Advanced Macroeconomics

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Correction to page 1085 in Chapter 30 (and page 1053 in Chapter 29)

In the break of today's lecture there was a good question about why on p. 1053 in Chapter 29 and p. 1085 in Chapter 30 the transformation $\tilde{U}_t \equiv (1+\rho)^t U_t$ is introduced. And I must admit that this clumsy $\tilde{U}_t$ is not needed and comes from an inconvenient implicit definition of U_t.

Here is a better way to proceed at page 1085 (and in Exercise X.7a).[1] The paragraph starting with "In every new period t" should be replaced by the following: .

Letting period t be an arbitrary period, i.e., $t \in \{0, 1, 2, ..., T-1\}$, we rewrite U_0 in the following way

$$
\begin{aligned}
U_0 &= \sum_{s=0}^{t-1} u(c_s)(1+\rho)^{-s} + \sum_{s=t}^{T-1} u(c_s)(1+\rho)^{-s} \\
&= \sum_{s=0}^{t-1} u(c_s)(1+\rho)^{-s} + (1+\rho)^{-t} \sum_{s=t}^{T-1} u(c_s)(1+\rho)^{-(s-t)} \\
&\equiv \sum_{s=0}^{t-1} u(c_s)(1+\rho)^{-s} + (1+\rho)^{-t} U_t.
\end{aligned}
$$

When deciding the "action" c_0, the household knows that in every new period, it has to solve the remainder of the problem in a similar way, given the information revealed up to and including that period. As seen from period t, the objective function is

$$E_t U_t = u(c_t) + (1+\rho)^{-1} E_t[u(c_{t+1}) + u(c_{t+2})(1+\rho)^{-1} + ...] \tag{30.12}$$

To solve the problem as seen from period t we will use the substitution method. First, from (30.10) we have

$$
\begin{aligned}
c_t &= (1+r_t)a_t + w_t n_t - a_{t+1}, \qquad \text{and} \qquad (30.13) \\
c_{t+1} &= (1+r_{t+1})a_{t+1} + w_{t+1} n_{t+1} - a_{t+2}.
\end{aligned}
$$

Substituting this into (30.12), the problem is reduced to an essentially unconstrained maximization problem, namely one of maximizing $E_t U_t$ w.r.t. $a_{t+1}, a_{t+2}, ..., a_T$ (thereby

[1] The unfortunate $\tilde{U}_t$ at page 1053 in Chapter 29 can be skipped in an analogue way.

indirectly choosing $c_t, c_{t+1}, .., c_{T-1}$). Hence, we first take the partial derivative w.r.t. a_{t+1} in (30.12) and set it equal to 0:

$$\frac{\partial E_t U_t}{\partial a_{t+1}} = u'(c_t) \cdot (-1) + (1+\rho)^{-1} E_t[u'(c_{t+1})(1+r_{t+1})] = 0.$$

Reordering gives the stochastic Euler equation,

$$u'(c_t) = (1+\rho)^{-1} E_t[u'(c_{t+1})(1+r_{t+1})], \qquad t = 0, 1, 2, ..., T-2. \tag{30.14}$$