Advanced Macroeconomics 2

08.10.2008.

Problem set for the midterm paper

General information: You are encouraged to do the problem set together with fellow students (max four students per group). Readable handwriting is OK, but you may also use a computer. You may answer in English or Scandinavian as you prefer. Do not make your answers too long, that is, write in a concise way. (Greg Mankiw has nice recommendations about how to write: http://gregmankiw.blogspot.com/2006/10/how-to-write-well.html.) For example, when you are asked to "comment", your comment should be a short remark - right to the point.

During working on the problems you are most welcome to consult me. You can ask questions by e-mail and I will answer by e-mail a.s.a.p. or you may consult me at my office (in that case we should make an appointment by e-mail first).

I evaluate your paper, i.e., evaluate whether it is "accepted" (corresponding to a grade $\geq$ 02, new scale) or "not accepted". To go in for the final written examination (four hours) at the end of the semester, it is required that the term paper is accepted.

Time table:

Wednesday 8/10 at 9:00 the midterm paper problem is announced here.

Friday 10/10 no class.

Thursday 23/10 (or before) you hand in your paper at the "Study Office" (Studiekontoret, Studiestræde 6). Opening hours are 10-13.

The front page of the paper must contain:

1) names of all the authors,

2) the first six digits in their cpr.numbers (not the last four)

3) date,

4) name of the course and teacher.

Good luck!

Christian Groth

The weighting of the four problems is:

Problem 1: 30 %, Problem 2: 30 %, Problem 3: 30 %, Problem 4: 10 %.

Problem 1.   The fundamental differential equations of the Blanchard OLG model for a closed economy are:

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{\omega + n + p}{n + p}\tilde{c}_t - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given,}$$

$$\dot{\tilde{c}}_t = \left[f'(\tilde{k}_t) - \delta - \rho + \omega - g\right]\tilde{c}_t - (n + p)(\rho + p)\tilde{k}_t.$$

Notation: $\tilde{k}_t \equiv K_t/(T_t L_t)$ and $\tilde{c}_t \equiv C_t/(T_t N_t) \equiv c_t/T_t$, where K_t and C_t are aggregate capital and aggregate consumption, respectively, N_t is population = labor supply, and T_t is the technology level, all at time t. Finally, f is a production function on intensive form, satisfying $f(0) = 0$, $f' > 0$, $f'' < 0$ and the Inada conditions. The remaining symbols stand for parameters and we assume all these are positive. Furthermore, $\rho \geq n$.

a) Interpret the parameters.

b) The model entails a simple theory of the rate of return, r^*, in the long run. For example, the model implies that r^* must belong to an open interval, defined by the parameters ρ, g, ω, δ, n, and p. Show this. (*Hint:* by drawing a phase diagram you get an idea of one of the end points of this interval; as to the other end point you may use the fact that the steady-state value of $\tilde{k}$ can be shown to be larger than some $\underline{\tilde{k}}$ satisfying the requirement $f'(\underline{\tilde{k}}) - \delta \leq \rho + g + n + p$.)

Although absent from simple macroeconomic models, uncertainty and risk of bankruptcy are significant features of reality. These features explain firms' unwillingness to finance all their investment by debt in spite of the lower rate of return on debt than on equity. In this way the excess of the rate of return on equity over that on debt, the equity premium, is sustained.

A rough, behavioral theory of the equity premium goes as follows. Firms prefer a capital structure where a fraction, s_f, is equity and the remaining fraction, $1 - s_f$, is debt (bonds). Households prefer a portfolio with the fraction s_h in equities and the fraction $1 - s_h$ in bonds. Because of households' risk aversion and memory of historical stock market crashes the theory assumes $s_h < s_f$.

As a crude adaptation of the Blanchard model to these features, we interpret its r^* as an average rate of return across firms. Let time be discrete. Then, given the aggregate financial wealth, A, the aggregate gross return, or payout, is $(1 + r^*)A$. Out of this, $(1 + r^*)As_f$ constitutes the gross return to the equity owners and $(1 + r^*)A(1 - s_f)$ the gross return to the bond owners. Let r_e denote the rate of return on equity and r_b the rate of return on bonds.

c) Find $1 + r_e$, $1 + r_b$, and the equity premium, $(1 + r_e)/(1 + r_b)$. *Hint:* consider how much the household sector has placed in equity and bonds, respectively.

d) It is generally believed that s_h is nowadays higher than in the long aftermath of the Great Depression. How has this affected the equity premium, according to this simple theory?

e) How do you think the recent financial crisis is likely to affect the equity premium? Why?

Problem 2. Consider a small open economy where domestic and foreign financial claims are perfect substitutes and there is perfect mobility of financial capital, but no mobility of labor. The real interest rate in the world financial market is a positive constant r. The dynamics of the economy are described (at least for some time) by the differential equation

$$\dot{\tilde{a}}_t = (r - g - n)\tilde{a}_t + \tilde{w}^* \frac{n + p}{\omega + n + p} - \tilde{c}_t, \qquad \text{where}$$

$$\dot{\tilde{c}}_t = (\rho + p)(\tilde{a}_t + \tilde{h}^*), \qquad \tilde{h}^* = \frac{(n + p)\tilde{w}^*}{(\omega + n + p)(r + \omega + p - g)}.$$

Notation: $\tilde{a} \equiv A/(TN) \equiv a/T$, $\tilde{c} \equiv C/(TN) \equiv c/T$, $A = $ national wealth, $T = $ technology level, $N = $ population, and $C = $ aggregate consumption. The following parameters are strictly positive: r, g, ω, p; the remaining are non-negative.

a) Briefly interpret the model, including the parameters.

Assume $r > \rho + g + n + p$.

b) Draw a phase diagram in $(\tilde{a}, \dot{\tilde{a}})$ space as well as $(\tilde{a}, \tilde{c})$ space. Illustrate in the diagram the path the economy follows for $t \geq 0$, given the initial condition: $\tilde{a}_0 > -\tilde{h}^*$. Comment.

c) Sign the current account surplus of the country. *Hint:* in the balance of payments accounting the current account surplus equals the increase in net foreign assets (whether positive or negative).

d) Suppose that at some point in time an unanticipated shift in the world interest rate occurs. If we imagine that this happens against the background of an international financial turmoil like the present one, what sign should we expect the shift to have? Why?

e) Assume agents rightly expect the new interest rate level to last for a long time, draw a phase diagram illustrating the effects of the shift. Comment.

f) Comment on the long-run development of the economy.

g) Briefly relate to the contemporary evolution of the Chinese economy.

Problem 3.   Consider the government budget in a small open economy. Time is continuous, the time unit is one year, and there is no uncertainty. Let g and n be non-negative constants and let

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\begin{aligned}
Y_t &= Y_0 e^{(g+n)t} = \text{ real GDP at time } t, \\
G_t &= \text{ real government spending on goods and services at time } t, \\
T_t &= \text{ real net tax revenue (= gross tax revenue } - \text{ transfer payments) at time } t, \\
B_t &= \text{ real public debt at time } t, \\
r &= \text{ real interest rate, a constant.}
\end{aligned}
$$

Assume that the budget deficit is exclusively financed by issuing debt.

a) Write down an equation showing how the increase in B_t per time unit is determined.

Consider a scenario with $B_0 > 0$, $r > g + n$, and $T_t/Y_t = \tau$, a positive constant less than one.

b) Find the maximum G/Y which is consistent with fiscal sustainability.

Consider another scenario, where there is a deficit rule saying that $\lambda \cdot 100$ per cent of the interest expenses on public debt plus the primary budget deficit must not be above $\alpha \cdot 100$ per cent of nominal GDP, i.e.

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\frac{\lambda i D_t + P_t(G_t - T_t)}{P_t Y_t} \le \alpha, \tag{*}
$$

where $0 < \lambda \le 1$, $\alpha > 0$, and

$$
\begin{aligned}
D_t &= \text{ nominal public debt at time } t, \\
P_t &= \text{ price level at time } t, \\
i &= r + \pi = \text{ nominal interest rate,} \\
\pi &= \text{ the inflation rate which we assume constant and non-negative.}
\end{aligned}
$$

c) Is the deficit rule of the SGP of the EMU a special case of (*)? Comment.

d) Let $b_t \equiv B_t/Y_t$. Derive the law of movement (differential equation) for b_t, assuming the deficit rule is always binding.

Suppose λ is such that $(1 - \lambda)r - \lambda\pi < g + n$.

e) Find the time path of b. *Hint:* the differential equation $\dot{x} + ax = k$, where a and k are constants, $a \ne 0$, has the solution $x_t = (x_0 - x^*)e^{-at} + x^*$, where $x^* = k/a$.

f) Let the steady-state value of b be denoted b^* and assume $b_0 > b^*$. Will b explode or converge towards b^* over time? Comment.

g) How does b^* depend on λ? Comment.

Problem 4.   *Short questions*

a) "The Barro model of families linked through bequests implies - as long as the bequest motive is operative - that family members pool their income across generations so that the amount any particular family member consumes depends only on the total present discounted value of the income of the infinitely-lived dynastic family, not on that member's share of the total." True or false? Why?

b) "In the Blanchard OLG model, the income effect on current consumption of an increase in the real interest rate exactly offsets the substitution effect." True or false? Why? *Hint:* it is helpful to write down the intertemporal budget constraint and the consumption function of the individual.

c) "When the steady state of a dynamic system is a saddle point, then the system is saddlepoint stable." True or false? Why?

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