situation in the considered country is as in the second from the top panel in Fig. 6.2. That is, initially, $b_0 > 0$ and

$$s = \tau - \gamma = \hat{s} = (r - g_Y)b_0 \leq \bar{s} \equiv \bar{\tau} - \bar{\gamma}, \tag{6.12}$$

with $b_0 > 0$. Define $\bar{r}$ to be the value of r satisfying

$$(\bar{r} - g_Y)b_0 = \bar{s}, \text{ i.e., } \bar{r} = \frac{\bar{s}}{b_0} + g_Y. \tag{6.13}$$

Thereby $\bar{r}$ is the maximum level of the interest rate consistent with absence of an explosive debt-income ratio.

According to (6.12), fundamentals (tax- and spending-income ratios, growth-corrected interest rate, and initial debt) are consistent with absence of an explosive debt-income ratio as long as r is unchanged. Nevertheless, financial investors may be worried about default if b_0 is high. Investors are aware that a rise in the actual interest rate, r, can always happen and that if it does, a situation with $r > \bar{r}$ is looming, in particular if the country has high debt. The larger is b_0, the lower is the critical interest rate, $\bar{r}$, as witnessed by (6.13).

The worrying scenario is that the fear of default triggers a risk premium, and if the resulting level of the interest rate on the debt, say r', exceeds $\bar{r}$, unpleasant debt dynamics like that in the top panel of Fig. 6.2 set in. To r' corresponds a new value of the primary surplus, say $\hat{s}'$, defined by $\hat{s}' = (r' - g_Y)b_0$. So $\hat{s}'$ is the

[11] Blanchard (2011).

minimum primary surplus (as a share of GDP) required for a non-accelerating debt-income ratio in the new situation. With $b_0 > 0$ and $r' > \bar{r}$, we get

$$\hat{s}' = (r' - g_Y)b_0 > (\bar{r} - g_Y)b_0 = \bar{s},$$

where $\bar{s}$ is given in (6.12). The government could possibly increase its primary surplus, s, but at most up to $\bar{s}$, and this will not be enough since the required primary surplus, $\hat{s}'$, exceeds $\bar{s}$. The situation would be as illustrated in the top panel of Fig. 6.2 with b^* given as $\bar{s}/(r' - g_Y) < b_0$.

That is, *if* the actual interest rate should rise above the critical interest rate, $\bar{r}$, runaway debt dynamics would take off and debt default follow. A fear that it *may* happen may be enough to trigger a fall in the market price of government bonds which means a rise in the actual interest rate, r. So financial investors' fear can be a self-fulfilling prophesy. Moreover, as we saw in connection with (6.13), the risk that r becomes greater than $\bar{r}$ is larger the larger is b_0.

It is not so that across countries there is a common threshold value for a "too large" public debt-to-income ratio. This is because variables like $\bar{\tau}$, $\bar{\gamma}$, r, and g_Y, as well as the net foreign debt position and the current account deficit (not in focus in this chapter), *differ* across countries. Late 2010 Greece had (gross) government debt of 148 percent of GDP and the interest rate on 10-year government bonds skyrocketed. Conversely Japan had (gross) government debt of more than 200 percent of GDP while the interest rate on 10-year government bonds remained very low.

Finer shades

1. As we have just seen, even when in a longer-run perspective a solvency problem is unlikely, self-fulfilling expectations can here and now lead to default. Such a situation is known as a *liquidity crisis* rather than a true *solvency crisis.* In a liquidity crisis there is an acute problem of insufficient cash to pay the next bill on time ("cash-flow insolvency") because borrowing is difficult due to actual and potential creditors' *fear* of default. A liquidity crisis can be braked by the central bank stepping in and acting as a "lender of last resort" by printing money. In a country with its own currency, the central bank can do so and thereby prevent a bad self-fulfilling expectations equilibrium to unfold.[12]

[12] In a monetary union which is not also a fiscal union (think of the eurozone), the situation is more complicated. A single member country with large government debt (or large debt in commercial banks for that matter) may find itself in an acute liquidity crisis without its own means to solve it. Indeed, the elevation of interest rates on government bonds in the Southern part of the eurozone in 2010-2012 can be seen as a manifestation of investors' fear of the governments running into difficulties of paying their way. The elevation was not reversed until the European Central Bank in September 2012 declared its willingness to effectively act as a "lender of last resort" (on a conditional basis), see Box 6.2 in Section 6.4.2.

2. In the above analysis we simplified by assuming that several variables, including γ, τ, and r, are constants. The upward trend in the old-age dependency ratio, due to a decreased birth rate and rising life expectancy, together with a rising request for medical care is likely to generate upward pressure on γ. Thereby a high initial debt-income ratio becomes *more* challenging.

3. On the other hand, rB_t is income to the private sector and can be taxed at the same average tax rate τ as factor income, Y_t. Then the benign inequality is no longer $r \leq g_Y$ but $(1-\tau)r \leq g_Y$, which is more likely to hold. Taxing interest income is thus supportive of fiscal sustainability (cf. Exercise B.28).

4. Having ignored seigniorage, there is an upward bias in our measure (6.10) of the *minimum* primary surplus as a share of GDP, $\hat{s}$, required for fiscal sustainability when $r > g_Y$. Imposing stationarity of the debt-income ratio at the level $\bar{b}$ into the general debt-accumulation formula (6.5), multiplying through by $1+g_Y$, and cancelling out, we find

$$\hat{s} = (r - g_Y)\bar{b} - \frac{\Delta M_{t+1}}{P_t Y_t} = (r - g_Y)\bar{b} - \frac{\Delta M_{t+1}}{M_t} \cdot \frac{M_t}{P_t Y_t}.$$

With $r = 0.04$, $g_Y = 0.03$, and $\bar{b} = 0.60$, we get $(r - g_Y)\bar{b} = 0.006$. With a seigniorage-income ratio even as small as 0.003, the "true" required primary surplus is 0.003 rather than 0.006. As long as the seigniorage-income ratio is approximately constant, our original formula, given in (6.10), for the required primary surplus as a share of GDP is in fact valid if we interpret τ as the (tax+seigniorage)-income ratio.

5. Having assumed a constant g_Y, we have ignored business cycle fluctuations. Allowing for booms and recessions, the *timing* of fiscal consolidation in a country with a structural primary surplus gap ($\hat{s} - s > 0$) becomes a crucial issue. The case study in the next section will be an opportunity to touch upon this issue.

6.4.2 Case study: The Stability and Growth Pact of the EMU

The European Union (EU) is approaching its aim of establishing a "single market" (unrestricted movement of goods and services, workers, and financial capital) across the territory of its member countries, 28 sovereign nations. Nineteen of these have joined the common currency, the euro. They constitute what is known as the Eurozone with the European Central Bank (ECB) as supranational institution responsible for conducting monetary policy in the Eurozone. The Eurozone countries as well as the nine EU countries outside the Eurozone (including UK, Denmark, Sweden, and Poland) are, with minor exceptions, required to abide with a set of *fiscal rules*, first formulated already in the Treaty of Maastrict from

1992. In that year a group of European countries decided a road map leading to the establishment of the euro in 1999 and a set of criteria for countries to join. These fiscal rules included a deficit rule as well as a debt rule. The *deficit rule* says that the annual nominal government budget deficit must not be above 3 percent of nominal GDP. The *debt rule* says that the government debt should not be above 60 percent of GDP. The fiscal rules were upheld and in minor respects tightened in the *Stability and Growth Pact* (SGP) which was implemented in 1997 as the key fiscal constituent of the Economic and Monetary Union (EMU). The latter name is a popular umbrella term for the fiscal and monetary legislation of the EU. The EU member countries that have adopted the euro are often referred to as "the full members of the EMU".

Some of the EU member states (Belgium, Italy, and Greece) had debt-income ratios above 100 percent since the early 1990s − and still have. Committing to the requirement of a gradual reduction of their debt-income ratios, they became full members of the EMU essentially from the beginning (that is, 1999 except Greece, 2001). The 60 percent debt rule of the SGP is to be understood as a long-run ceiling that, by the stock nature of debt, can not be accomplished here and now if the country is highly indebted.

The deficit and debt rules (with associated detailed contingencies and arrangements including ultimate pecuniary fines for defiance) are meant as discipline devices aiming at "sound budgetary policy", alternatively called "fiscal prudence". The motivation is protection of the ECB against political demands to loosen monetary policy in situations of fiscal distress. A fiscal crisis in one or more of the Eurozone countries, perhaps "too big to fail", could set in and entail a state of affairs approaching default on government debt and chaos in the banking sector with rising interest rates spreading to neighboring member countries (a negative externality). This could lead to open or concealed political pressure on the ECB to inflate away the real value of the debt, thus challenging the ECB's one and only concern with "price stability".[13] Or a fiscal crisis might at least result in demands on the ECB to curb soaring interest rates by purchasing government bonds from the country in trouble. In fact, such a scenario is close to what we have seen in southern Europe in the wake of the Great Recession triggered by the financial crisis starting 2007. Such "bailing out" could give governments incentives to be relaxed about deficits and debts (a "moral hazard" problem). And the lid on deficit spending imposed by the SGP should help to prevent needs for "bailing out" to arise.

[13] In recent years the ECB has interpreted "price stability" as a consumer price inflation rate "below, but close to, 2 percent per year over the medium term".

The link between the deficit and the debt rule

Whatever the virtues or vices of the design of the deficit and debt rules, one may ask the plain question: what is the arithmetical relationship, if any, between the 3 percent and 60 percent tenets?

First a remark about measurement. The measure of government debt, called the EMU debt, used in the SGP criterion is based on the book value of the financial liabilities rather than the market value. In addition, the EMU debt is more of a *gross* nature than the theoretical net debt measure represented by our D. The EMU debt measure allows fewer of the government financial assets to be subtracted from the government financial liabilities.[14] In our calculation and subsequent discussion we ignore these complications.

Consider a deficit rule saying that the (total) nominal budget deficit must never be above $\alpha \cdot 100$ percent of nominal GDP. By (6.3) with ΔM_{t+1} "small" enough to be ignored, this deficit rule is equivalent to the requirement

$$D_{t+1} - D_t = GBD_t = i_t D_t + P_t(G_t - T_t) \leq \alpha P_t Y_t. \tag{6.14}$$

In the SGP, $\alpha = 0.03$. Here we consider the general case: $\alpha > 0$. To see the implication for the (public) debt-to-income ratio in the long run, let us first imagine a situation where the deficit ceiling, α, is always *binding* for the economy we look at. Then $D_{t+1} = D_t + \alpha P_t Y_t$ and so

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} \equiv \frac{D_{t+1}}{P_t Y_{t+1}} = \frac{D_t}{(1+\pi)P_{t-1}(1+g_Y)Y_t} + \frac{\alpha}{1+g_Y},$$

assuming constant output growth rate, g_Y, and inflation rate π. This reduces to

$$b_{t+1} = \frac{1}{(1+\pi)(1+g_Y)} b_t + \frac{\alpha}{1+g_Y}. \tag{6.15}$$

Assuming that $(1+\pi)(1+g_Y) > 1$ (as is normal over the medium run), this linear difference equation has the stable solution

$$b_t = (b_0 - b^*)\left(\frac{1}{(1+\pi)(1+g_Y)}\right)^t + b^* \to b^* \text{ for } t \to \infty, \tag{6.16}$$

where

$$b^* = \frac{(1+\pi)}{(1+\pi)(1+g_Y) - 1}\alpha. \tag{6.17}$$

[14] For instance for Denmark the difference between the EMU and the net debt is substantial. In 2013 the Danish EMU debt was 44.6% of GDP while the government net debt was 5.5% of GDP (Danish Ministry of Finance, 2014).

Consequently, if the deficit rule (6.14) is always binding, the debt-income ratio tends in the long run to be proportional to the deficit bound α. The factor of proportionality is a decreasing function of the long-run growth rate of real GDP and the inflation rate. This result confirms the general tenet that if there is economic growth, perpetual budget deficits need not lead to fiscal problems.

If on the other hand the deficit rule is *not* always binding, then the budget deficit is on average smaller than above so that the debt-income ratio will in the long run be *smaller* than b^*.

The conclusion is the following. With one year as the time unit, suppose the deficit rule has $\alpha = 0.03$ and that $g_Y = 0.03$ (which by the architects of the Maastrict Treaty was considered the "natural" GDP growth rate) and $\pi = 0.02$ (which is the upper end of the inflation interval aimed at by the ECB). Suppose further the deficit rule is never violated. Then in the long run the debt-income ratio will be *at most* $b^* = 1.02 \times 0.03/(1.02 \times 1.03 - 1) \approx 0.60$. This is in agreement with the debt rule of the SGP according to which the maximum value allowed for the debt-income ratio is 60%.

Although there is nothing sacred about either of the numbers 0.60 or 0.03, they are mutually consistent, given $\pi = 0.02$ and $g_Y = 0.03$.

We observe that the deficit rule (6.14) implies that:

- The upper bound, b^*, on the long-run debt income ratio is lower the higher is inflation. The reason is that the growth factor $\beta \equiv [(1 + \pi) \;\; (1 + g_Y)]^{-1}$ for b_t in (6.15) depends negatively on the inflation rate, π. So does therefore b^* since, by (6.16), $b^* \equiv \alpha(1 + g_Y)^{-1}(1 - \beta)^{-1}$.

- For a *given* π, the upper bound on the long-run debt income ratio is *independent* of both the nominal and real interest rate (this follows from the indicated formula for the growth factor for b_t and the fact that $(1+i)(1+r)^{-1} = 1 + \pi$).

The debate about the design of the SGP

In addition to the aimed long-run implications, by its design the SGP has short-run implications for the economy. Hence an evaluation of the SGP cannot ignore the way the economy functions in the short run. How changes in government spending and taxation affects the economy depends on the "state of the business cycle": is the economy in a boom with full capacity utilization or in a slump with slack aggregate demand?

Much of the debate about the SGP has centered around the consequences of the deficit rule in an economic recession triggered by a collapse of aggregate demand (for instance due to private deleveraging in the wake of a banking crisis).

Although the Eurozone countries are economically quite different, they are subject to the same one-size-fits-all monetary policy. Facing dissimilar shocks, the single member countries in need of aggregate demand stimulation in a recession have by joining the euro renounced on both interest rate policy and currency depreciation.[15] The only policy tool left for demand stimulation is therefore fiscal policy. Instead of a supranational fiscal authority responsible for handling the problem, it is up to the individual member countries to act − and to do so within the constraints of the SGP.

On this background, the critiques of the deficit rule of the SGP include the following points. (It may here be useful to have at the back of one's mind the simple Keynesian income-expenditure model, where output is below capacity and demand-determined whereas the general price level is sticky.)

Critiques 1. When considering the need for fiscal stimuli in a recession, a ceiling at 0.03 is too low unless the country has almost no government debt in advance. Such a deficit rule gives too little scope for counter-cyclical fiscal policy, including the free working of the *automatic fiscal stabilizers* (i.e., the provisions, through tax and transfer codes, in the government budget that automatically cause tax revenues to fall and spending to rise when GDP falls).[16] As an economy moves towards recession, the deficit rule may, bizarrely, force the government to tighten fiscal policy although the situation calls for stimulation of aggregate demand. The pact has therefore sometimes been called the "Instability and Depression Pact" − it imposes a *wrong timing* of fiscal consolidation.[17]

2. Since what really matters is long-run fiscal sustainability, a deficit rule should be designed in a more flexible way than the 3% rule of the SGP. A meaningful deficit rule would relate the deficit to the *trend* nominal GDP, which we may denote $(PY)^*$. Such a criterion would imply

$$GBD \leq \alpha(PY)^*. \tag{6.18}$$

[15] Denmark is in a similar situation. In spite of not joining the euro after the referendum in 2000, the Danish krone has been linked to the euro through a fixed exchange rate since 1999.

[16] Over the first 13 years of existence of the euro even Germany violated the 3 percent rule five of the years.

[17] The SGP has an exemption clause referring to "exceptional" circumstances. These circumstances were originally defined as "severe economic recession", interpreted as an annual fall in real GDP of at least 1-2%. By the reform of the SGP in March 2005, the interpretation was changed into simply "negative growth". Owing to the international economic crisis that broke out in 2008, the deficit rule was thus suspended in 2009 and 2010 for most of the EMU countries. But the European Commission brought the rule into effect again from 2011, which according to many critics was much too early, given the circumstances.

Then

$$\frac{GBD}{PY} \leq \alpha \frac{(PY)^*}{PY}.$$

In recessions the ratio $(PY)^*/(PY)$ is high, in booms it is low. This has the advantage of allowing more room for budget deficits when they are needed − without interfering with the long-run aim of stabilizing government debt below some specified ceiling.

3. A further step in this direction is a rule directly in terms of the *structural* or *cyclically adjusted* budget deficit rather than the actual year-by-year deficit. The cyclically adjusted budget deficit in a given year is defined as the value the deficit would take in case actual output were equal to trend output in that year. Denoting the cyclically adjusted budget deficit GBD^*, the rule would be

$$\frac{GBD^*}{(PY)^*} \leq \alpha.$$

In fact, in its original version as of 1997 the SGP contained an *additional* rule like that, but in the very strict form of $\alpha \approx 0$. This requirement was implicit in the directive that the cyclically adjusted budget "should be close to balance or in surplus". By this requirement it is imposed that the debt-income ratio should be close to zero in the long run. Many EMU countries certainly had − and have − larger cyclically adjusted deficits. Taking steps to comply with such a low structural deficit ceiling may be hard and endanger national welfare by getting in the way of key tasks of the public sector. The minor reform of the SGP endorsed in March 2005 allowed more contingencies, also concerning this structural bound. By the more recent reform in 2012, the Fiscal Pact, the lid on the cyclically adjusted deficit-income ratio was raised to 0.5% and to 1.0% for members with a debt-income ratio "significantly below 60%". These are still quite small numbers. Abiding by the 0.5% or 1.0% rule implies a long-run debt-income ratio of at most 10% or 20%, respectively, given structural inflation and structural GDP growth at 2% and 3% per year, respectively.[18]

4. Regarding the *composition* of government expenditure, critics have argued that the SGP pact entails a problematic disincentive for public investment. The view is that a fiscal rule should be based on a proper accounting of public investment instead of simply ignoring the composition of government expenditure. We consider this issue in Section 6.6 below.

5. At a more general level critics have contended that policy rules and surveillance procedures imposed on sovereign nations will hardly be able to do their job unless they encompass stronger incentive-compatible elements. Enforcement mechanisms are bound to be week. The SGP's threat of pecuniary fines to a

[18] Again apply (6.17).

country which during a recession has difficulties to reduce its budget deficit lacks credibility and has, at the time of writing (June 2015), not been made use of so far. Moreover, abiding by the fiscal rules of the SGP prior to the Great Recession was certainly no guarantee of not ending up in a fiscal crisis in the wake of a crisis in the banking sector, as witnessed by Ireland and Spain. A seemingly strong fiscal position can vaporize fast, particularly if banks, "too big to fail", need be bailed out.

Counter-arguments Among the counter-arguments raised against the criticisms of the SGP has been that the potential benefits of the proposed alternative rules are more than offset by the costs in terms of reduced simplicity, measurability, and transparency. The lack of flexibility may even be a good thing because it helps "tying the hands of elected policy makers". Tight rules are needed because of a *deficit bias* arising from short-sighted policy makers' temptation to promise spending without ensuring the needed financing, especially before an upcoming election. These points are sometimes linked to the view that market economies are generally self-regulating: Keynesian stabilization policy is not needed and may do more harm than good.

Box 6.1. The 2010-2012 debt crisis in the Eurozone

What began as a banking crisis became a deep economic recession combined with a government debt crisis.

At the end of 2009, in the aftermath of the global economic downturn, it became evident that Greece faced an acute debt crisis driven by three factors: high government debt, low ability to collect taxes, and lack of competitiveness due to cost inflation. Anxiety broke out about the debt crisis spilling over to Spain, Portugal, Italy, and Ireland, thus widening bond yield spreads in these countries vis-a-vis Germany in the midst of a serious economic recession. Moreover, the solvency of big German and French banks that were among the prime creditors of Greece was endangered. The major Eurozone governments and the International Monetary Fund (IMF) reached an agreement to help Greece (and thereby its creditors) with loans and guarantees for loans, conditional on the government of Greece imposing yet another round of harsh fiscal austerity measures. The elevated bond interest rates of Greece, Italy, and Spain were not convincingly curbed, however, until in August-September 2012 the president of the ECB, Mario Draghi, launched the "Outright Monetary Transactions" (OMT) program according to which, under certain conditions, the ECB will buy government bonds in the secondary market with the aim of "safeguarding an appropriate monetary policy transmission and the singleness of the monetary policy" and with "no ex ante quantitative limits". Considerably reduced government bond spreads followed and so the sheer announcement of the program seemed effective in its own right. Doubts raised by the German Constitutional Court about its legality vis-à-vis Treaties of the European Union were finally repudiated by the European Court of Justice mid-June 2015. At the time of writing (late June 2015) the OMT program has not been used in practice. Early 2015, a different massive program for purchases of government bonds, including long-term bonds, in the secondary market as well as private asset-backed bonds was decided and implemented by the ECB. The declared aim was to brake threatening deflation and return to "price stability", by which is meant inflation close to 2 percent per year.

So much about the monetary policy response. What about fiscal policy? On the basis of the SGP, the EU Commission imposed "fiscal consolidation" initiatives to be carried out in most EU countries in the period 2011-2013 (some of the countries were required to start already in 2010). With what consequences? By many observers, partly including the research department of the IMF, the initiatives were judged self-defeating. When at the same time comprehensive deleveraging in the private sector is going on, "austerity" policy deteriorates aggregate demand further and raises unemployment. Thereby, instead of budget deficits being decreased, it is the denominator of the debt-income ratio, $D/(PY)$, that is decreased. Fiscal multipliers are judged to be large ("in the 0.9 to 1.7 range since the Great Recession", according to IMF's *World Economic Outlook*, Oct. 2012) in a situation of idle resources where monetary policy aims at low interest rates; and negative spillover effects through trade linkages when "fiscal consolidation" is synchronized across countries. The unemployment rate in the Eurozone countries was elevated from 7.5 percent in 2008 to 12 percent in 2013. The British economists, Holland and Portes (2012), concluded: "It is ironic that, given that the EU was set up in part to avoid coordination failures in economic policy, it should deliver the exact opposite".

The whole crisis has pointed to a basic difficulty faced by the Eurozone. In spite of the member countries being economically very different sovereign nations, they are subordinate to the same one-size-fits-all monetary policy without sharing a federal government ready to use fiscal instruments to mitigate regional consequences of country-specific shocks. Adverse demand shocks may lead to sharply rising budget deficits in some countries, and financial investors may loose confidence and so elevate government bond interest rates. A liquidity crisis may arise, thereby amplifying adverse shocks. Even when a common negative demand shock hits all the member countries in a similar way, and a general relaxation of both monetary and fiscal policy is called for, there is the problem that the individual countries, in fear of boosting their budget deficit and facing the risk of exceeding the deficit or debt limit, may wait for the others to initiate fiscal expansion. The possible consequence of this "free rider" problem is general under-stimulation of the economies.

The dismal experience regarding the ability of the Eurozone to handle the Great Recession has incited proposals along at least two dimensions. One dimension is about allowing the ECB greater scope for acting as a "lender of last resort". The other dimension is about centralizing a larger part of the national budgets into a common union budget (see, e.g., De Grauwe, 2014). (END OF BOX)

6.5 Solvency, the NPG condition, and the intertemporal government budget constraint

Up to now we have considered the issue of government solvency from the perspective of dynamics of the government debt-to-income ratio. It is sometimes useful to view government solvency from another angle – the intertemporal budget constraint (GIBC). Under a certain condition stated below, the intertemporal budget constraint is, essentially, as relevant for a government as for private agents.

A simple condition closely linked to whether the government's intertemporal budget constraint is satisfied or not is what is known as the government's No-Ponzi-Game (NPG) condition. It is convenient to first focus on this condition. We concentrate on government *net* debt, measured in *real* terms, and ignore seigniorage.

6.5.1 When is the NPG condition necessary for solvency?

Consider a situation with a given constant interest rate, r. Suppose taxes are lump sum or at least that there is no tax on interest income from owning government bonds. Then the government's *NPG condition* is that the present discounted value of the public debt in the far future is not positive, i.e.,

$$\lim_{t\to\infty} B_t(1+r)^{-t} \leq 0. \tag{NPG}$$

This condition says that government debt is not allowed to grow in the long run at a rate as high as (or even higher than) the interest rate.[19] That is, a fiscal policy satisfying the NPG condition rules out a permanent debt rollover. Indeed, as we saw in Section 6.3.1, with $B_0 > 0$, a permanent debt rollover policy (financing all interest payments and perhaps even also part of the primary government spending) by debt issue leads to $B_t \geq B_0(1+r)^t$ for $t = 0, 1, 2, \ldots$. Substituting into (NPG) gives $\lim_{t\to\infty} B_t \geq B_0(1+r)^t(1+r)^{-t} = B_0 > 0$, thus violating (NPG).

The designation No-Ponzi-Game condition refers to a guy from Boston, Charles Ponzi, who in the 1920s made a fortune out of an investment scam based on the

[19] If there is effective taxation of interest income at the rate $\tau_r \in (0,1)$, then the after-tax interest rate, $(1-\tau_r)r$, is the relevant discount rate, and the NPG condition would read $\lim_{t\to\infty} B_t\left[1+(1-\tau_r)r\right]^{-t} \leq 0$.

chain-letter principle. The principle is to pay off old investors with money from new investors, keeping the remainder of that money to oneself. Ponzi was sentenced to many years in prison for his transactions; he died poor − and without friends!

To our knowledge, this kind of financing behavior is nowhere forbidden for the government as it generally is for private agents. But under "normal" circumstances a government *has* to plan its expenditures and taxation so as to comply with its NPG condition since otherwise not enough lenders will be forthcoming.

As the state is in principle infinitely-lived, however, there is no final date where all government debt should be over and done with. Indeed, the NPG condition does not even require that the debt has ultimately to be non-increasing. The NPG condition "only" says that the debtor, here the government, can not let the debt grow forever at a rate as high as (or higher than) the interest rate. For instance the U.K. as well as the U.S. governments have had positive debt for centuries − and high debt after both WW I and WW II.

Suppose Y (GDP) grows at the given constant rate g_Y (actually, for most of the following results it is enough that $\lim_{t\to\infty} Y_{t+1}/Y_t = 1 + g_Y$). We have:

PROPOSITION 1 Interpret "solvency" as absence of an for ever accelerating debt-income ratio, $b_t \equiv B_t/Y_t$. Then:

(i) if $r > g_Y$, solvency requires (NPG) satisfied;

(ii) if $r \leq g_Y$, the government can remain solvent without (NPG) being satisfied.

Proof. When $b_t \neq 0$,

$$\lim_{t\to\infty} \frac{b_{t+1}}{b_t} \equiv \lim_{t\to\infty} \frac{B_{t+1}/Y_{t+1}}{B_t/Y_t} = \lim_{t\to\infty} \frac{B_{t+1}/B_t}{Y_{t+1}/Y_t} = \lim_{t\to\infty} \frac{B_{t+1}/B_t}{1+g_Y}. \tag{6.19}$$

Case (i): $r > g_Y$. If $\lim_{t\to\infty} B_t \leq 0$, then (NPG) is trivially satisfied. Assume $\lim_{t\to\infty} B_t > 0$. For this situation we prove the statement by contradiction. Suppose (NPG) is not satisfied. Then, $\lim_{t\to\infty} B_t(1+r)^{-t} > 0$, implying that $\lim_{t\to\infty} B_{t+1}/B_t \geq 1+r$. In view of (6.19) this implies that $\lim_{t\to\infty} b_{t+1}/b_t \geq (1+r)/(1+g_Y) > 1$. Thus, $b_t \to \infty$, which violates solvency. By contradiction, this proves that solvency implies (NPG) when $r > g_Y$.

Case (ii): $r \leq g_Y$. Consider the permanent debt roll-over policy $T_t = G_t$ for all $t \geq 0$, and assume $B_0 > 0$. By (DGBC) of Section 6.2 this policy yields $B_{t+1}/B_t = 1+r$; hence, in view of (6.19), $\lim_{t\to\infty} b_{t+1}/b_t = (1+r)/(1+g_Y) \leq 1$. The policy consequently implies solvency. On the other hand the solution of the difference equation $B_{t+1} = (1+r)B_t$ is $B_t = B_0(1+r)^t$. Thus $B_t(1+r)^{-t} = B_0 > 0$ for all t, thus violating (NPG). □

Hence imposition of the NPG condition on the government relies on the interest rate being in the long run higher than the growth rate of GDP. If instead $r \leq g_Y$, the government can cut taxes, run a budget deficit, and postpone the tax burden indefinitely. In that case the government can thus run a Ponzi Game and still stay solvent. Nevertheless, as alluded to earlier, if uncertainty is added to the picture, there will be many different interest rates, and matters become more complicated. Then qualifications to Proposition 1 are needed (Blanchard and Weil, 2001). The prevalent view among macroeconomists is that imposition of the NPG condition on the government is generally warranted.

While in the case $r > g_Y$, the NPG condition is *necessary* for solvency, it is *not sufficient*. Indeed, we could have

$$1 + g_Y < \lim_{t\to\infty} B_{t+1}/B_t < 1 + r. \tag{6.20}$$

Here, by the upper inequality, (NPG) is satisfied, yet, by the lower inequality together with (6.19), we have $\lim_{t\to\infty} b_{t+1}/b_t > 1$ so that the debt-income ratio explodes.

EXAMPLE 1 Let GDP $= Y$, a constant, and $r > 0$; so $r > g_Y = 0$. Let the budget deficit in real terms equal $\varepsilon B_t + \alpha$, where $0 \leq \varepsilon < r$ and $\alpha > 0$. Assuming no money-financing of the deficit, government debt evolves according to $B_{t+1} - B_t = \varepsilon B_t + \alpha$ which implies a simple linear difference equation:

$$B_{t+1} = (1 + \varepsilon)B_t + \alpha. \tag{*}$$

Case 1: $\varepsilon = 0$. Then the solution of (*) is

$$B_t = B_0 + \alpha t, \tag{**}$$

B_0 being historically given. Then $B_t(1+r)^{-t} = B_0(1+r)^{-t} + \alpha t(1+r)^{-t} \to 0$ for $t \to \infty$. So, (NPG) is satisfied. Yet the debt-GDP ratio, B_t/Y, goes to infinity for $t \to \infty$. That is, in spite of (NPG) being satisfied, solvency is not present. For $\varepsilon = 0$ we thus get the insolvency result even though the lower *strict* inequality in (6.20) is *not* satisfied. Indeed, (**) implies $B_{t+1}/B_t = 1 + \alpha/B_t \to 1$ for $t \to \infty$ and $1 + g_Y = 1$.

Case 2: $0 < \varepsilon < r$. Then the solution of (*) is

$$B_t = (B_0 + \frac{\alpha}{\varepsilon})(1 + \varepsilon)^t - \frac{\alpha}{\varepsilon} \to \infty \text{ for } t \to \infty,$$

if $B_0 > -\alpha/\varepsilon$. So $B_t/Y \to \infty$ for $t \to \infty$ and solvency is violated. Nevertheless $B_t(1 + r)^{-t} \to 0$ for $t \to \infty$ so that (NPG) holds.

The example of this case fully complies with both strict inequalities in (6.20) because $B_{t+1}/B_t = 1 + \varepsilon + \alpha/B_t \to 1 + \varepsilon$ for $t \to \infty$. □

An approach to fiscal budgeting that *ensures* debt stabilization and thereby solvency is the following. First impose that the cyclically adjusted primary budget surplus as a share of GDP equals a constant, s. Next adjust taxes and/or spending such that $s \geq \hat{s} = (r - g_Y)b_0$, ignoring short-run differences between Y_{t+1}/Y_t and $1 + g_Y$ and between r_t and its long-run value, r. As in (6.10), $\hat{s}$ is the minimum primary surplus as a share of GDP required to obtain $b_{t+1}/b_t \leq 1$ for all $t \geq 0$ (Example 2 below spells this out in detail). This $\hat{s}$ is a measure of the burden that the government debt imposes on tax payers. If the policy steps needed to realize at least $\hat{s}$ are not taken, the debt-income ratio will grow, thus worsening the fiscal position in the future by increasing $\hat{s}$.

6.5.2 Equivalence of NPG and GIBC

The condition under which the NPG condition is necessary for solvency is also the condition under which the government's intertemporal budget constraint is necessary. To show this we let t denote the current period and $t + i$ denote a period in the future. As above, we ignore seigniorage. Debt accumulation is then described by

$$B_{t+1} = (1 + r)B_t + G_t + X_t - \tilde{T}_t, \qquad \text{where } B_t \text{ is given.} \tag{6.21}$$

The *government intertemporal budget constraint* (GIBC), as seen from the beginning of period t, is the requirement

$$\sum_{i=0}^{\infty}(G_{t+i} + X_{t+i})(1 + r)^{-(i+1)} \leq \sum_{i=0}^{\infty} \tilde{T}_{t+i}(1 + r)^{-(i+1)} - B_t. \tag{GIBC}$$

This condition requires that the present value (PV) of current and expected future government spending does not exceed the government's net wealth. The latter equals the PV of current and expected future tax revenue minus existing government debt. By the symbol $\sum_{i=0}^{\infty} x_i$ we mean $\lim_{I\to\infty} \sum_{i=0}^{I} x_i$. Until further notice we assume this limit exists.

What connection is there between the dynamic accounting relationship (6.21) and the intertemporal budget constraint, (GIBC)? To find out, we rearrange

(6.21) and use forward substitution to get

$$
\begin{aligned}
B_t &= (1+r)^{-1}(\tilde{T}_t - X_t - G_t) + (1+r)^{-1}B_{t+1} \\
&= \sum_{i=0}^{j}(1+r)^{-(i+1)}(\tilde{T}_{t+i} - X_{t+i} - G_{t+i}) + (1+r)^{-(j+1)}B_{t+j+1} \\
&= \sum_{i=0}^{\infty}(1+r)^{-(i+1)}(\tilde{T}_{t+i} - X_{t+i} - G_{t+i}) + \lim_{j\to\infty}(1+r)^{-(j+1)}B_{t+j+1} \\
&\le \sum_{i=0}^{\infty}(1+r)^{-(i+1)}(\tilde{T}_{t+i} - X_{t+i} - G_{t+i}), \qquad (6.22)
\end{aligned}
$$

if and only if the government debt ultimately grows at a rate less than r so that

$$\lim_{j\to\infty}(1+r)^{-(j+1)}B_{t+j+1} \le 0. \qquad (6.23)$$

This latter condition is exactly the NPG condition above (replace t in (6.23) by 0 and j by $t-1$). And the condition (6.22) is just a rewriting of (GIBC). We conclude:

PROPOSITION 2 Given the book-keeping relation (6.21), then:

(i) (NPG) is satisfied if and only if (GIBC) is satisfied;

(ii) there is strict equality in (NPG) if and only if there is strict equality in (GIBC).

We know from Proposition 1 that in the "normal case" where $r > g_Y$, (NPG) is needed for government solvency. The message of (i) of Proposition 2 is then that also (GIBC) need be satisfied. Given $r > g_Y$, to appear solvent a government has to realistically plan taxation and spending profiles such that the PV of current and expected future primary budget surpluses matches the current debt, cf. (6.22). Otherwise debt default is looming and forward-looking investors will refuse to buy government bonds or only buy them at a reduced price, thereby aggravating the fiscal conditions.[20]

In view of the remarks around the inequalities in (6.20), however, satisfying the condition (6.22) is only a necessary condition (if $r > g_Y$), not in itself a sufficient condition for solvency. A simple condition under which satisfying the

[20] Government debt defaults have their own economic as well as political costs, including loss of credibility. Yet, they occur now and then. Recent examples include Russia in 1998 and Argentina in 2001-2002. During 2010-12, Greece was on the brink of debt default. At the time of writing (June 2015) such a situation has turned up again for Greece.

condition (6.22) *is* sufficient for solvency is that both G_t and T_t are proportional to Y_t, cf. Example 2.

EXAMPLE 2 Consider a small open economy facing an exogenous constant real interest rate r. Suppose that at time t government debt is $B_t > 0$, GDP is growing at the constant rate g_Y, and $r > g_Y$. Assume $G_t = \gamma Y_t$ and $T_t \equiv \tilde{T}_t - X_t = \tau Y_t$, where γ and τ are positive constants. What is the minimum size of the primary budget surplus as a share of GDP required for satisfying the government's intertemporal budget constraint as seen from time t? Inserting into the formula (6.22), with strict equality, yields $\sum_{i=0}^{\infty}(1+r)^{-(i+1)}(\tau-\gamma)Y_{t+i} = B_t$. This gives $\frac{\tau-\gamma}{1+g_Y}Y_t\sum_{i=0}^{\infty}\left(\frac{1+g_Y}{1+r}\right)^{(i+1)} = \frac{\tau-\gamma}{r-g_Y}Y_t = B_t$, where we have used the rule for the sum of an infinite geometric series. Rearranging, we conclude that the required primary surplus as a share of GDP is

$$\tau - \gamma = (r - g_Y)\frac{B_t}{Y_t}.$$

This is the same result as in (6.10) above if we substitute $\hat{s} = \tau - \gamma$ and $t = 0$. Thus, maintaining G_t/Y_t and T_t/Y_t constant while satisfying the government's intertemporal budget constraint ensures a constant debt-income ratio and thereby government solvency. □

On the other hand, if $r \leq g_Y$, it follows from propositions 1 and 2 together that the government can remain solvent *without* satisfying its intertemporal budget constraint (at least as long as we ignore uncertainty).[21] The background for this fact may become more apparent when we recognize how the condition $r \leq g_Y$ affects the constraint (GIBC). Indeed, to the extent that the tax revenue tends to grow at the same rate as national income, we have $\tilde{T}_{t+i} = \tilde{T}_t(1+g_Y)^i$. Then

$$\sum_{i=0}^{\infty}\tilde{T}_{t+i}(1+r)^{-(i+1)} = \frac{\tilde{T}_t}{1+g_Y}\sum_{i=0}^{\infty}\left(\frac{1+g_Y}{1+r}\right)^{(i+1)},$$

which is clearly infinite if $r \leq g_Y$. The PV of expected future tax revenues is thus unbounded in this case. Suppose that also government spending, $G_{t+i} + X_{t+i}$, grows at the rate g_Y. Then the evolution of the primary surplus is described by $\tilde{T}_{t+i} - X_{t+i} - G_{t+i} = (\tilde{T}_t - (G_t + X_t))(1+g_Y)^i$, $i = 1, 2, \ldots$. Although in this case also the PV of future government spending is infinite, (6.22) shows that any

[21] Of course, this statement is a contradiction in terms if one thinks of "solvency" in the standard *financial* sense where solvency requires that the debt does not exceed the present value of future surplusses, i.e., that (6.22) holds. As noted in Section 6.3 we use the term *solvency* in the broader meaning of "being able to meet the financial commitments as they fall due".

positive initial primary budget surplus, $\tilde{T}_t - (G_t + X_t)$, ever so small can repay any level of initial debt in finite time.

In (GIBC) and (6.23) we allow strict inequalities to obtain. What is the interpretation of a strict inequality here? The answer is:

COROLLARY OF PROPOSITION 2 Given the book-keeping relation (6.21), then strict inequality in (GIBC) is equivalent to the government in the long run accumulating positive net financial wealth.

Proof. Strict inequality in (GIBC) is equivalent to strict inequality in (6.22), which in turn, by (ii) of Proposition 2, is equivalent to strict inequality in (6.23), which is equivalent to $\lim_{j\to\infty}(1+r)^{-(j+1)}B_{t+j+1} < 0$. This latter inequality is equivalent to $\lim_{j\to\infty} B_{t+j+1} < 0$, that is, positive net financial wealth in the long run. Indeed, by definition, $r > -1$, hence $\lim_{j\to\infty}(1+r)^{-(j+1)} \geq 0$. □

It is common to consider as the *regular case* the case where the government does not attempt to accumulate positive net financial wealth in the long run and thereby become a net creditor vis-à-vis the private sector. Returning to the assumption $r > g_Y$, in the regular case fiscal solvency thus amounts to the requirement

$$\sum_{i=0}^{\infty} \tilde{T}_{t+i}(1+r)^{-(i+1)} = \sum_{i=0}^{\infty}(G_{t+i} + X_{t+i})(1+r)^{-(i+1)} + B_t, \tag{GIBC'}$$

which is obtained by rearranging (GIBC) and replacing weak inequality with strict equality. It is certainly *not* required that the budget is balanced all the time. The point is "only" that for a given planned expenditure path, a government should plan realistically a stream of future tax revenues the PV of which matches the PV of planned expenditure *plus* the current debt.

We may rewrite (GIBC') as

$$\sum_{i=0}^{\infty} \left(\tilde{T}_{t+i} - (G_{t+i} + X_{t+i})\right)(1+r)^{-(i+1)} = B_t. \tag{GIBC"}$$

This expresses the basic principle that when $r > g_Y$, solvency requires that *the present value of planned future primary surpluses equals the initial debt.* We have thus shown:

PROPOSITION 3 Consider the regular case. Assume $r > g_Y$. Then:

(i) if debt is positive today, the government has to run a positive primary budget surplus for a sufficiently long time in the future;

(ii) if an unplanned deficit arises so as to create an unexpected rise in public debt, then higher taxes than otherwise must be levied in the future.

Finer shades

1. If the real interest rate varies over time, all the above formulas remain valid if $(1+r)^{-(i+1)}$ is replaced by $\Pi_{j=0}^{i}(1+r_{t+j})^{-1}$.

2. We have essentially ignored seigniorage. Under "normal" circumstances seigniorage is present and this relaxes (GIBC") somewhat. Indeed, as noted in Section 6.2, the money-nominal income ratio, M/PY, tend to be roughly constant over time, reflecting that money and nominal income tend to grow at the same rate. So a rough indicator of g_M is the sum $\pi + g_Y$. Seigniorage is $S \equiv \Delta M/P = g_M M/P = sY$, where s is the seigniorage-income ratio. Taking seigniorage into account amounts to subtracting the present value of expected future seigniorage, PV(S), from the right-hand side of (GIBC"). With s constant and Y growing at the constant rate $g_Y < r$, PV(S) can be written

$$\begin{aligned} \text{PV(S)} &= \sum_{i=0}^{\infty} S_{t+i}(1+r)^{-(i+1)} = s\sum_{i=0}^{\infty} Y_{t+i}(1+r)^{-(i+1)} = \frac{sY_t}{1+g_Y}\sum_{i=0}^{\infty}\left(\frac{1+g_Y}{1+r}\right)^{(i+1)} \\ &= \frac{sY_t}{1+g_Y}\frac{1+g_Y}{1+r}\frac{1}{1-\frac{1+g_Y}{1+r}} = \frac{sY_t}{r-g_Y}, \end{aligned}$$

where the second to last equality comes from the rule for the sum of an infinite geometric series. So the right-hand side of (GIBC") becomes $B_t - sY_t/(r-g_Y) \equiv [b_t - s/(r-g_Y)]\, Y_t$.[22]

3. Should a public deficit rule not make a distinction between public consumption and public investment? This issue is taken up in the next section.

6.6 A proper accounting of public investment*

Public investment as a share of GDP has been falling in the EMU countries since the middle of the 1970s, in particular since the run-up to the euro 1993-97. This later development is seen as in part induced by the deficit rule of the Maastrict Treaty (1992) and the Stability and Growth Pact (1997) which, like the customary government budget accounting we have considered up to now, attributes government gross investment as an expense in a single year's operating account instead of just the depreciation of the public capital. Already Musgrave (1939) recommended applying separate capital and operating budgets. Thereby government net investment will be excluded from the definition of the public "budget deficit". And more meaningful deficit rules can be devised.

[22] In a recession where the economy is in a liquidity trap, the non-conventional monetary policy called Quantitative Easing may partly take the form of seigniorage. This is taken up in Chapter 24.

To see the gist of this, we partition G into public consumption, C^g, and public investment, I^g, that is, $G = C^g + I^g$. Public investment produces public capital (infrastructure etc.). Denoting the public capital K^g we may write

$$\Delta K^g = I^g - \delta K^g, \tag{6.24}$$

where δ is a (constant) capital depreciation rate. Let the annual (direct) financial return per unit of public capital be r_g. This is the sum of user fees and the like. Net government revenue, T', now consists of net tax revenue, T, plus the direct financial return $r_g K^g$.[23] In that now only interest payments and the capital depreciation, δK^g, along with C^g, enter the operating account as "true" expenses, the "true" budget deficit is $rB + C^g + \delta K^g - T'$, where $T' = T + r_g K^g$.

We impose a rule requiring balancing the "true structural budget" in the sense that on average over the business cycle

$$T' = rB + C^g + \delta K^g \tag{6.25}$$

should hold. The spending on public investment of course enters the debt accumulation equation which now takes the form

$$\Delta B = rB + C^g + I^g - T'.$$

Substituting (6.25) into this, we get

$$\Delta B = I^g - \delta K^g = \Delta K^g, \tag{6.26}$$

by (6.24). So the balanced "true structural budget" implies that public net investment is financed by an increase in public debt. Other public spending is tax financed.

Suppose public capital keeps pace with trend GDP, Y_t^*, that is, $\Delta K^g / K^g = g_Y > 0$. So the ratio K^g/Y^* remains constant at some level $h > 0$. Then (6.26) implies

$$B_{t+1} - B_t = K_{t+1}^g - K_t^g = g_Y K_t^g = g_Y h Y_t^*. \tag{6.27}$$

What is the implication for the evolution of the debt-to-trend-income ratio, $\hat{b}_t \equiv B_t/Y_t^*$, over time? By (6.27) together with $Y_{t+1}^* = (1+g_Y)Y_t^*$ follows

$$\hat{b}_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}^*} = \frac{B_t}{(1+g_Y)Y_t^*} + \frac{g_Y h}{1+g_Y} \equiv \frac{1}{1+g_Y}\hat{b}_t + \frac{g_Y h}{1+g_Y}.$$

[23] There is also an indirect financial return deriving from the fact that better infrastructure may raise efficiency in the supply of public services and increase productivity in the private sector and thereby the tax base. While such expected effects matter for a cost-benefit analysis of a public investment project, from an accounting point of view they will be included in the net tax revenue, T, in the future.

This linear first-order difference equation has the solution

$$\hat{b}_t = (\hat{b}_0 - \hat{b}^*)(1 + g_Y)^{-t} + \hat{b}^*, \qquad \text{where } \hat{b}^* = \frac{1}{1+g_Y}\hat{b}^* + \frac{g_Y h}{1+g_Y} = h,$$

assuming $g_Y > 0$. Then $\hat{b}_t \to h$ for $t \to \infty$. Run-away debt dynamics is precluded.[24] Moreover, the ratio B_t/K_t^g, which equals $\hat{b}_t/h$, approaches 1. Eventually the public debt is in relative terms thus backed by the accumulated public capital.

Fiscal sustainability is here ensured *in spite of* a positive "budget deficit" in the *traditional* sense of Section 6.2 and given by ΔB in (**??**). This result holds even when $r_g < r$, which is perhaps the usual case. Still, the public investment may be worthwhile in view of indirect financial returns as well as non-financial returns in the form of the utility contribution of public goods and services.

Additional remarks

1. The deficit rule described says only that the "true structural budget" should be balanced "on average" over the business cycle. This invites deficits in slumps and surpluses in booms. Indeed, in economic slumps government borrowing is usually cheap. As Harvard economist Lawrence Summers put it: "Idle workers + Low interest rates = Time to rebuild infrastructure" (Summers, 2014).

2. When separating government consumption and investment in budget accounting, a practical as well as theoretical issue arises: where to draw the border between the two? A sizeable part of what is investment in an economic sense is in standard public sector accounting categorized as "public consumption": spending on education, research, and health are obvious examples. Distinguishing between such categories and public consumption in a narrower sense (administration, judicial system, police, defence) may be important when economic growth policy is on the agenda. Apart from noting the issue, we shall not pursue the matter here.

3. That *time lags*, cf. point (iii) in Section 6.1, are a constraining factor for fiscal policy is especially important for macroeconomic stabilization policy aiming at dampening business cycle fluctuations. If the lags are ignored, there is a risk that government intervention comes too late and ends up amplifying the fluctuations instead of dampening them. In particular the *monetarists*, lead by Milton Friedman (1912-2006), warned against this risk, pointing out the "long and variable lags". Other economists find awareness of this potential problem relevant but point to ways to circumvent the problem. During a recession there is for instance the option of reimbursing a part of last year's taxes, a policy that can be quickly implemented. In addition, the ministries of Economic affairs

[24] This also holds if $g_Y = 0$. Indeed, in this case, (6.27) implies $B_{t+1} = B_t = B_0$.

can have plans concerning upcoming public investment ready for implementation and carry them out when expansive fiscal policy is called for. More generally, legislation concerning taxation, transfers, and other spending can be designed with the aim of strengthening the automatic fiscal stabilizers.

6.7 Ricardian equivalence?

Having so far concentrated on the issue of fiscal sustainability, we shall now consider how budget policy affects resource allocation and intergenerational distribution. The role of budget policy for economic activity within a time horizon corresponding to the business cycle is not the issue here. The focus is on the longer run: does it matter for aggregate consumption and aggregate saving in an economy with full capacity utilization whether the government finances its current spending by (lump-sum) taxes or borrowing?

There are two opposite answers in the literature to this question. Some macroeconomists tend to answer the question in the negative. This is the *debt neutrality* view, also called the *Ricardian equivalence* view. The influential American economist Robert Barro is in this camp. Other macroeconomists tend to answer the question in the positive. This is the *debt non-neutrality* view or *absence of Ricardian equivalence* view. The influential French-American economist Olivier Blanchard is in this camp.

The two different views rest on two different models of the economic reality. Yet the two models have a *common* point of departure:

1) $r > g_Y$;

2) fiscal policy satisfies the intertemporal budget constraint with strict equality:
$$\sum_{t=0}^{\infty} \tilde{T}_t (1+r)^{-(t+1)} = \sum_{t=0}^{\infty} (G_t + X_t)(1+r)^{-(t+1)} + B_0, \tag{6.28}$$
where the initial debt, B_0, and the planned path of $G_t + X_t$ are given;

3) agents have rational (model consistent) expectations;

4) at least some of the taxes are lump sum and only these are varied in the thought experiment to be considered;

5) no financing by money;

6) credit market imperfections are absent.

For a given planned time path of $G_t + X_t$, equation (6.28) implies that a tax cut in any period has to be met by an increase in future taxes of the same present discounted value as the tax cut.

6.7.1 Two differing views

Ricardian equivalence

The *Ricardian equivalence* view is the conception that government debt is *neutral* in the sense that for a given time path of future government spending, aggregate private consumption is unaffected by a temporary tax cut. The temporary tax cut does not make the households feel richer because they expect that the ensuing rise in government debt will lead to higher taxes in the future. The essential claim is that the timing of (lump-sum) taxes does not matter. The name *Ricardian equivalence* comes from a – seemingly false – association of this view with the early nineteenth-century British economist David Ricardo. It is true that Ricardo articulated the possible logic behind debt neutrality. But he suggested several reasons that debt neutrality would not hold in practice and in fact he warned against high public debt levels (Ricardo, 1969, pp. 161-164). Therefore it is doubtful whether Ricardo was a Ricardian.

Debt neutrality was rejuvenated, however, by Robert Barro in a paper entitled "Are government bonds net wealth [of the private sector]?", a question which Barro answered in the negative (Barro 1974). Barro's debt neutrality view rests on a representative agent model, that is, a model where the household sector is described as consisting of a fixed number of infinitely-lived forward-looking "dynasties". With perfect financial markets, a change in the timing of taxes does not change the PV of the infinite stream of taxes imposed on the individual dynasty. A cut in current taxes is offset by the expected higher future taxes. Though current government saving $(T - G - rB)$ goes down, private saving and bequests left to the members of the next generation go up equally much.

More precisely, the logic of the debt neutrality view is as follows. Suppose, for simplicity, that the government waits only 1 period to increase taxes and then does so in one stroke. Then, for each unit of account current taxes are reduced, taxes next period are increased by $(1+r)$ units of account. The PV as seen from the end of the current period of this future tax increase is $(1+r)/(1+r) = 1$. As $1-1 = 0$, the change in the time profile of taxation will make the dynasty feel neither richer nor poorer. Consequently, its current and planned future consumption will be unaffected. That is, its current saving goes up just as much as its current taxation is reduced. In this way the altruistic parents make sure that the next generation is fully compensated for the higher future taxes. Current private consumption in

Ricardian non-equivalence The old saying that "in life only death and tax are certain" fits the Ricardian non-equivalence view well. Many economists dissociate themselves from representative agent models because of their problematic description of the household sector. Instead attention is drawn to overlapping generations models which emphasize finite lifetime and life-cycle behavior of human beings and lead to a refutation of Ricardian equivalence. The essential point is that those individuals who benefit from lower taxes today will only be a fraction of those who bear the higher tax burden in the future. As taxes levied at different times are thereby levied at partly different sets of agents, the timing of taxes generally matters. The current tax cut makes current tax payers feel wealthier and so they increase their consumption and decrease their saving. The present generations benefit and future tax payers (partly future generations) bear the cost in the form of access to less national wealth than otherwise. With another formulation: under full capacity utilization government deficits have a crowding-out effect because they compete with private investment for the allocation of saving.

The next subsection provides an example showing in detail how a change in the timing of taxes affects aggregate private consumption in an overlapping generations life-cycle framework.

6.7.1 A small open OLG economy with a temporary budget deficit

We consider a Diamond-style overlapping generations (OLG) model of a small open economy (henceforth named SOE) with a government sector. The relationship between SOE and international markets is described by the same four assumptions as in Chapter 5.3:

(a) Perfect mobility of goods and financial capital across borders.

(b) No uncertainty and domestic and foreign financial claims are perfect substitutes.

(c) No need for means of payment, hence no need for a foreign exchange market.

(d) No labor mobility across borders.

The assumptions (a) and (b) imply *real interest rate equality.* That is, in equilibrium the real interest rate in SOE must equal the real interest rate, r, in the world financial market. By saying that SOE is "small" we mean it is small enough to not affect the world market interest rate as well as other world market factors. We imagine that all countries trade one and the same homogeneous good. International trade will then be only *intertemporal* trade, i.e., international borrowing and lending of this good.

We assume that r is constant over time and that $r > n \geq 0$. We let L_t denote the size of the young generation and assume $L_t = L_{-1}(1+n)^{t+1}$, $t = 0, 1, 2, \ldots$. Each young supplies one unit of labor inelastically, hence L_t is aggregate labor supply. Assuming full employment and ignoring technical progress, gross domestic product, GDP, is $Y_t = F(K_t, L_t)$.

Firms' behavior and the equilibrium real wage

GDP is produced by an aggregate neoclassical production function with CRS:

$$Y_t = F(K_t, L_t) = L_t F(k_t, 1) \equiv L_t f(k_t),$$

where K_t and L_t are input of capital and labor, respectively, and $k_t \equiv K_t/L_t$. Technological change is ignored. Imposing perfect competition, profit maximization gives $\partial Y_t/\partial K_t = f'(k_t) = r+\delta$, where δ is a constant capital depreciation rate, $0 \leq \delta \leq 1$. When f satisfies the condition $\lim_{k\to 0} f'(k) > r + \delta > \lim_{k\to\infty} f'(k)$, there is always a solution for k_t in this equation and it is unique (since $f'' < 0$) and constant over time (as long as r and δ are constant). Thus,

$$k_t = f'^{-1}(r + \delta) \equiv k, \text{ for all } t \geq 0, \tag{6.29}$$

where k is the desired capital-labor ratio, given r. The endogenous stock of capital, K_t, is determined by the equation $K_t = kL_t$, where, in view of clearing in the labor market, L_t can be interpreted as both employment and labor supply (exogenous).

The desired capital-labor ratio, k, also determines the equilibrium real wage before tax:

$$w_t = \frac{\partial Y_t}{\partial L_t} = f(k_t) - f'(k_t)k_t = f(k) - f'(k)k \equiv w, \tag{6.30}$$

a constant. GDP will evolve over time according to

$$Y_t = f(k)L_t = f(k)L_0(1+n)^t = Y_0(1+n)^t.$$

The growth rate of Y thus equals the growth rate of the labor force, i.e., $g_Y = n$.

Some national accounting for an open economy with a public sector

Since we ignore labor mobility across borders, gross national product (= gross national income) in SOE is

$$GNP_t = GDP_t + r \cdot NFA_t = Y_t + r \cdot NFA_t,$$

where NFA_t is net foreign assets at the beginning of period t. If $NFA_t > 0$, SOE has positive net claims on resources in the rest of the world, it may be in the form of direct ownership of production assets or in the form of net financial claims. If $NFA_t < 0$, the reason may be that part of the capital stock, K_t, in SOE is directly owned by foreigners or these have on net financial claims on the citizens of SOE (in practice usually a combination of the two).

Gross national saving is

$$S_t = Y_t + rNFA_t - C_t - G_t = Y_t + rNFA_t - (c_{1t}L_t + c_{2t}L_{t-1}) - G_t, \qquad (6.31)$$

where G_t is government consumption in period t, and c_{1t} and c_{2t} are consumption by a young and an old in period t, respectively. In the open economy, generally, gross investment, I_t, differs from gross saving.

National wealth, V_t, of SOE at the beginning of period t is, by definition, national assets minus national liabilities,

$$V_t \equiv K_t + NFA_t.$$

National wealth is also, by definition, the sum of private financial (net) wealth, A_t, and government financial (net) wealth, $-B_t$. We assume the government has no physical assets and B_t is government (net) debt. Thus,

$$V_t \equiv A_t + (-B_t). \qquad (6.32)$$

We may also view *national* wealth from the perspective of national *saving. First*, when the young save, they accumulate *private* financial wealth. The private financial wealth at the start of period $t+1$ must in our Diamond framework equal the (net) saving by the young in the previous period, S_{1t}^N, and the latter must equal *minus* the (net) saving by the old in the next period, S_{2t+1}^N :

$$A_{t+1} = s_t L_t \equiv S_{1t}^N = -S_{2t+1}^N. \qquad (6.33)$$

The notation in this section of the chapter follows the standard notation for the Diamond model, and so s_t stands for the saving by the young individual in period t, not the primary budget surplus as in the previous sections.

Second, the increase in *national* wealth equals by definition net *national* saving, S_t^N, which in turn equals the sum of net saving by the private sector, $S_{1t}^N + S_{2t}^N$, and the net saving by the public sector, S_{gt}^N. So

$$\begin{aligned} V_{t+1} - V_t &= S_t - \delta K_t = S_t^N \equiv S_{1t}^N + S_{2t}^N + S_{gt}^N = A_{t+1} + (-A_t) + (-GBD_t) \\ &= A_{t+1} - A_t - (B_{t+1} - B_t), \end{aligned}$$

where the second to last equality comes from (6.33) and the identity $S_{gt}^N \equiv -GBD_t$, while the last equality reflects the maintained assumption that budget deficits are fully financed by debt issue.

Government and household behavior

We assume that the role of the government sector is to deliver public goods and services in the amount G_t in period t. Think of non-rival goods like "rule of law", TV-transmitted theatre, and other public services free of charge. Suppose G_t grows at the same rate as Y_t :

$$G_t = G_0(1+n)^t,$$

where G_0 is given, $0 < G_0 < F(K_0, L_0)$. We may think of G_t as being produced by the same technology as the other components of GDP, thus involving the same unit production costs. We ignore that the public good may affect productivity in the private sector (otherwise G should in principle appear as a third argument in the production function F).

To get explicit solutions, we specify the period utility function to be CRRA: $u(c) = (c^{1-\theta} - 1)/(1-\theta)$, where $\theta > 0$. To keep things simple, the utility of the public good enters individuals' life-time utility additively. Thereby it does not affect marginal utilities of private consumption. There is a tax on the young as well as the old in period t, τ_1 and τ_2, respectively. These taxes are *lump sum* (levied on individuals irrespective of their economic behavior). Until further notice, the taxes are time-independent. Possibly, τ_1 or τ_2 is negative, in which case there is a transfer to either the young or the old.

The consumption-saving decision of the young will be the solution to the following problem:

$$\begin{aligned}
\max U(c_{1t}, c_{2t+1}) &= \frac{c_{1t}^{1-\theta}-1}{1-\theta} + v(G_t) + (1+\rho)^{-1}\left[\frac{c_{2t+1}^{1-\theta}-1}{1-\theta} + v(G_{t+1})\right] \text{ s.t.} \\
c_{1t} + s_t &= w - \tau_1, \\
c_{2t+1} &= (1+r)s_t - \tau_2, \\
c_{1t} &\geq 0, c_{1t+1} \geq 0,
\end{aligned}$$

where the function v represents the utility contribution of the public good. The implied Euler equation can be written

$$\frac{c_{2t+1}}{c_{1t}} = \left(\frac{1+r}{1+\rho}\right)^{1/\theta}.$$

Inserting the two budget constraints and solving for s_t, we get

$$s_t = \frac{w - \tau_1 + \left(\frac{1+\rho}{1+r}\right)^{1/\theta}\tau_2}{1+(1+\rho)\left(\frac{1+r}{1+\rho}\right)^{(\theta-1)/\theta}} \equiv s_0 = s(w, r, \tau_1, \tau_2), \qquad t = 0, 1, 2, \ldots,$$

This shows how saving by the young depends on the preference parameters θ and ρ and on labor income and the interest rate. Further, saving by the young is constant over time.

Before considering the solution for c_{1t} and c_{2t+1}, it is convenient to introduce the *intertemporal* budget constraint of an individual belonging to generation t and consider the value of the individual's after-tax *human wealth*, h_t, evaluated at the end of period t. This is the present (discounted) value, as seen from the end of period t, of *disposable lifetime income* (the "endowment").obtainable by a member of generation t. In the present case we get

$$c_{1t} + \frac{c_{2t+1}}{1+r} = w_t - \tau_1 - \frac{\tau_2}{1+r} \equiv h, \tag{6.34}$$

where h on the right-hand side is the time independent value of h_t under the given circumstances.[26] To ensure that $h > 0$, we must assume that τ_1 and τ_2 in combination are of "moderate" size.

The solutions for consumption in the first and the second period, respectively, can then be written

$$c_{1t} = w - \tau_1 - s_t = \hat{c}_1(r)h \tag{6.35}$$

and

$$c_{2t+1} = \hat{c}_2(r)h, \tag{6.36}$$

where

$$\hat{c}_1(r) \equiv \frac{1+\rho}{1+\rho+\left(\frac{1+r}{1+\rho}\right)^{(1-\theta)/\theta}} \in (0,1) \text{ and} \tag{6.37}$$

$$\hat{c}_2(r) \equiv \left(\frac{1+r}{1+\rho}\right)^{1/\theta}\hat{c}_1(r) = \frac{1+r}{1+(1+\rho)\left(\frac{1+r}{1+\rho}\right)^{(\theta-1)/\theta}} \tag{6.38}$$

are the marginal (= average) propensities to consume out of wealth.[27]

Given r, both in the first and the second period of life is individual consumption proportional to individual human wealth. This is as expected in view of the homothetic lifetime utility function. If $\rho = r$, then $\hat{c}_1(r) = \hat{c}_2(r) = (1+r)/(2+r)$, that is, there is complete consumption smoothing.

The tax revenue in period t is $T_t = \tau_1 L_t + \tau_2 L_{t-1} = (\tau_1 + \tau_2/(1+n))L_t$. Let $B_0 = 0$ and let the "benchmark path" be a path along which the budget is and remains *balanced* for all t, i.e.,

$$T_t = (\tau_1 + \frac{\tau_2}{1+n})L_0(1+n)^t = G_t = G_0(1+n)^t.$$

[26] With technical progress, the real wage would be rising over time and so would h_t.

[27] By calculating backwards from (6.38) to (6.37) to (??), the reader will be able to confirm that the calculated s, c_{1t} and c_{2t+1} are consistent.

In this "benchmark policy regime" the tax code (τ_1, τ_2) thus satisfies $(\tau_1+\tau_2/(1+n))L_0 = G_0$. Given L_0, consistency with $h > 0$ in (6.34) requires a "not too large" G_0.

Along the benchmark path, aggregate private consumption grows at the same constant rate as GDP and public consumption, the rate n. Indeed,

$$C_t = c_{1t}L_t + \frac{c_{2t}}{1+n}L_t = (c_{1t} + \frac{c_{2t}}{1+n})L_0(1+n)^t = C_0(1+n)^t.$$

In view of (6.33) and the absence of government debt, also *national wealth* grows at the rate n :

$$V_t = A_t - B_t = A_t - 0 = s_{t-1}L_{t-1} = s_0 L_{t-1} = s_0 L_{-1}(1+n)^t = V_0(1+n)^t, \qquad t = 0, 1, \ldots. \tag{6.39}$$

Consequently, national wealth per old, V_t/L_{t-1}, is constant over time (recall, we have ignored technical progress).

6.7.2 A one-off tax cut

As an alternative to the benchmark path, consider the case where an unexpected one-off cut in taxation by z units of account takes place in period 0 for every individual, whether young or old. What are the consequences of this? The tax cut amounts to creating a budget deficit in period 0 equal to

$$GBD_0 = rB_0 + G_0 - T_0' = G_0 - T_0' = T_0 - T_0' = (L_0 + L_{-1})z,$$

where the value taken by a variable along this *alternative path* is marked with a prime. At the start of period 1, there is now a government debt $B_1' = (L_0+L_{-1})z$. In the benchmark path we had $B_1 = 0$. Since we assume $r > n = g_Y$, government solvency requires that the present value of future taxes, as seen from the beginning of period 1, rises by $(L_0 + L_{-1})z$, cf. (6.28). Suppose this is accomplished by raising the tax on all individuals from period 1 onward by m. Then

$$\Delta T_t = (L_t + L_{t-1})m = (L_0 + L_{-1})(1+n)^t, \qquad t = 1, 2, \ldots.$$

Suppose the government in period 0 credibly announces that the way it will tackle the arisen debt is by his policy. So also the young in period 0 are aware of the future tax rise.

As solvency requires that the present value of future taxes, as seen from the beginning of period 1, rises by $(L_0 + L_{-1})z$, the required value of m will satisfy

$$\sum_{t=1}^{\infty} \Delta T_t(1+r)^{-t} = \sum_{t=1}^{\infty}(L_0 + L_{-1})(1+n)^t m(1+r)^{-t} = (L_0 + L_{-1})z.$$

This gives

$$m \sum_{t=1}^{\infty} \left(\frac{1+n}{1+r} \right)^t = z.$$

As $r > n$, from the rule for the sum of an infinite geometric series follows that

$$m = \frac{r-n}{1+n} z \equiv \bar{m}. \tag{6.40}$$

As an example, let $r = 0,02$ and $n = 0.005$ per year. Then $\bar{m} \simeq 0.015 \cdot z$.

The needed rise in future taxes is thus higher the higher is the interest rate r. This is because the interest burden of the debt will be higher. On the other hand, a higher population growth rate, n, reduces the needed rise in future taxes. This is because the interest burden per capita is mitigated by population growth. Finally, a greater tax cut, z, in the first period implies greater tax rises in future periods. (It is assumed throughout that z is of "moderate" size in the sense of not causing $\bar{m}$ to violate the condition $h'_t > 0$. The requirement is $0 < z < (1+r)(1+n)h/\left[(2+r)(r-n)\right]$.)

Effect on the consumption path

In period 0 the tax cut unambiguously benefits the old. Their increase in consumption equals the saved tax:

$$c'_{20} - c_{20} = z > 0. \tag{6.41}$$

The young in period 0 know that per capita taxes next period will be increased by $\bar{m}$. In view of the tax cut in period 0, the young nevertheless experience an increase in after-tax human wealth equal to

$$\begin{aligned} h'_0 - h_0 &= \left(w - \tau_1 + z - \frac{\tau_2 + \bar{m}}{1+r} \right) - \left(w - \tau_1 - \frac{\tau_2}{1+r} \right) \\ &= \left(1 - \frac{r-n}{(1+r)(1+n)} \right) z \qquad \text{(by (6.40))} \\ &= \frac{1 + (2+r)n}{(1+r)(1+n)} z > 0. \end{aligned} \tag{6.42}$$

Consequently, through the *wealth effect* this generation enjoys increases in consumption through life equal to

$$c'_{10} - c_{10} = \hat{c}_1(r)(h'_0 - h_0) > 0, \qquad \text{and} \tag{6.43}$$

$$c'_{21} - c_{21} = \hat{c}_2(r)(h'_0 - h_0) > 0, \tag{6.44}$$

by (6.35) and (6.36), respectively. The two generations alive in period 0 thus gain from the temporary budget deficit.

All *future* generations are worse off, however. These generations do not benefit from the tax relief in period 0, but they have to bear the future cost of the tax relief by a *reduction* in individual after-tax human wealth. Indeed, for $t = 1, 2, \ldots$,

$$\begin{aligned} h_t' - h_t &= h_1' - h = w - \tau_1 - \bar{m} - \frac{\tau_2 + \bar{m}}{1+r} - \left(w - \tau_1 - \frac{\tau_2}{1+r} \right) \\ &= -\left(\bar{m} + \frac{\bar{m}}{1+r} \right) = -\frac{2+r}{1+r}\bar{m} < 0. \end{aligned} \tag{6.45}$$

All things considered, since both the young and the old in period 0 increase their consumption, aggregate consumption in period 0 rises. Ricardian equivalence thus *fails*.

Effect on wealth accumulation*

How does aggregate *private* saving in period 0 respond to the temporary tax cut? Consider first the old in period 0. Along both the benchmark path and the alternative path the old entered period 0 with the financial wealth A_0 and they leave the period with zero financial wealth. So their aggregate net saving is $S_{20}^N = -A_0$ in both fiscal regimes. The young in period 0 increase their consumption in response to the temporary tax cut. At the same time they *increase* their period-0 saving. Indeed, from (6.44) and the period budget constraint as old follows

$$\begin{aligned} 0 &< c_{21}' - c_{21} = (1+r)s_0' - (\tau_2 + \bar{m}) - ((1+r)s_0 - \tau_2) \\ &= (1+r)(s_0' - s_0) - \bar{m} < (1+r)(s_0' - s_0), \end{aligned}$$

thus implying $s_0' - s_0 > 0$. The explanation is that the individuals have a preference for consumption smoothing in that $\theta > 0$. So the young in period 0 want to smooth out the increased consumption possibilities resulting from the increase in their human wealth. To be able to increase consumption as old, their extra saving, with interest, must exceed what is needed to pay the extra tax $\bar{m}$ in period 1. It is the tax cut that makes it possible for the young to increase both consumption and saving in period 0.

The impact on national wealth in period 1 The higher saving by the young in period 0 implies higher aggregate *private* financial wealth per old at the beginning of period 1, since $A_1'/L_0 = s_0' > s_0 = A_1/L_0$. Nevertheless, gross *national* saving, cf. (6.31), is clearly lower than in the benchmark case. Indeed, $C_0' > C_0$ implies

$$S_0' = F(K_0, L_0) + r \cdot NFA_0 - C_0' - G_0 < F(K_0, L_0) + r \cdot NFA_0 - C_0 - G_0 = S_0.$$

That gross national saving is lower is not inconsistent with the just mentioned rise in *private* saving in period 0 compared to the benchmark path. A counterpart of the increased *private* saving is the *public dissaving*, reflecting that the tax cut in period 0 creates a budget deficit one-to-one. Since the increased disposable income implied by the tax cut is used partly to increase private saving *and* partly to increase private consumption, the rise in private saving is *smaller* than the public dissaving. So *total* or *national* saving in period 0 is reduced.

Consequently, we have:

(i) *National wealth* at the start of period 1 is lower in the debt regime than in the no-debt regime.

By how much? In the benchmark regime the national wealth at the start of period 1 is $V_1 = V_0 + S_0^N = V_0 + S_0 - \delta K_0$. This exceeds national wealth in the debt regime by

$$\begin{aligned} V_1 - V_1' &= S_0 - S_0' = C_0' - C_0 = c_{10}' L_0 + c_{20}' L_{-1} - (c_{10} L_0 + c_{20} L_{-1}) \\ &= (c_{10}' - c_{10}) L_0 + (c_{20}' - c_{20}) L_{-1} \\ &= \hat{c}_1(r)(h_0' - h_0) L_0 + z L_{-1} \quad \text{(by (6.43) and (6.41))} \\ &= \left(\hat{c}_1(r) \frac{1 + (2+r)n}{1+r} + 1 \right) \frac{1}{1+n} L_0 z > 0. \quad \text{(by (6.42))} \end{aligned} \tag{6.46}$$

Later consequences As revealed by (6.45), all future generations (those born in period $1, 2, \ldots$) are worse off along the alternative path. This gives rise to two further claims:

(ii) *National wealth per old* along the alternative path, V_t'/L_{t-1}, will remain constant from period 2 onward at a level below that along the path without government debt.

(iii) The constant level along the alternative path from period 2 onward will even be below the level in period 1.

To substantiate these two claims, consider $V_t' \equiv A_t' - B_t'$. In Appendix A it is shown that *government debt* per old will from period 1 onward satisfy

$$\frac{B_t'}{L_{t-1}} = \frac{B_1'}{L_0} = \frac{(L_0 + L_{-1})z}{L_0} = \frac{2+n}{1+n} z, \qquad t = 1, 2, \ldots,$$

and thus be constant. So government debt grows at the rate of population growth. In addition, Appendix A shows that *private* financial wealth per old is constant from period 2 onward and satisfies

$$\frac{A'_t}{L_{t-1}} = s'_{t-1} = s_0 - \left(1 - \hat{c}_1(r)\frac{2+r}{1+r}\right)\frac{r-n}{1+n}z, \qquad t = 2, 3, \ldots.$$

It follows that *national* wealth per old from period 2 onward will be

$$\begin{aligned}\frac{V'_t}{L_{t-1}} &\equiv \frac{A'_t}{L_{t-1}} - \frac{B'_t}{L_{t-1}} = s'_{t-1} - \frac{2+n}{1+n}z = s_0 - \left(1 - \hat{c}_1(r)\frac{2+r}{1+r}\right)\frac{r-n}{1+n}z - \frac{2+n}{1+n}z \\ &= s_0 - \left(1 - \hat{c}_1(r)\frac{r-n}{1+r}\right)\frac{2+r}{1+n}z = \frac{V'_2}{L_1} < s_0 = \frac{V_2}{L_1} = \frac{V_1}{L_0} \qquad t = 2, 3, \ldots \end{aligned} \tag{6.47}$$

where the last two equalities follow from (6.39). This proves our claim (ii).

National wealth per old in period 1 of the debt path is, by (6.46),

$$\begin{aligned}\frac{V'_1}{L_0} &= \frac{V_1}{L_0} - \left(\hat{c}_1(r)\frac{1+(2+r)n}{1+r} + 1\right)\frac{z}{1+n} \\ &= s_0 - \left(\hat{c}_1(r)\frac{1+(2+r)n}{1+r} + 1\right)\frac{z}{1+n} > \frac{V'_2}{L_1},\end{aligned}$$

where the inequality follows by comparison with (6.47). This proves our claim (iii).

Period 1 is special compared to the subsequent periods. While there is a per capita tax increase by $\bar{m}$ like in the subsequent periods, period 1's old generation still benefits from the higher disposable income in period 0. Hence, in period 2 national wealth per old is even lower than in period 1 but remains constant henceforth.

A closed economy Also in a closed economy would a temporary lump-sum tax cut make the future generations worse off. Indeed, in view of reduced national saving in period 0, national wealth (which in the closed economy equals K) would from period 1 onward be smaller than along the no-debt path. The precise calculations are more complicated because the rate of interest will no longer be a constant.

6.7.3 Widening the perspective

The fundamental point underlined by OLG models is that there is a difference between the public sector's future tax base, including the resources of individuals yet to be born, and the future tax base emanating from individuals alive today.

This may be called the *composition-of-tax-base argument* for a tendency to non-neutrality of shifting the timing of (lump-sum) taxation.[28]

The conclusion that under full capacity utilization budget deficits imply a burden for future generations may be seen in a somewhat different light if persistent technological progress is included in the model. In that case, everything else equal, future generations will generally be better off than current generations. Then it might seem less unfair if the former carry some public debt forward to the latter. In particular this is so if a part of G_t represents spending on infrastructure, education, research, health, and environmental protection. As future generations directly benefit from such investment, it seems fair that they also contribute to the financing. This is the "benefits received principle" known from public finance theory.

A further concern is whether the economy is in a state of full capacity utilization or serious unemployment and idle capital. The above analysis assumes the first. What if the economy in period 0 is in economic depression with high unemployment due to insufficient aggregate demand? Some economists maintain that also in this situation is a cut in (lump-sum) taxes to stimulate aggregate demand futile because it has no real effect. The argument is again that foreseeing the higher taxes needed in the future, people will save more to prepare themselves (or their descendants through higher bequests) for paying the higher taxes in the future. The opposite view is, first, that the composition-of-tax-base argument speaks against this as usual. Second, there is in a depression an additional and quantitatively important factor. The "first-round" increase in consumption due to the temporary tax cut raises aggregate demand. Thereby production and income is stimulated and a further (but smaller) rise in consumption occurs in the "second round" and so on (the Keynesian multiplier process).

This Keynesian mechanism is important for the debate about effects of budget deficits because there are limits to how *large* deviations from Ricardian equivalence the composition-of-tax-base argument can deliver in the long-run life-cycle perspective of OLG models. Indeed, taking into account the sizeable life expectancy of the average citizen, Poterba and Summers (1987) point out that the composition-of-tax-base argument by itself delivers only modest deviations if the issue is timing of taxes over the business cycle. They find that to comply with the data on private saving responses to supposedly exogenous shifts in taxation should be combined with the hypothesis that households are "myopic" than what standard OLG models assume.

Another concern is that in the real world, taxes tend to be distortionary and

[28] In Exercise 6.?? the reader is asked how the burden of the public debt is distributed across generations if the debt should be completely wiped out through a tax increase in only periods 1 and 2.

not lump sum. On the one hand, this should not be seen as an argument against the possible *theoretical* validity of the Ricardian equivalence proposition. The reason is that Ricardian equivalence (in its strict meaning) claims absence of allocational effects of changes in the timing of *lump-sum* taxes.

On the other hand, in a wider perspective the interesting question is, of course, how changes in the timing of *distortionary* taxes is likely to affect resource allocation. Consider first *income taxes.* When taxes are proportional to income or progressive (average tax rate rising in income), they provide insurance through reducing the volatility of after-tax income. The fall in taxes in a recession thus helps stimulating consumption through *reduced precautionary saving* (the phenomenon that current saving tends to rise in response to increased uncertainty, cf. Chapter ??). In this way, replacing lump-sum taxation by income taxation underpins the positive wealth effect on consumption, arising from the composition-of-tax-base channel, of a debt-financed tax-cut in an economic recession.

What about *consumption taxes*? A debt-financed temporary cut in consumption taxes stimulates consumption through a positive wealth effect, arising from the composition-of-tax-base channel. On top of this comes a positive intertemporal substitution effect on current consumption caused by the changed consumer price time profile.

The question whether Ricardian non-equivalence is important from a quantitative and empirical point of view pops up in many contexts within macroeconomics. We shall therefore return to the issue several times later in this book.

6.8 Concluding remarks

(incomplete)

Point (iv) in Section 6.1 hints at the fact that when outcomes depend on forward-looking expectations in the private sector, governments may face a time-inconsistency problem. In this context *time inconsistency* refers to the possible temptation of the government to deviate from its previously announced course of action once the private sector has acted. An example: With the purpose of stimulating private saving, the government announces that it will not tax financial wealth. Nevertheless, when financial wealth has reached a certain level, it constitutes a tempting base for taxation and so a tax on wealth might be levied. To the extent the private sector anticipates this, the attempt to affect private saving in the first place fails. This raises issues of *commitment* and *credibility.* We return to this kind of problems in later chapters.

Finally, point (v) in Section 6.1 alludes to the fact that political processes, bureaucratic self-interest, rent seeking, and lobbying by powerful interest groups interferes with fiscal policy.[29] This is a theme in the branch of economics called *political economy* and is outside the focus of this chapter.

6.9 Literature notes

(incomplete)

Sargent and Wallace (1981) study consequences of − and limits to − a shift from debt financing to money financing of sustained government budget deficits in response to threatening increases in the government debt-income ratio.

How the condition $r > g_Y$, for prudent debt policy to be necessary, is modified when the assumption of no uncertainty is dropped is dealt with in Abel et al. (1989), Bohn (1995), Ball et al. (1998), and Blanchard and Weil (2001). On self-fulfilling sovereign debt crises, see, e.g., Cole and Kehoe (2000).

Readers wanting to go more into detail with the policy-oriented debate about the design of the EMU and the Stability and Growth Pact is referred to the discussions in for example Buiter (2003), Buiter and Grafe (2004), Fogel and Saxena (2004), Schuknecht (2005), and Wyplosz (2005). As to discussions of the actual functioning of monetary and fiscal policy in the Eurozone in response to the Great Recession, see for instance the opposing views by De Grauwe and Ji (2013) and Buti and Carnot (2013). Blanchard and Giavazzi (2004) discuss how proper accounting of public investment would modify the deficit and debt rules of the EMU. Beetsma and Giuliodori (2010) survey recent research of costs and benefits of the EMU.

On the theory of *optimal currency areas,* see Krugman, Obstfeld, and Melitz (2012).

In addition to the hampering of Keynesian stabilization policy discussed in Section 6.4.2, also demographic staggering (due to baby booms succeeded by baby busts) may make rigid deficit rules problematic. In Denmark for instance demographic staggering is prognosticated to generate considerable budget deficits during several decades after 2030 where younger and smaller generations will succeed older and larger ones in the labor market. This is prognosticated to take place, however, without challenging the long-run sustainability of current fiscal policy as assessed by the Danish Economic Council (see the English Summary in De Økonomiske Råd, 2014). This phenomenon is in Danish known as "hængekøjeproblemet" (the "hammock problem").

Sources for last part of Section 6.7 ....

[29] *Rent seeking* refers to attempts to gain by increasing one's share of existing wealth, instead of trying to *produce* wealth.

6.10 Appendix A

In Section 6.7.2 we asserted that along the alternative path the government debt will grow at the same rate as the population. The proof is as follows.

The law of motion of the debt is, for $t = 1, 2, \dots,$

$$\begin{aligned} B'_{t+1} &= (1+r)B'_t + G_t - T'_t = (1+r)B'_t + G_t - \left(\tau_1 + \frac{\tau_2}{1+n} + \bar{m} + \frac{\bar{m}}{1+n}\right) L_t \\ &= (1+r)B'_t - \left(\bar{m} + \frac{\bar{m}}{1+n}\right) L_t = (1+r)B'_t - \frac{2+n}{1+n}\bar{m}L_t, \end{aligned}$$

where the second line follows from $G_t - (\tau_1 + \tau_2(1+n))L_t = 0$ in view of the balanced budget along the benchmark path. It is convenient to rewrite the law of motion in terms of $x_t \equiv B'_t/L_{t-1}$, i.e., government debt per old. We get

$$x_{t+1} \equiv \frac{B'_{t+1}}{L_t} = \left(\frac{1+r}{1+n}\right) x_t - \frac{2+n}{1+n}\bar{m}, \qquad t = 1, 2, \dots,$$

where we have used that $L_t = (1+n)L_{t-1}$. The solution of this first-order difference equation with constant coefficients is

$$x_t = (x_1 - x^*) \left(\frac{1+r}{1+n}\right)^{t-1} + x^*,$$

with

$$\begin{aligned} x_1 &= \frac{B'_1}{L_0} = \frac{(L_0 + L_{-1})z}{L_0} = \frac{2+n}{1+n}z, \qquad \text{and} \\ x^* &= -\frac{2+n}{1+n}\bar{m}\left(1 - \frac{1+r}{1+n}\right)^{-1} = \frac{2+n}{r-n}\bar{m} = \frac{2+n}{1+n}z, \end{aligned}$$

using the solution (6.40) for the tax rise $\bar{m}$. It follows that x_t is constant over time and equals x^*. Hence, from period 1 onward $B'_t/L_{t-1} = (2+n)z/(1+n)$ where z is the per capita tax cut in period 0. □

In Section 6.7.2 we also asserted that along the alternative path the private financial wealth per old will from period 2 onward be constant. The proof is as follows:

For $t = 2, 3, \dots,$

$$\begin{aligned} \frac{A'_t}{L_{t-1}} &= s'_{t-1} = w - (\tau_1 + \bar{m}) - c'_{1t-1} = w - (\tau_1 + \bar{m}) - \hat{c}_1(r)\left(w - \tau_1 - \bar{m} - \frac{\tau_2 + \bar{m}}{1+r}\right) \\ &= w - \tau_1 - \hat{c}_1(r)(w - \tau_1 - \frac{\tau_2}{1+r}) - \bar{m} + \hat{c}_1(r)\bar{m} + \hat{c}_1(r)\frac{\bar{m}}{1+r} \\ &= s_0 - \left(1 - \hat{c}_1(r)\left(1 + \frac{1}{1+r}\right)\right)\bar{m} = s_0 - \left(1 - \hat{c}_1(r)\frac{2+r}{1+r}\right)\frac{r-n}{1+n}z, \end{aligned}$$

where we have used (6.33), the period budget constraint of the young along the alternative path, (6.35), (6.34), the period budget constraint of the young along the benchmark path, the constancy of saving by the young along the benchmark path, and finally the solution for the tax rise $\bar{m}$. We see that private financial wealth per old is constant from period 2 onward. □

6.11 Exercises

6.? Consider the OLG model of Section 6.7. a) Show that if the temporary per capita tax cut, z, is sufficiently small, the debt can be completely wiped out through a per capita tax increase in only periods 1 and 2. b) Investigate how in this case the burden of the debt is distributed across generations. Compare with the alternative debt policy described in the text.