Advanced macroeconomics 2 Short Note 4.

28.11.2008. Christian Groth

Notes to Chapter 16

1 The UIP

Useful comments by participants have made me aware that the UIP is strongly rejected at short prediction horizons although it does somewhat better at horizons longer than a year. Some texts about these problems:

Isard, P., 2008, "Uncovered interest parity". In: The new Palgrave Dictionary of Economics. Second edition.

Online: http://www.econ.ku.dk/english/libraries/links/

Lewis, K. K., 1995, Puzzles in international financial markets. In: Handbook of International Economics, vol. III, Elsevier, Amsterdam.

Obstfeld, M., and K. Rogoff, 1996, Foundations of International Macroeconomics, The MIT Press, London, Ch. 8.7.

Wickens, M., 2008, Macroeconomic Theory. A Dynamic General Equilibrium Approach, Princeton University Press, Oxford, Ch. 11.4

2 The exchange rate as a forward-looking variable

This is an addition to Ch. 16, p. 577, right before "Anticipated rise in the money supply":

It helps the interpretation of the dynamics if one recognizes that the exchange rate is an asset price, hence forward-looking. The uncovered interest parity implies that the exchange rate satisfies the differential equation

$$\dot{X}_t = (i_t - i^*)X_t \tag{1}$$

except at points of discontinuity of X_t. For fixed $t > t_0$ we can write the solution of this linear differential equation as

$$X_\tau = X_t e^{\int_t^\tau (i_s - i^*)ds} \equiv X_t e^{(\bar{i}_{t,\tau} - i^*)(\tau - t)}, \qquad \text{for } \tau > t,$$

where $\bar{i}_{t,\tau}$ is the mean of the interest rates between time t and time τ, i.e., $\bar{i}_{t,\tau} \equiv (\int_t^\tau i_s ds)/(\tau - t)$. Being a forward-looking variable, X_t is not predetermined. It is therefore more natural to write the solution on the form

$$X_t = X_\tau e^{-\int_t^\tau (i_s - i^*)ds} \equiv X_\tau e^{-(\bar{i}_{t,\tau} - i^*)(\tau - t)}, \tag{2}$$

where X_τ and i_s can be interpreted as the *expected* future values. Thus, under the UIP hypothesis the exchange rate today equals the expected future exchange rate discounted by the mean interest differential $\bar{i}_{t,\tau} - i^*$ expected to be in force in the meantime.[1] Letting $\tau \to \infty$, (2) gives

$$X_t = (\lim_{\tau \to \infty} X_\tau) e^{-\int_t^\infty (i_s - i^*)ds} = \bar{X}' e^{-\int_t^\infty (i_s - i^*)ds} > \bar{X}', \tag{3}$$

where the inequality is due to $i_s < i^*$ during the adjustment process. As time proceeds, the excess of the foreign over the domestic interest rate decreases and the exchange rate converges to its long-run value from above.

In (2) and (3) we consider the value of the foreign currency. Similar expressions of course hold for the value of the domestic currency. Thus, inverting (2) gives

$$X_t^{-1} = X_\tau^{-1} e^{-\int_t^\tau (i^* - i_s)ds} \equiv X_\tau^{-1} e^{-(i^* - \bar{i}_{t,\tau})(\tau - t)}.$$

That is, the value of the domestic currency today equals its expected future value discounted by the mean interest differential $i^* - \bar{i}_{t,\tau}$ expected to be in force in the meantime. Inverting (3) yields

$$X_t^{-1} = \bar{X}'^{-1} e^{-\int_t^\infty (i^* - i_s)ds} < \bar{X}'^{-1}.$$

As time proceeds, the excess of the foreign over the domestic interest rate decreases and the value of the domestic currency converges to its long-run value from below.

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[1] Note that since (1) is not valid at points of discontinuity of X_t, the solution formula (2) is valid only as long as no jump in X is expected. In turn arbitrage prevents any such expected jump. Ex post the formula (2) is valid only if no jumps in X *did* occur in the time interval considered.