Two major kinds of strategies have been suggested. One kind of strategy is the *pre-funding strategy.* The idea is to prevent sharp future tax increases by ensuring a fiscal consolidation prior to the expected future demographic changes. Another strategy (alternative or complementary to the former) is to attempt a gradual increase in the labor force by letting the age limits for retirement and pension increase along with expected lifetime − this is the *indexed retirement strategy.* The first strategy implies that current generations bear a large part of the adjustment cost. In the second strategy the costs are shared by current and future generations in a way more similar to the way the benefits in the form of increasing life expectancy are shared. We shall not here go into detail about these matters, but refer the reader to the growing literature about securing fiscal sustainability in the ageing society, see Literature notes.

6.4 Debt arithmetic

A key tool for evaluating fiscal sustainability is debt arithmetic, i.e., the analytics of debt dynamics. The previous section described the important role of the growth-corrected interest rate. The next subsection considers the minimum primary budget surplus required for fiscal sustainability in different situations.

6.4.1 The required primary budget surplus

Let the spending-income ratio, G/Y, and the (net) tax-income ratio, T/Y, be constants, γ and τ, respectively. Then from the dynamic government budget constraint with $\Delta M = 0$,

$$B_{t+1} = (1+r)B_t + G_t - T_t, \tag{DGBC}$$

follows that the debt-income ratio $b_t \equiv B_t/Y_t$ changes over time according to

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{1+r}{1+g_Y}b_t + \frac{\gamma - \tau}{1+g_Y}, \tag{6.6}$$

where we have assumed that also g_Y and r are constant.

There are three cases to consider. *Case 1:* $r > g_Y$. As emphasized above this case is generally considered the one of most practical relevance. And it is in this case that *latent debt instability* is present and the government has to pay attention to the danger of explosive debt dynamics. To see this, note that the solution of the linear difference equation (6.6) is

$$b_t = (b_0 - b^*)\left(\frac{1+r}{1+g_Y}\right)^t + b^*, \quad \text{where} \tag{6.7}$$

$$b^* = \frac{\gamma-\tau}{1+g_Y}\left(1-\frac{1+r}{1+g_Y}\right)^{-1} = \frac{\tau-\gamma}{r-g_Y}. \tag{6.8}$$

Here b_0 is historically given. But the steady-state debt-income ratio, b^*, depends on fiscal policy. The important feature is that the growth-corrected interest factor is in this case higher than 1 and has the exponent t. Therefore, if fiscal policy is such that $b^* < b_0$, the debt-income ratio explodes. The solid curve in the topmost panel in Fig. 6.2 shows a case where fiscal policy is such that $\tau - \gamma < (r - g_Y)b_0$ whereby we get $b^* < b_0$ when $r > g_Y$, so that the debt-income ratio, b_t, grows without bound.

With reference to the fiscal stance of the US government, economist and Nobel laureate George Akerlof remarked:

> "It takes some time after running off the cliff before you begin to fall. But the law of gravity works, and that fall is a certainty" (Akerlof 2004, p. 6).

Somewhat surprisingly, perhaps, when $r > g_Y$, there can be debt explosion in the long run even if $\tau > \gamma$, namely if $0 < \tau - \gamma < (r - g_Y)b_0$. Debt explosion can even arise if $b_0 < 0$, namely if $\tau - \gamma < (r - g_Y)b_0 < 0$.

The only way to avoid the snowball effects of compound interest when the growth-corrected interest rate is positive is to ensure a primary budget surplus as a share of GDP, $\tau - \gamma$, high enough such that $b^* \geq b_0$. So the *minimum* primary surplus as a share of GDP, $\hat{s}$, required for fiscal sustainability is the one implying $b^* = b_0$, i.e., by (6.8),

$$\hat{s} = (r - g_Y)b_0 \equiv \hat{s}(r - g_Y, b_0). \tag{6.9}$$

If by adjusting τ and/or γ, the government obtains $\tau - \gamma = \hat{s}$, then $b^* = b_0$ whereby $b_t = b_0$ for all $t \geq 0$ according to (6.7), cf. the second from the top panel in Fig. 6.2.

Note that $\hat{s}$ will be larger:

- the higher is the initial level of debt, b_0; and,
- when $b_0 > 0$, the higher is the growth-corrected interest rate, $r - g_Y$.

For fixed spending-income ratio γ, the minimum tax-income ratio needed for fiscal sustainability is

$$\hat{\tau} = \gamma + (r - g_Y)b_0. \tag{6.10}$$

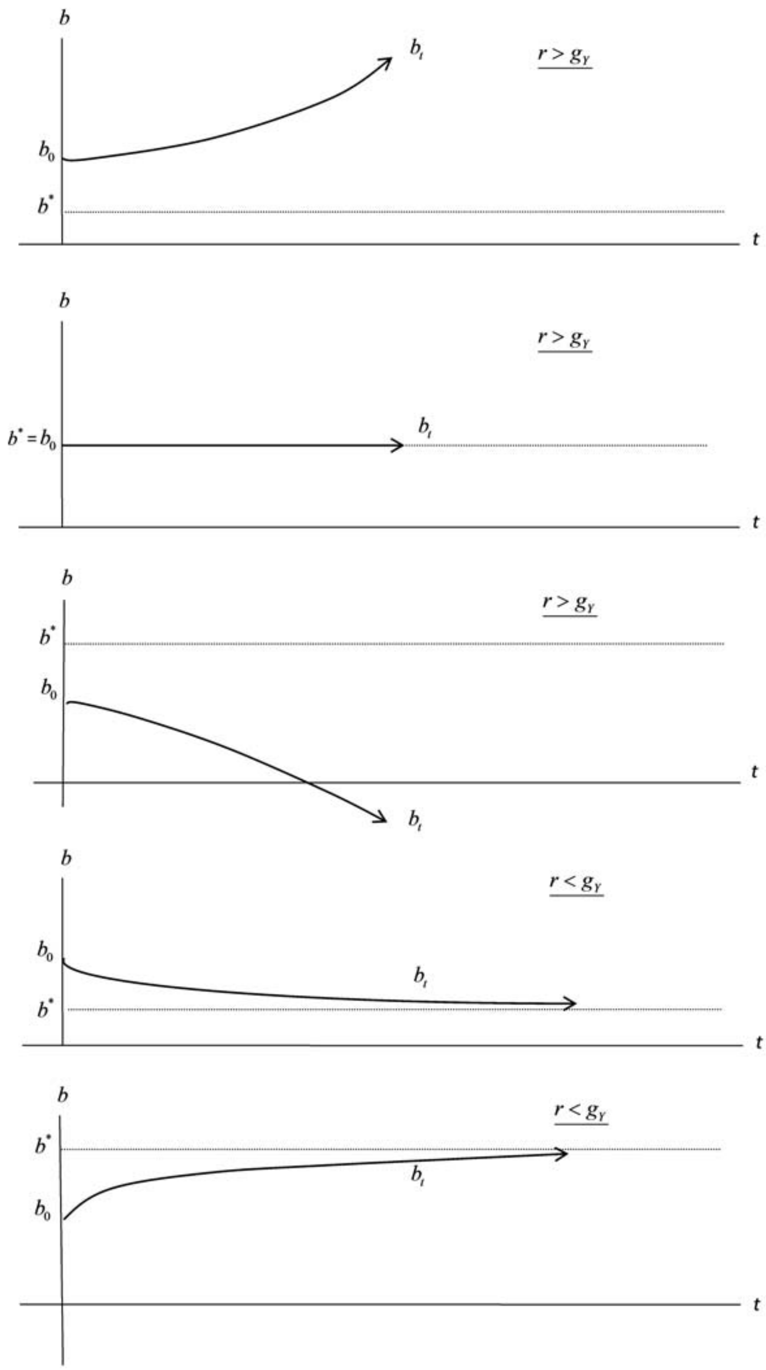

Figure 6.2: Evolution of the debt-income ratio in the cases $r > g_Y$ (the three upper panels) and $r < g_Y$ (the two lower panels), respectively.

The difference, $\hat{\tau} - \tau$, between this needed tax rate and the actual one indicates the size of the needed adjustment, were it to take place at time 0. A more involved indicator of the *sustainability gap*, taking the room for manoeuvre into account, is $(\hat{\tau} - \tau)/(1 - \tau)$.[9] Delaying the adjustment increases the size of the needed policy action, since the debt-income ratio, and thereby $\hat{\tau}$, has become higher in the meantime.

Suppose that the debt build-up can be - and is - prevented already from time 0 by ensuring that the primary surplus as a share of income, $\tau - \gamma$, at least equals $\hat{s}$ so that $b^* \geq b_0$. The solid curve in the midmost panel in Fig. 6.2 illustrates the resulting evolution of the debt-income ratio if b^* is at the level corresponding to the hatched horizontal line while b_0 is unchanged compared with the top panel. Presumably, the government would in such a state of affairs relax its fiscal policy after a while in order not to accumulate large government financial net wealth. Yet, the pre-funding strategy vis-a-vis the fiscal challenge of population ageing (referred to above) is in fact based on accumulating some positive public financial net wealth as a buffer before the substantial effects of population ageing set in. In this context, the higher the growth-corrected interest rate, the shorter the time needed to reach a given positive net wealth position.

Case 2: $r = g_Y$. In this knife-edge case there is still a danger of runaway dynamics, but less explosive. The formula (6.7) is no longer valid. Instead the solution of (6.6) is $b_t = b_0 + [(\gamma - \tau)/(1 + g_Y)]\, t = b_0 - [(\tau - \gamma)/(1 + g_Y)]\, t$. Here, a non-negative primary surplus is both necessary and sufficient to avoid $b_t \to \infty$ for $t \to \infty$.

Case 3: $r < g_Y$. This is the case of stable debt dynamics. The formula (6.7) is again valid, but now implying that the debt-income ratio is non-explosive. Indeed, $b_t \to b^*$ for $t \to \infty$, whatever the level of the initial debt-income ratio and whatever the sign of the budget surplus. Moreover, when $r < g_Y$,

$$b^* = \frac{\tau - \gamma}{r - g_Y} \lesseqqgtr 0 \text{ for } \tau - \gamma \gtreqqless 0. \qquad (*)$$

So, if there is a forever positive primary surplus, the result is a negative long-run debt, i.e., a positive government financial net wealth in the long run. And if there is a forever negative primary surplus, the result is not debt explosion but just convergence toward some positive long-run debt-income ratio. The second from bottom panel in Fig. 6.2 illustrates this case for a situation where $b_0 > b^*$ and $b^* > 0$, i.e., $\tau - \gamma < 0$, by (*). When the

[9] With a Laffer curve in mind, in this formula one should in principle replace the number 1 by a tax rate estimated to maximize the tax revenue in the country considered. This is easier said than done, however, because as noted in the first section of this chapter there are many uncertainties and contingencies involved in the construction of a Laffer curve.

GDP growth rate continues to exceed the interest rate on government debt, a large debt-income ratio can be brought down quite fast, as witnessed by the evolution of both UK and US government debt in the first three decades after the second world war. Indeed, if the growth-corrected interest rate remains negative, permanent debt roll-over can handle the financing, and taxes need never be levied.[10]

Finally, the bottom panel in Fig. 6.2 shows the case where, with a *large* primary deficit ($\tau - \gamma < 0$ but large in absolute value), excess of output growth over the interest rate still implies convergence towards a constant debt-income ratio, albeit a high one.

The level of the debt-income ratio and self-fulfilling expectations of default

We return to Case 1: $r > g_Y$. There will generally be an upper bound for the tax-income ratio deemed feasible by the government (think of the limits for the tax revenue implied by the Laffer curve, say). Similarly, there is a lower bound for the spending-income ratio be it for economic or political reasons. In the present framework we therefore let the government face the constraints $\tau \leq \bar{\tau}$ and $\gamma \geq \bar{\gamma}$, where $\bar{\tau}$ is the least upper bound for the tax-income ratio and $\bar{\gamma}$ is the greatest lower bound for the spending-income ratio. Then the actual primary surplus, s, can at most be $\bar{s} \equiv \bar{\tau} - \bar{\gamma}$.

Suppose that initially the situation in the considered country is as in the second from the top panel in Fig. 6.2. So, initially,

$$s = \tau - \gamma = (r - g_Y)b_0 \leq \bar{s} \equiv \bar{\tau} - \bar{\gamma}, \tag{6.11}$$

with $b_0 > 0$. Define $\bar{r}$ to be the value of r satisfying

$$(\bar{r} - g_Y)b_0 = \bar{s}, \quad \text{i.e., } \bar{r} = \frac{\bar{s}}{b_0} + g_Y. \tag{6.12}$$

So $\bar{r}$ is the maximum interest rate on government bonds consistent with absence of an explosive debt-income ratio.

According to (6.11), some of the fundamentals (the spending- and tax-income ratios) are consistent with no explosion in the debt-income ratio as long as r is unchanged. Nevertheless financial investors may be worried about default if b_0 is high. Investors are aware that a rise in the actual interest rate, r, may always happen and that if it does, a situation with $r - \bar{r} > 0$ is looming, in particular if the country has high debt. Indeed, the larger is b_0 the lower is the critical interest rate, $\bar{r}$, as witnessed by (6.12).

[10]Conditions under which this can theoretically be consistent with general equilibrium are given in connection with the "rational bubbles theme" in Chapter 27.

The worrying scenario is that if $r - \bar{r}$ becomes positive, the unpleasant debt arithmetic in (**) sets in. With $\hat{s}$ still denoting the minimum primary surplus as a share of GDP required for fiscal sustainability, we have

$$r - \bar{r} > 0 \Rightarrow b^*(= \frac{s}{r - g_Y}) \leq \frac{\bar{s}}{r - g_Y} < \frac{\hat{s}}{r - g_Y} = b_0 \Rightarrow b_t \text{ takes off}, \qquad (**)$$

where the second inequality follows from $s \leq \bar{s}$, cf. (6.11), and the last from $(\bar{r} - g_Y)b_0 = \bar{s} < (r - g_Y)b_0 = \hat{s}$, an implication of $r > \hat{r}$ in combination with (6.12) and (6.9). That is, *if* the actual interest rate should rise above the critical interest rate, $\bar{r}$, $\hat{s}$ will exceed $\bar{s}$, thus leading to $b^* < b_0$ whereby runaway debt dynamics takes off, i.e., default is threatening. Moreover, as we saw in connection with (6.12), the risk that $r - \bar{r}$ becomes positive is larger the larger is b_0. The sudden fear that it may happen may be enough to trigger a fall in the market price of government bonds which means a rise in the actual interest rate, r.[11] So financial investors' fear can be a self-fulfilling prophesy. Especially for countries with high $\bar{\gamma}$ or low $\bar{\tau}$ (low capability of collecting taxes as in Greece for instance) a high b_0 is therefore problematic.

Discussion

In the above analysis we have simplified by assuming that several variables, including γ, τ, and r, are constants. The upward trend in the dependency ratio, due to a decreased birth rate and rising life expectancy, together with a rising request for medical care is likely to generate upward pressure on γ. Thereby a high initial debt-income ratio becomes *more* challenging.[12]

On the other hand, as rB_t is income to the private sector, it can, along with the factor income, Y_t, be taxed at the average tax rate τ. Then the benign inequality is no longer $r \leq g_Y$ but $(1 - \tau)r \leq g_Y$, which is more likely to hold (cf. Exercise 6.?).

6.4.2 The Stability and Growth Pact of the EMU

The Maastrict criteria, after the Treaty of Maastrict 1992, for joining the Economic and Monetary Union (EMU) of the EU specified both a government deficit rule and a government debt rule. The first is the rule saying that the

[11] Several observers see the events in the South European part of the Eurozone in 2010-2012 as a manifestation of such a process (De Grauwe and Ji, 2013). The process came to a halt when the European Central Bank in September 2012 declared its willingness to effectively act as a "lender of last resort" (on a conditional basis).

[12] A sustained government budget deficit may also endogenously raise the interest rate in the economy, a topic to which we return in Chapter 13.