Chapter 8

Choice of social discount rate

With an application to the climate change problem

A controversial issue within economists' debate on long-term public investment and the climate change problem is the choice of discount rate. This choice matters a lot for the present value of a project which involves costs that begin now and benefits that occur only after many years, say 75-100-200 years from now, as is the case with the measures against global warming.

Compare the present value of receiving 1000 inflation-corrected euros a hundred years from now under two alternative discount rates, $r = 0.07$ and $r = 0.01$ per year:

$$PV_0 = 1000 * e^{-r*100} = \begin{cases} 0.9 \text{ if } r = 0.07, \\ 368 \text{ if } r = 0.01. \end{cases}$$

So when evaluated at a 7 percent discount rate the 1000 inflation-corrected euros a hundred years from now are worth less than 1 euro today. But with a discount rate at 1 percent they are worth 368 euros today.

In this chapter we discuss different aspects of social discounting, that is, discounting from a policy maker's point of view. We shall set up the theoretical framework around the concept of *optimal capital accumulation* described in Acemoglu, Chapter 8, Section 8.3. In the final sections we apply the framework to an elementary discussion of the climate change problem from an economic perspective.

Unfortunately it is not always recognized that "discount rate" can mean several different things. This sometimes leads to serious confusion, even within academic debates about policies addressing climate change. We therefore start with the ABC of discounting.

A *discount rate* is an interest rate applied in the construction of a *discount factor.* The latter is a factor by which a project's future costs or benefits, measured in some unit of account, are converted into present equivalents. Applying a discount factor thus allows economic effects occurring at different times to be compared. The lower the discount factor the higher the associated discount rate.

Think of period t as running from date t to date $t+1$. More precisely, think of period t as the time interval $[t, t+1)$ on a continuous time axis with time unit equal to the period length. With time t thus referring to the beginning of period t, we speak of "date t" as synonymous with time t. This timing convention is common in discrete-time growth and business cycle theory and is convenient because it makes switching between discrete and continuous time analysis fairly easy.[1] Unless otherwise indicated, our period length, hence our time unit, will be one year.

8.1 Basic distinctions relating to discounting

A basic reason that we have to distinguish between different types of discount rates is that there is a variety of possible units of account.

To simplify matters, in this section we assume there is no uncertainty unless otherwise indicated. Future market interest rates will thus with probability one be equal to the ex ante expected future interest rates.

8.1.1 The unit of account

Money as the unit of account

When the unit of account is *money*, we talk about a *nominal discount rate.* More specifically, if the money unit is euro, we talk about an euro discount rate. Consider a one-period bond promising one euro at date one to the investor buying the bond at date 0. If the market interest rate is i_0, the present value at date 0 of the bond is

$$\frac{1}{1+i_0} \quad \text{euro.}$$

In this calculation the (nominal) *discount factor* is $1/(1+i_0)$ and tells how many euro need be invested in the bond at time 0 to obtain 1 euro at time 1. When the interest rate in this way appears as a constituent of a discount factor, it is called a (nominal) *discount rate.* Like any interest rate it tells

[1] Note, however, that this timing convention is different from that in the standard finance literature where, for example, K_t would denote the *end-of-period* t stock that begins to yield its services *next* period.

how many additional units of account (here euros) are returned after one period of unit length, if one unit of account (one euro) is invested in the asset at the beginning of the period.[2]

A payment stream, $z_0, z_1, \ldots, z_t, \ldots, z_T$, where z_t ($\gtrless 0$) is the net payment in euro due at the *end* of period t, has present value (in euro as seen from the beginning of period 0)

$$PV_0 = \frac{z_0}{1+i_0} + \frac{z_1}{(1+i_0)(1+i_1)} + \cdots + \frac{z_{T-1}}{(1+i_0)(1+i_1)\cdots(1+i_{T-1})}, \quad (8.1)$$

where i_t is the nominal interest rate in euro on a one-period bond from date t to date $t+1$, $t = 0, 1, \ldots, T-1$.

The *average nominal discount rate* from date T to date 0 is the number $\bar{i}_{0,T-1}$ satisfying

$$1 + \bar{i}_{0,T-1} = \left((1+i_0)(1+i_1)\cdots(1+i_{T-1})\right)^{1/T}. \quad (8.2)$$

The corresponding nominal discount factor is

$$(1 + \bar{i}_{0,T-1})^{-T} = \frac{1}{(1+i_0)(1+i_1)\cdots(1+i_{T-1})}. \quad (8.3)$$

If i is constant, the average nominal discount rate is of course the same as i and the nominal discount factor is simply $1/(1+i)^T$.

If the stream of z's in (8.1) represents expected but uncertain dividends to an investor as seen from date 0, we may ask: What is the *relevant* discount rate to be applied on the stream by the investor? The answer is that the relevant discount rate is that rate of return the investor can obtain generally on investments with a similar risk profile. So the relevant discount rate is simply the *opportunity cost* faced by the investor.

In continuous time with continuous compounding the formulas corresponding to (8.1), (8.2), and (8.3) are

$$PV_0 = \int_0^T z(t)\, e^{-\int_0^t i(\tau)d\tau} dt, \quad (8.4)$$

$$\bar{i}(0,T) \equiv \frac{\int_0^T i(\tau)d\tau}{T}, \quad \text{and} \quad (8.5)$$

$$e^{-\bar{i}(0,T)T} = e^{-\int_0^T i(\tau)d\tau}. \quad (8.6)$$

And as above, if i is constant, the nominal discount factor takes the simple form e^{-iT}.

[2] A discount factor is by definition a non-negative number. Hence, a discount rate in discrete time is by definition greater than -1.

Consumption as the unit of account

When the unit of account is a basket of consumption goods or, for simplicity, just a homogeneous consumption good, we talk about a *consumption discount rate* (or a *real* discount rate). Let the consumption good's price in terms of euros be P_t, $t = 0, 1, \ldots, T$. A consumption stream $c_0, c_1, \ldots, c_t, \ldots, c_T$, where c_t is available at the end of period t, has present value (as seen from the beginning of period 0)

$$PV_0 = \frac{c_0}{1+r_0} + \frac{c_1}{(1+r_0)(1+r_1)} + \cdots + \frac{c_{T-1}}{(1+r_0)(1+r_1)\cdots(1+r_{T-1})}. \quad (8.7)$$

Instead of the nominal interest rate, the proper discount rate is now the *real* interest rate, r_t, on a one-period bond from date t to date $t+1$. Ignoring indexed bonds, the real interest rate is not directly observable, but can be calculated in the following way from the observable nominal interest rate i_t :

$$1 + r_t = \frac{P_{t-1}(1+i_t)}{P_t} = \frac{1+i_t}{1+\pi_t},$$

where P_{t-1} is the price (in terms of money) of a period-$(t-1)$ consumption good paid for at the *end* of period $t-1$ (= the beginning of period t) and $\pi_t \equiv P_t/P_{t-1} - 1$ is the inflation rate from period $t-1$ to period t.

The *consumption discount factor* (or *real* discount factor) from date $t+1$ to date t is $1/(1+r_t)$. This discount factor tells how many consumption goods' worth need be invested in the bond at time t to obtain one consumption good's worth at time $t+1$. The stream $c_0, c_1, \ldots, c_t, \ldots, c_T$ could alternatively represent an income stream measured in current consumption units. Then the real interest interest rate, r_t, would still be the relevant real discount rate and (8.7) would give the present real value of the income stream.

The *average consumption discount rate* and the corresponding consumption discount factor are defined in a way analogous to (8.2) and (8.3), respectively, but with i_t replaced by r_t. Similarly for the continuous time versions (8.4), (8.5), and (8.6).

Utility as the unit of account

Even though "utility" is not a measurable entity but just a convenient mathematical devise used to represent preferences, a utility discount rate is in many cases a meaningful concept.

Suppose intertemporal preferences can be represented by a sum of period utilities discounted by a constant rate, ρ :

$$U(c_0, c_1, \cdots, c_{T-1}) = u(c_0) + \frac{u(c_1)}{1+\rho} + \cdots + \frac{u(c_{T-1})}{(1+\rho)^{T-1}}, \quad (8.8)$$

where $u(\cdot)$ is the period utility function. Here ρ appears as a *utility discount rate.* The associated *utility discount factor* from date T to date 0 is $1/(1+\rho)^{T-1}$. We may alternatively write the intertemporal utility function as $\tilde{U}(c_0, c_1, \cdots, c_{T-1}) \equiv (1+\rho)^{-1}U(c_0, c_1, \cdots, c_{T-1})$. Then the utility discount factor from date T to date 0 appears instead as $1/(1+\rho)^T$, which in form corresponds exactly to (8.3); this difference is, however, immaterial, since $\tilde{U}(\cdot)$ and $U(\cdot)$ represent the same preferences and will imply the same choices. In continuous time (with continuous compounding) the "sum" of discounted utility is

$$U_0 = \int_0^T u(c(t))e^{-\rho t}dt,$$

where $e^{-\rho t}$ is the utility discount factor from time t to time 0.[3]

8.1.2 The economic context

Along with the unit of account the economic context of the investment project to be evaluated matters for the choice of discount rate. Here is a brief list of important distinctions:

1. It matters whether the circumstances of relevance for the investment project are endowed with *certainty, computable risk,* or *non-computable risk*, also called *fundamental uncertainty.* In the latter case, the probability distribution is unknown (or scientists deeply disagree about it) and, typically, the full range of possible outcomes is unknown.

2. *Length of the time horizon.* Recently several countries have decided to draw a line between less than vs. more than 30-50 years, choosing a lower discount rate for years on the other side of the line. This is in accordance with recommendations from economists and statisticians arguing that the further ahead in time the discount rate applies, the smaller should it be. With longer time horizons systematic risk and fundamental uncertainty, about both the socio-economic environment as such and the results of the specific project, play a larger role, thus motivating precautionary saving.

3. A *single* or several *different kinds consumption goods.* As we shall see below, the relevant consumption discount rate in a given context

[3]Note that a first-order Taylor approximation of e^x around $x = 0$ gives $e^x \approx e^0 + e^0(x-0) = 1+x$ for x "small"; hence, $x \approx \ln(1+x)$ for x "small". Replacing x by ρ and taking powers, we see the analogy between $e^{-\rho t}$ and $(1+\rho)^{-t}$. Because of the continuous compounding, we have $e^{-\rho t} < (1+\rho)^{-t}$ whenever $\rho > 0$ and $t > 0$ and the difference increases with rising t.

depends on several factors, including the growth rate of consumption. When fundamentally different consumption goods enter the utility function - for instance an ordinary produced commodity versus services from the eco-system - then a disaggregate setup is needed and the relevant consumption discount rate may become an intricate matter. Sterner and Persson (2008) give an introduction to this issue.

4. *Private vs. social.* Discounting from an individual household's or firm's point of view, as it occurs in private investment analysis, is one thing. Discounting from a government's point of view is another, and in connection with evaluation of government projects we speak of *social* cost-benefit analysis. Here externalities and other market failures should be taken into account. Whatever the unit of account, a discount rate applied in social cost-benefit analysis is called a *social discount rate.*

5. *Micro vs. macro.* Social cost-benefit analysis may be concerned with a *microeconomic* project and policy initiatives that involve only marginal changes. In this case a lot of circumstances are exogenous (like in partial equilibrium analysis). Alternatively social cost-benefit analysis may be concerned with a *macroeconomic* project and involve over-all changes. At this level more circumstances are *endogenous*, including possibly the rate of economic growth and the quality of the natural environment on a grand scale. In macroeconomic cost-benefit analysis intra- and intergenerational ethical issues are thus important.

8.2 Criteria for choice of a social discount rate

There has been some disagreement among both economists and policy makers about how to discount in *social* cost-benefit analysis, in particular when the economy as a whole and a long time horizon are involved. At one side we have the *descriptive approach* to social discounting, sometimes called the *opportunity cost* view:

> According to this view, even when considering climate change policy evaluation and caring seriously about future generations, the average market rate of return, before taxes, is the relevant discount rate. This is because funds used today to pay the cost of, say, mitigating greenhouse gas emissions, could be set aside and invested in other things and thereby accumulate at the market rate of return for the benefit of the future generations.

At the other side we find a series of opinions that are not easily lumped together apart from their scepticism about the descriptive approach (in its narrow sense as defined above). These "*other views*" are commonly grouped together under the label the *normative* or *prescriptive* approach. This terminology has become standard. With some hesitation we adopt it here (the reason for the hesitation should become clear below).

One reason that the descriptive approach is by some considered inappropriate is the presence of *market failures.*[4] Another is the presence of *conflicting interests*: those people who benefit may not be the same as those who bear the costs. And where as yet unborn generations are involved, difficult ethical and coordination issues arise.

Amartia Sen (1961) pointed at the *isolation paradox.* Suppose each old has an altruistic concern for *all* members of the next generation. Then a transfer from any member of the old generation to the heir entails an externality that benefits all other members of the old generation. A nation-wide coordination (political agreement) that internalizes these externalities would raise intergenerational transfers (bequests etc.) and this corresponds to a lowering of the intergenerational utility discount rate, ρ, cf. (8.8).

More generally, members of the present generations may be willing to join in a collective contract of more saving and investment by all, though unwilling to save more in isolation.

Other reasons for a relatively low social discount rate have been proposed. One is based on the *super-responsibility argument:* the government has responsibility over a longer time horizon than those currently alive. Another is based on the *dual-role argument*: the members of the currently alive generations may in their political or public role be more concerned about the welfare of the future generations than they are in their private economic decisions.

Critics of the descriptive approach may agree about the relevance of asking: "To what extent will investments made to reduce greenhouse gas emissions displace investments made elsewhere?". They may be inclined to add that there is no guarantee that the funds in question *are* set aside for investment benefitting generations alive two hundred years ahead, say.

Another point against the descriptive approach is that the future damages of global warming could easily be underestimated. If nothing is done now, the risk of the damage being irreparable at any cost becomes higher. Applying the current market rate of return as discount rate for damages occurring

[4]Intervening into the debate about the suitable discount rate for climate change projects, Heal (2008) asks ironically: "Is it appropriate to assume no market failure in evaluating a consumption discount rate for a model of climate change?".

say 200 years from now on may imply that these damages become almost imperceptible and so action tends to be postponed. This may be problematic if there is a positive albeit low probability that a tipping point with disastrous consequences is reached.

The reason for hesitation to lump together these “other views" under the labels *normative* or *prescriptive* approach is that the contraposition of “descriptive” versus “normative” in this context may be misleading. In the final analysis also the *descriptive approach* has a normative element namely the view that the social discount rate *ought to* be that implied by the market behavior of the current generations as reflected in the current market interest rate - the alternative is seen as paternalism.[5]

Anyway, in practice there seems to be a kind of convergence in the sense that elements from the descriptive and the prescriptive way of thinking tend to be combined. Nevertheless, there is considerable diversity across countries regarding the governments’ official “social consumption discount rate” (sometimes just called the “social discount rate”) to be applied for public investment projects. Even considering only West-European countries and Western Offshoots, including the U.S., the range is roughly from 8% to 2% per year. An increasing fraction of these countries prescribe a lower rate for benefits and costs accruing more than 30-50 years in the future (Harrison, 2010). The Danish Ministry of Finance recently (May 2013) reduced its social consumption discount rate from 5% per year to 4% per year for the first 35 years of the time horizon of the project, 3% for the years in the interval 36 to 69 years, and 2% for the remainder of the time horizon if relevant.[6] Among economists involved in climate change policy evaluation there is a wide range regarding what the recommended social discount rate should be (from 1.4% to 8.0%).[7] An evaluation of the net worth of the public involvement in the Danish wind energy sector in the 1990s gives opposite conclusions depending on whether the discount rate is 5-6% (until recently the official Danish discount rate) or 3-4% (Hansen, 2010).

This diversity notwithstanding, let us consider some examples of social cost-benefit problems of a macroeconomic nature and with a long time horizon. Our first example will be the standard neoclassical problem of optimal capital accumulation.

[5] Here the other side of the debate may respond that such “paternalism” need not be illigitimate but rather the responsibility of democratically elected governments.

[6] Finansministeriet (2013) .

[7] Harrison (2010).

8.3 Optimal capital accumulation

The perspective is that of an "all-knowing and all-powerful" social planner facing a basic intertemporal allocation problem in a closed economy: how much should society save? The point of departure for this problem is the prescriptive approach. The only discount rate which is decided in advance is the *utility* discount rate, ρ. No consumption discount rate is part of either the objective function or the constraints. Instead, a long-run consumption discount rate applicable to a class of public investment problems comes out as a *by-product* of the steady-state *solution* to the problem.

8.3.1 The setting

We place our social planner in the simplest neoclassical set-up with exogenous Harrod-neutral technical change. Uncertainty is ignored. Although time is continuous, for simplicity we date the variables by sub-indices, thus writing Y_t etc. The aggregate production function is neoclassical and has CRS:

$$Y_t = F(K_t, T_t L_t) \equiv T_t L_t f(\tilde{k}_t), \tag{8.9}$$

where Y_t is output, K_t physical capital input, and L_t labor input which equals the labor force which in turn equals the population and grows at the constant rate n. The argument in the production function on intensive form is defined by $\tilde{k}_t \equiv K_t/(T_t L_t)$. The factor T_t represents the economy-wide level of technology and grows exogenously according to

$$T_t = T_0 e^{gt}, \tag{8.10}$$

where $T_0 > 0$ and $g \geq 0$ are given constants. Population grows at the constant rate $n \geq 0$. Output is used for consumption and investment so that

$$\dot{K}_t = Y_t - c_t L_t - \delta K_t, \tag{8.11}$$

where c_t is per capita consumption and $\delta \geq 0$ a constant capital depreciation rate.

The social planner's objective is to maximize a *social welfare function*, W. We assume that this function is time separable with (i) an instantaneous utility function $u(c)$ with $u' > 0$ and $u'' < 0$ and where c is per capita consumption; (ii) a constant utility discount rate $\rho \geq 0$, often named "the pure rate of time preference"; and (iii) an infinite time horizon. The social planner's optimization problem is to choose a plan $(c_t)_{t=0}^{\infty}$ so as to maximize

$$W = \int_0^{\infty} u(c_t) L_t e^{-\rho t} dt \qquad \text{s.t.} \tag{8.12}$$

$$c_t \geq 0, \tag{8.13}$$

$$\dot{\tilde{k}}_t = f(\tilde{k}_t) - \frac{c_t}{T_t} - (\delta + g + n)\tilde{k}_t, \qquad \tilde{k}_0 > 0 \text{ given}, \tag{8.14}$$

$$\tilde{k}_t \geq 0 \qquad \text{for all } t \geq 0. \tag{8.15}$$

Comments

1. If there are technically feasible paths along which the improper integral W goes to $+\infty$, a maximum of W does not exist (in the CRRA case, $u(c) = c^{1-\theta}/(1-\theta)$, $\theta > 0$, this will happen if and only if the parameter condition $\rho - n > (1-\theta)g$ is *not* satisfied). By "optimizing" we then mean finding an "overtaking optimal" solution or a "catching-up optimal" solution, assuming one of either exists (cf. Sydsæter et al. 2008).

2. The long time horizon should be seen as involving many successive and as yet unborn generations. Comparisons across time should primarily be interpreted as comparisons across generations.

3. The model abstracts from inequality within generations.

4. By weighting per capita utility by L_t and thereby effectively taking population growth, n, into account, the social welfare function (8.12) respects the principle of *discounted classical utilitarianism.* A positive *pure* rate of time preference, ρ, implies discounting the utility of future people just because they belong to the future. Some analysts defend this discounting of the future by the argument that it is a typical characteristic of an individual's preferences. Others find that this is not a valid argument for long-horizon evaluations because these involve different persons and even as yet unborn generations. For example Stern (2007) argues that the only ethically defensible reason for choosing a positive ρ is that there is always a small risk of extinction of the human race due to for example a devastating meteorite or nuclear war. This issue aside, in (8.12) the *effective* utility discount rate will be $\rho - n$. This implies that the larger is n, the more weight is assigned to the future because more people will be available.[8] We shall throughout assume that the size of population is exogenous although this may not accord entirely well with large public investment projects, like climate change mitigation, that have implication for health and mortality. With endoge-

[8] In contrast, the principle of discounted *average* utilitarianism is characterized by population growth *not* affecting the effective utility discount rate. This corresponds to eliminating the factor L_t in the integrand in (8.12).

nous population very difficult ethical issues arise (Dasgupta (2001), Broome (2005)).

8.3.2 First-order conditions and their economic interpretation

To characterize the solution to the problem, we use the Maximum Principle. The current-value Hamiltonian is

$$H(\tilde{k}, c, \lambda, t) = u(c) + \lambda \left[f(\tilde{k}) - \frac{c}{T} - (\delta + g + n)\tilde{k} \right],$$

where λ is the adjoint variable associated with the dynamic constraint (8.14). An interior optimal path $(\tilde{k}_t, c_t)_{t=0}^{\infty}$ will satisfy that there exists a continuous function $\lambda = \lambda_t$ such that, for all $t \geq 0$,

$$\frac{\partial H}{\partial c} = 0, \text{ i.e., } u'(c) = \frac{\lambda}{T}, \quad \text{and} \tag{8.16}$$

$$\frac{\partial H}{\partial \tilde{k}} = \lambda(f'(\tilde{k}) - \delta - g - n) = (\rho - n)\lambda - \dot{\lambda} \tag{8.17}$$

hold along the path and the transversality condition,

$$\lim_{t \to \infty} \tilde{k}_t \lambda_t e^{-(\rho - n)t} = 0, \tag{8.18}$$

is satisfied.

By taking logs on both sides of (8.16) and differentiating w.r.t. t we get

$$\frac{du'(c_t)/dt}{u'(c_t)} = \frac{u''(c_t)}{u'(c_t)}\dot{c}_t = \frac{\dot{\lambda}_t}{\lambda_t} - g = \rho - (f'(\tilde{k}_t) - \delta),$$

where the last equality comes from (8.17). Reordering gives

$$f'(\tilde{k}_t) - \delta = \rho + \left(-\frac{u''(c_t)}{u'(c_t)} \right) \dot{c}_t, \tag{8.19}$$

where the term $(-u''(c_t)/u'(c_t)) > 0$ indicates the rate of decline in marginal utility when consumption is increased by one unit. So the right-hand side of (8.19) exceeds ρ when $\dot{c}_t > 0$.

A technically feasible path satisfying both (8.19) and the transversality condition (8.18) with $\lambda_t = T_t u'(c_t)$ will be an optimal path and there are no other optimal paths.[9]

[9] This follows from *Mangasarian's sufficiency theorem* and the fact that the Hamiltonian is *strictly concave* in $(\tilde{k}, \tilde{c})$. The implied resource allocation will be the same as that of a competitive conomy with the same technology as that given in (8.9) and with a representative household that has the same intertemporal preferences as those of the social planner given in (8.12) (this is the *Equivalence theorem*).

The optimality condition (8.19) could of course be written on the standard Keynes-Ramsey rule form, where $\dot{c}_t/c_t$ is isolated on one side of the equation. But from the perspective of rates of return, and therefore discount rates, the form (8.19) is more useful, however. The condition expresses the general principle that in the optimal plan the marginal unit of per capita output is equally valuable whether used for investment or current consumption. When used for investment, it gives a rate of return equal to the net marginal productivity of capital indicated on the left-hand side of (8.19). When used for current consumption, it raises current utility. Doing this to an extent just enough so that no further postponement of consumption is justified, the *required* rate of return is exactly obtained. The condition (8.19) says that in the optimal plan the *actual* marginal rate of return (the left-hand side) equals the *required* marginal rate of return (the right-hand side).

Reading the optimality condition (8.19) from the right to the left, there is an analogy between this condition and the general microeconomic principle that the consumer equates the marginal rate of substitution, MRS, between any two consumption goods with the price ratio given from the market. In the present context the two goods refer to the same consumption good delivered in two successive time intervals. And instead of a price ratio we have the marginal rate of transformation, MRT, between consumption in the two time intervals as given by technology. The analogy is only partial, however, because this MRT is, from the perspective of the optimizing agent (the social planner) not a given but is endogenous just as much as the MRS is endogenous.

8.3.3 The social consumption discount rate

More specifically, (8.19) says that the social planner will sacrifice per capita consumption today for more per capita consumption tomorrow only up to the point where this saving for the next generations is compensated by a rate of return sufficiently above ρ. Naturally, the required compensation is higher, the faster marginal utility declines with rising consumption, i.e., the larger is $(-u''/u')\dot{c}$. Indeed, every extra unit of consumption handed over to future generations delivers a smaller and smaller marginal utility to these future generations. So the marginal unit of investment today is only warranted if the marginal rate of return is sufficiently above ρ, as indicated by (8.19).

Letting the required marginal rate of return be denoted r_t^{SP} and letting the values of the variables along the optimal time path be marked by a bar, we can write the right-hand side of (8.19) as

$$r_t^{SP} = \rho + \theta(\bar{c}_t)\frac{\dot{\bar{c}}_t}{\bar{c}_t}, \tag{8.20}$$

where $\theta(c) \equiv -cu''(c)/u'(c) > 0$ (the absolute elasticity of marginal utility of consumption). For a given $\theta(\bar{c}_t)$, a higher per capita consumption growth rate implies a higher required rate of return on marginal saving. In other words, the higher the standard of living of future generations compared with current generations, the higher is the required rate of return on current marginal saving. Indeed, less should be saved for the future generations. Similarly, for a given per capita consumption growth rate, $\dot{\bar{c}}_t/\bar{c}_t > 0$, the required rate of return on marginal saving is higher, the larger is $\theta(\bar{c}_t)$. This is because $\theta(\bar{c}_t)$ reflects *aversion towards consumption inequality across time and generations* (in a context with uncertainty $\theta(\bar{c}_t)$ also reflects what is known as the *relative risk aversion,* see below). Indeed, $\theta(\bar{c}_t)$ indicates the percentage fall in marginal utility when per capita consumption is raised by one percent. So a higher $\theta(\bar{c}_t)$ contributes to more consumption smoothing over time.

So far these remarks are only various ways of interpreting an optimality condition. Worth emphasizing is:

- The *required* marginal rate of return (the right-hand side of (8.20)) at time t is not something given in advance, but an endogenous and time-dependent variable which along the optimal path must equal the *actual* marginal rate of return (the endogenous rate of return on investment represented by the left-hand side of equation (8.20)). Indeed, both the required and the actual marginal rates of return are endogenous because they depend on the endogenous variables c_t and $\dot{c}_t$ and on what has been decided up to time t and is reflected in the current value of the state variable, $\tilde{k}_t$. As we know from phase diagram analysis in the $(\tilde{k}, c/T)$ plane, there are infinitely many technically feasible paths satisfying the inverted Keynes-Ramsey rule in (8.20) for all $t \geq 0$. What is lacking up to now is to select among these paths one that satisfies the transversality condition (8.18).

- In the present problem the only discount rate which is decided in advance is the *utility* discount rate, ρ. No consumption discount rate is part of either the objective function or the constraints. We shall now see, however, that a long-run consumption discount rate applicable to (less-inclusive) public investment problems comes out as a *by-product* of the steady-state *solution* to the problem.

Steady state

To help existence of a steady state we now assume that the instantaneous utility, $u(c)$, belong to the CRRA family so that $\theta(c) = \theta$, a positive constant. Then

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta}, & \text{when } \theta > 0, \theta \neq 1, \\ \ln c, & \text{when } \theta = 1. \end{cases} \tag{8.21}$$

We know that if the parameter condition $\rho - n > (1-\theta)g$ holds and f satisfies the Inada conditions, then there *exists* a *unique* path satisfying the necessary and sufficient optimality conditions, including the transversality condition (8.18). Moreover, this path *converges* to a balanced growth path with a constant effective capital-labor ratio, $\tilde{k}^*$, satisfying $f'(\tilde{k}^*) - \delta = \rho + \theta g$. So, at least for the *long run,* we may replace $\dot{\bar{c}}_t/\bar{c}_t$ in (8.20) with the constant rate of exogenous technical progress, g. Then (8.20) reduces to a required consumption rate of return that is now *constant* and *given* by the parameters in the problem:

$$r^{SP} = \rho + \theta g. \tag{8.22}$$

This r^{SP} is the prevalent suggestion for the choice of a social consumption discount rate. Note that as long as $g > 0$, r^{SP} will be positive even if $\rho = 0$. A higher θ will imply stronger discounting of additional consumption in the future because higher θ means faster decline in the marginal utility of consumption in response to a given rise in consumption. So with g equal to, say, 1.5% per year, the social discount rate r_{SP} is in fact more sensitive to the value of θ than to the value of ρ. Note also that a higher g raises r_{sp} and thereby reduces the incentive to save and invest.

Now consider a potential public investment project with time horizon T ($\leq \infty$) which comes at the expense of an investment in capital in the "usual" way as described above. Suppose the project is "minor" or "local" in the sense of not affecting the structure of the economy as a whole, like for instance the long-run per capita growth rate, g. Let the project involve an initial investment outlay of k_0 and a stream of real net revenues, $(z_t)_{t=0}^{T}$, assuming that both costs and benefits are measurable in terms of current consumption equivalents.[10] Letting r^{SP} serve to convert future consumption into current consumption equivalents, we calculate the present value of the project,

$$PV_0 = -k_0 + \int_0^T z_t e^{-r^{SP}t} dt.$$

[10] We bypass all the difficult issues involved in converting non-marketed goods like environmental qualities, biodiversity, health, and mortality risk etc. into consumption equivalents.

The project is worth undertaking if $PV_0 > 0$.

Limitations of the Ramsey formula $r^{SP} = \rho + \theta g$

For a closed economy, reasonably well described by the model, it makes sense to choose the r^{SP} given in (8.22) as discount rate for public investment projects if the economy is not "far" from its steady state. Yet there are several cases where modification is needed:

1. Assuming the model still describes the economy reasonably well, if the actual economy is initially "far" from its steady state and T is of moderate size, g in (8.22) should be replaced by a somewhat larger value if $\tilde{k}_0 < \tilde{k}$ (since in that case $\dot{c}/c > g$) and somewhat smaller value if $\tilde{k}_0 > \tilde{k}$ (since in that case $\dot{c}/c < g$).

2. The role of natural resources, especially non-renewable natural resources, has been ignored. If they are essential inputs, the parameter g needs reinterpretation and a negative value can not be ruled out *apriori*. In that case the social discount rate can in principle be negative.

3. Global problems like the climate change problem has an important international dimension. As there is great variation in the standard of living, c, and to some extent also in g across developed and developing countries, it might be relevant to include not only a parameter, θ_1, reflecting aversion towards consumption inequality over time and generations but also a parameter, θ_2, reflecting aversion towards *spatial* consumption inequality, i.e., inequality across countries.

4. Another limitation of the Ramsey formula (8.22), as it stands, is that it ignores *uncertainty*. In particular with a long planning horizon uncertainty both concerning the results of the investment project and concerning the socio-economic environment are important and should of course be incorporated in the analysis.

5. Finally, for "large" macroeconomic projects, the long-run technology growth rate may not be given, but dependent on the chosen policy. In that case, neither g nor r^{SP} are given. This is in fact the typical situation within the macroeconomic theory of *endogenous productivity growth.* Then formulation of a "broader" optimization problem is necessary and only parameters like the utility discount rate, ρ, and the elasticity of marginal utility of consumption, θ, will in this case serve as points of departure.

In connection with the climate change problem we shall in the next section apply a brief article by Arrow (2007)[11] to illustrate at least one way to deal with the problems 4 and 5.

8.4 The climate change problem from an economic point of view

There is now overwhelming agreement among scientists that man-made global warming is a reality. Mankind faces a truly large-scale and global economic problem with potentially dramatic consequences for economic and social development in centuries. Future economic evolution is *uncertain* and *depends* on policies chosen now. A series of possible "act now" measures has been described in detail in the voluminous *The economics of climate change. The Stern Review,* made by a team of researchers lead by the prominent British economist Nicholas Stern (Stern 2007).

The mentioned article by Arrow is essentially a comment on the *Stern Review* and on the debate about discount rates it provoked among climate economists as well as in the general public. It is Arrow's view that taking *risk aversion* properly into account implies that the conclusion of the Stern Review goes through: Mankind is better off to act *now* to reduce CO_2 emissions substantially rather than to risk the consequences of failing to meet this challenge. In many areas of life, high insurance premia are willingly paid to reduce risks. It is in such a perspective that part of the costs of mitigation should be seen.

8.4.1 Damage projections

As asserted by the Stern Review, the CO_2 problem is "the greatest and widest-ranging market failure ever seen" (Stern 2007, p.). The current level of CO_2 (including other greenhouse gases, in CO_2 equivalents) is today (i.e., in 2007) about 430 parts per million (ppm), compared with 280 ppm before the industrial revolution. Under a "business as usual" assumption the level will likely be around 550 ppm by 2035 and will continue to increase. The level 550 ppm is almost twice the pre-industrial level, and a level that has not been reached for several million years.

Most climate change models predict this would be associated with a rise in temperature of at least two degrees Centigrade, probably more. A continuation of "business as usual" is likely to lead to a trebling of CO_2 by the end

[11] Arrow won the Nobel Prize in Economics in 1972.

of the century and to a 50% likelihood of a rise in temperature of more than five degrees Centigrade. Five degrees Centigrade are about the same as the increase from the last ice age to the present.

The full consequences of such rises are not known. But drastic negative effects on agriculture in the heavily populated tropical regions due to changes in rainfall patterns are certain. The rise in the sea level will wipe out small island countries, and for example Bangladesh will loose much of its land area. A reversal of the Gulf Stream is possible, which could cause climate in Europe to resemble that of Greenland. Tropical storms and other kinds of extreme weather events will become severe and many glaciers will disappear and with them, valuable water supplies.

The challenging factors are that the emissions of CO_2 and other gases are *almost irreversible.* They constitute a global *negative externality at a grand scale.* The Stern Review assesses that avoiding such an outcome is possible by a series of concrete measures (carbon taxes, technology policy, international collective action) aimed at stopping or at least reducing the emission of green-house gases and mitigate their consequences. Out of the Stern Review's suggested range of the estimated costs associated with this, in his evaluation of an "act now" policy Arrow chooses a cost level of 1% of GNP every year forever (see below).

According to many observers, postponing action is likely to increase both risks and costs. The Stern Review suggests that the costs of action *now* are less than the costs of inaction because the marginal damages of rising temperature increase strongly as temperatures rise. In the words by Nobel Laureate, Joseph Stiglitz: "[The Stern Review] makes clear that the question is not whether we can afford to act, but whether we can afford not to act" (Stiglitz, 2007).

8.4.2 Uncertainty, risk aversion, and the certainty-equivalent loss

Since there is *uncertainty* about the size of the future damages, we follow Arrow's attempt to convert this uncertainty into the *certainty-equivalent* damage.

Given preferences involving risk aversion, an uncertain gain can be evaluated as being equivalent to a single gain *smaller* than the expected value (the "average") of the possible outcomes. With the green-house gas effect mankind is facing an uncertain *damage* which should be evaluated as being equivalent to a single loss *greater* than the expected value of the possible damages. For the so-called High-climate Scenario (considered by Arrow to

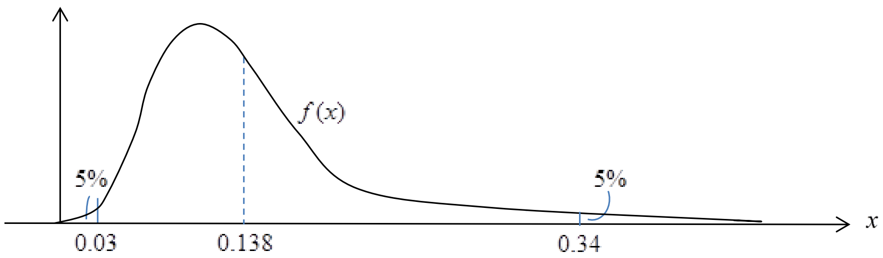

Figure 8.1: The density function of the per capita consumption loss X in year 2200.

be the best-substantiated scenario) the Stern Review estimates that by year 2200 the losses in global GNP per capita, by following a "business as usual" policy compared with , have an *expected* value of 13.8% of what global GNP per capita would be if green-house gas concentration is prevented from exceeding 550 ppm. The estimated loss distribution has a 0.05 percentile of about 3% and a 0.95 percentile of about 34%.

Assuming consumption per capita, c, in year 2200 is proportional to GNP per capita in year 2200, let us recapitulate:

$$\begin{aligned} &\text{under "mitigation now" policy (MNP):} && c = c_1, \\ &\text{under "business as usual" (BAU):} && c = (1-X)c_1 \equiv c_0, \end{aligned}$$

where c_1 is considered given while X is a stochastic variable measuring the fraction of c_1 lost in year 2200 due to the damage occurring under BAU. A probability density function of X according to the High-climate Scenario is represented by $f(x)$ in Figure 8.1. The expected loss of $EX = \int_0^1 xf(x)dx = 0.138$ is indicated and so are the 5th and 95th percentiles of 0.03 and 0.34, respectively.[12] The distribution is right-skew.

Let x_0 denote the *certainty-equivalent loss*, that is, the number x_0 satisfying

$$u((1-x_0)c_1) = Eu((1-X)c_1) = Eu(c_0). \tag{8.23}$$

This means that an agent with preferences expressed by u is indifferent between facing a certain loss of size x_0 or an uncertain loss, X, that has density function f.

The condition (8.23) is illustrated in Figure 8.2. The density function for the stochastic BAU consumption level, c_0, is indicated in the lower panel of

[12] The Stern Review estimates that $X < 0$ has zero probability.

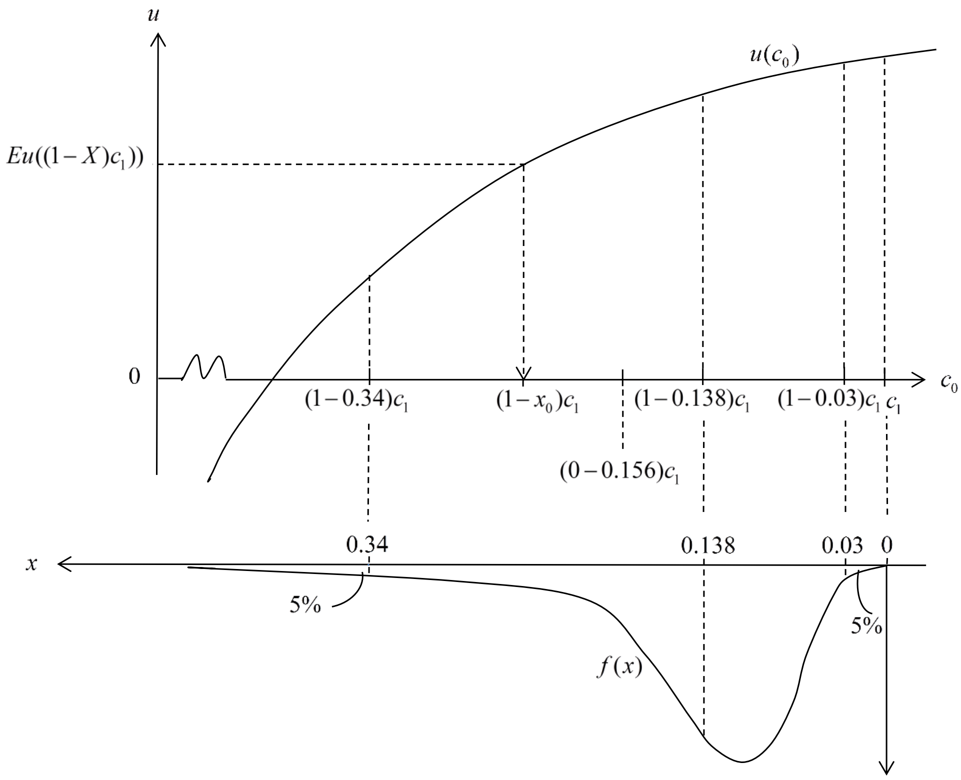

Figure 8.2: The certanty-equivalent loss, x_0, assuming expected utility is known.

the figure by a reversed coordinate system. If the utility function is specified and one knows the complete density function, then $Eu((1-X)c_1)$ is known and the certainty-equivalent BAU consumption level $(1-x_0)c_1$, can be read off the diagram.

The instantaneous utility function chosen by Arrow as well as Stern is of the CRRA form (8.21). Arrow proposes the value 2 for θ, while the Stern Review relies on $\theta = 1$ which by many critics was considered "too low" from a descriptive-empirical point of view. As mentioned above, in a context of uncertainty, θ not only measures the aversion towards consumption inequality across time and generations but also the level of *relative risk aversion*.

The problem now is that the loss density function $f(x)$ is *not* known. The Stern Review only reports an estimated mean of 0.138 together with estimated 5th and 95th percentiles of 0.03 and 0.34, respectively. This does not suffice for calculation of a good estimate of expected utility, $Eu((1-X)c_1)$. At best one can give a rough approximation. Arrow's approach to this problem is to split the probability mass into two halves and place them on the 5th and 95th percentiles, respectively, assuming this gives a reasonable approximation:

$$Eu((1-X)c_1) \approx u((1-0.03)c_1)0.5 + u((1-0.34)c_1)0.5. \tag{8.24}$$

With $u(c)$ given as CRRA, by (8.23) and (8.24) we thus have

$$\frac{[(1-x_0)c_1]^{1-\theta}}{1-\theta} \approx \frac{[(1-0.03)c_1]^{1-\theta}}{1-\theta}0.5 + \frac{[(1-0.34)c_1]^{1-\theta}}{1-\theta}0.5,$$

since the additive constant $-1/(1-\theta)$ cancels out on both sides. We see that also $c_1^{1-\theta}/(1-\theta)$ cancels out on both sides so that we are left with

$$(1-x_0)^{1-\theta} \approx (1-0.03)^{1-\theta}0.5 + (1-0.34)^{1-\theta}0.5.$$

With $\theta = 2$ the approximative estimate of the certainty-equivalent loss is $\hat{x}_0 = 0.21$, that is "about 20%" (of GNP per capita in year 2200) as Arrow says (Arrow 2007, p. 5).[13]

Here we shall proceed with this estimate of the certainty-equivalent loss while in the appendix we briefly discuss the quality of the estimate. On average the estimated certainty-equivalent loss corresponds to a decrease of the expected growth rate per year of GNP per capita between year 2001 and year 2200 from $g_1 = 1.3\%$ (the base rate of GNP per capita growth before the damages by further "business as usual") to $g_0 = 1.2\%$ per year.

[13] Although the calculation behind these "about 20%" is not directly reported in Arrow's brief article, he has in an e-mail to me confirmed that (8.24) *is* the applied method.

8.4.3 Comparing benefit and costs

Avoiding the projected fall in average per capita consumption growth is thus the *benefit* of the "mitigation now" policy while the *costs* amount to the above-mentioned 1% of GNP every year forever.

The criterion for assessing whether the "mitigation now" policy is worth the costs is the social (in fact "global") welfare function presented in (8.12) above with instantaneous utility being of CRRA form.[14] Following Arrow we let θ equal 2 (while Stern has $\theta = 1$).

The Stern Review has been criticized by several economic analysts for adopting "too low" values of both the two intergenerational preference parameters, θ and ρ. As to the rate of time preference, ρ, following the "descriptive approach", these critics argue that a level about 1-3% per year is better in line with a backward calculation from observed market rates of return. Anticipating such criticism, the Stern Review fights back by claiming that such high values are not ethically defensible since they amount to discriminating future generations for the only reason that they belong to the future. As mentioned in Section 8.3.1, Stern argues that the only ethically defensible reason for choosing a positive ρ is that there always is a small risk of extinction of the human race due to for example a devastating meteorite or nuclear war. Based on this view, Stern chooses a value of ρ close to zero, namely $\rho = 0.001$.[15] As Arrow argues and as we shall see in a moment, this disagreement as to the size of ρ is not really crucial given the involved benefit and costs.

The break-even utility discount rate

Assuming balanced growth with some constant productivity growth rate, g, consumption per capita will also grow at the rate g, i.e., $c_t = c(0)e^{gt}$ for all $t \geq 0$.[16] Then

$$u(c_t) = \frac{(c(0)e^{gt})^{1-\theta}}{1-\theta} - \frac{1}{1-\theta},$$

[14] We ignore the minor difference vis-a-vis the Stern Review that it brings in a so-called scrap value function subsuming discounted utility from year 2200 to infinity.

[15] This is in fact a relatively high value of ρ in the sense that it suggests that the probability of extinction within one hundred years from now is as high as 9.5% ($1-P(X < x)) = 1 - e^{-0.1} = 0.095$). But as the Stern Review (p. 53) indicates, the term "extinction" is meant to include "partial extinction by som exogenous or man-made force which has little to do with climate change".

[16] To avoid confusion with the above c_0, we write initial per capita consumption $c(0)$ rather than c_0.

along the balanced growth path. As adding or subtracting a constant from the utility function changes neither the preferences nor the economic behavior, from now we skip the constant $(1-\theta)^{-1}$. Under the BAU policy the social welfare function then takes the value[17]

$$\begin{aligned} W_0 &= \frac{c(0)^{1-\theta}}{1-\theta}\int_0^\infty (e^{g_0 t})^{1-\theta}e^{-(\rho-n)t}dt = \frac{c(0)^{1-\theta}}{1-\theta}\int_0^\infty e^{[(1-\theta)g_0-(\rho-n)]t}dt \\ &= \frac{c(0)^{1-\theta}}{1-\theta}\frac{1}{\rho-n-(1-\theta)g_0}. \end{aligned}$$

Let the value of the welfare outcome under the "mitigation now" policy be denoted W_1. According to the numbers mentioned above, the latter policy involves a *cost* whereby $c(0)$ is replaced by $c(0)' = 0.99c(0)$ and a *benefit* whereby $g_0 = 0.012$ is replaced by $g_1 = 0.013$.[18] We get

$$W_1 = \frac{(0.99c(0))^{1-\theta}}{1-\theta}\frac{1}{\rho-n-(1-\theta)g_1}.$$

Since the benefits of the "mitigation now" policy come in the future and the costs are there from date zero, we have $W_1 > W_0$ only if the effective utility discount rate, $\rho - n$, is below some upper bound. Let us calculate the least upper bound. With $\theta = 2$, we have

$$\begin{aligned} W_1 &= -(0.99c(0))^{-1}\frac{1}{\rho-n+g_1} > W_0 = -(c(0))^{-1}\frac{1}{\rho-n+g_0} \\ &\Rightarrow \frac{1}{0.99(\rho-n+g_1)} < \frac{1}{\rho-n+g_0} \\ &\Rightarrow 0.99(\rho-n+g_1) > \rho-n+g_0 \\ &\Rightarrow 0.01(\rho-n) < 0.99g_1 - g_0 = 0.00087 \\ &\Rightarrow \rho-n < 0.087 \text{ or } \rho-n < 8.7\% \text{ per year.} \end{aligned}$$

The *break-even level* for $\rho - n$ at which $W_1 = W_0$ is thus 8.7% per year.

As Arrow remarks, "no estimate of the pure rate of time preference even by those who believe in relatively strong discounting of the future has ever approached 8.5%".[19] The conclusion is that given the estimated certainty-

[17]The transversality condition holds and the utility integral W_0 is convergent if $\rho - n > (1-\theta)g_0$. In the present case where $\rho = 0.001$, $\theta = 2$ and $g_0 = 0.012$, W_0 is thus convergent for $n < \rho - (1-\theta)g_0 = \rho + g_0 = 0.013$. This inequality seems likely to hold.

[18]By taking $g_1 = 0.013 > g_0$ also after year 2200, we deviate a little from both Arrow and Stern in a direction favoring the Stern conclusion slightly.

[19]Possibly the difference between Arrow's 8.5% and our result is due to the point mentioned in the previous footnote. Another minor difference is that Arrow seemingly takes n to be zero since he speaks of the "pure rate of time preference" rather than the "effective rate of time preference", $\rho - n$.

equivalent loss, the "mitigation now" policy passes the cost-benefit test for *any* reasonable value of the pure rate of time preference.

It should be mentioned that there has been considerably disagreement also about other aspects of the Stern Review's investigation, not the least the time profiles for the projected benefits and costs.[20] So it is fair to say that "further sensitivity analysis is called for", as Arrow remarks. He adds: "Still, I believe there can be little serious argument over the importance of a policy of avoiding major further increases in combustion by-products" (Arrow 2007, p. 5)

8.5 Conclusion

In his brief analysis of the economics of the climate change problem Arrow (2007) finds the fundamental conclusion of the Stern Review justified even if one, unlike the Stern Review, heavily discounts the utility of future generations. In addition to discounting, risk aversion plays a key role in the argument. A significant part of the costs of mitigation is like an insurance premium society should be ready to pay.

The analysis above took a *computable risk approach.* For more elaborate accounts about uncertainty issues, also involving situations with systematic uncertainty, about c_1 for instance, increasing with the length of the time horizon as well as fundamental uncertainty, see the list of references, in particular the papers by Gollier and Weitzman.

We have been tacit concerning the difficult political economy problems about how to obtain coordinated international action vis-a-vis global warming. About this, see, e.g., Gersbach (2008) and Roemer (2010).

8.6 Appendix: A closer look at Arrow's estimate of the certainty loss

In this appendix we briefly discuss Arrow's estimate of the certainty-equivalent loss based on (8.24). The applied procedure would be accurate if the density function $f(x)$ were *symmetric* and the utility function $u(c)$ were *linear.*

So let us first consider the case of a linear utility function, $\tilde{u}(c)$, cf. the stippled positively sloped line in Figure 8.3. With $f(x)$ symmetric, EX coincides with the median of the distribution. Given the estimated 5th and 95th percentiles of 0.03 and 0.34, respectively, we would thus have EX

[20] See for example: http://en.wikipedia.org/wiki/Stern_Review#cite_ref-5

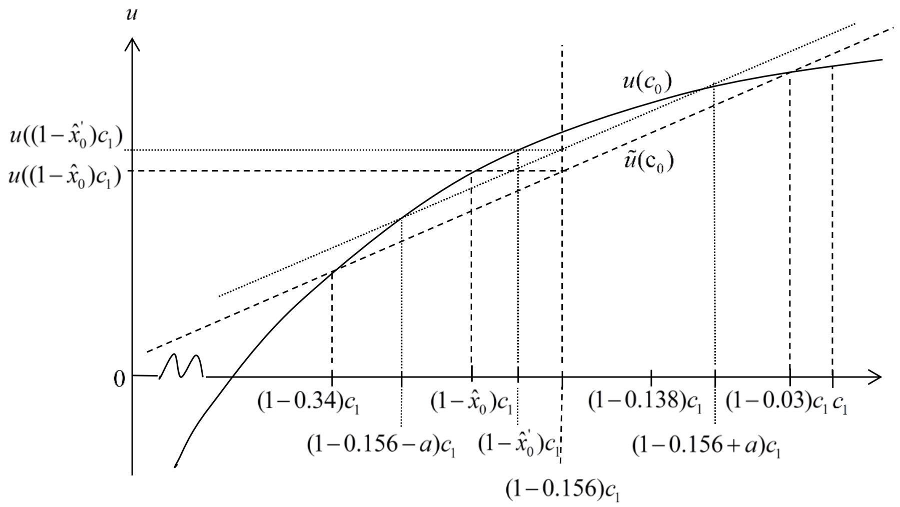

Figure 8.3: The case of symmetric density. Comparison of linear and strictly concave utility function.

$= (0.34 - 0.03)/2 = 0.156$. So $E((1-X)c_1) = (1-0.156)c_1$. In view of $\tilde{u}(c)$ being linear, we then get $\tilde{u}((1-0.156)c_1) = E\tilde{u}((1-X)c_1)$. And for this case an estimate of the certainty-equivalent loss, x_0, of course equals $EX = 0.156$.

The "true" density function, $f(x)$, is right-skew, however, and has $EX = 0.138$. In combination with the linear utility function, $\tilde{u}(c)$, this implies an estimate of x_0 equal to 0.138, that is, we get a *lower* value for the certainty-equivalent loss than with a symmetric density function.

Now let us consider the "true" utility function, $u(c)$. In Figure 8.3 it is represented by the solid strictly concave curve $u(c_0)$. Let us again imagine for a while that the density function is symmetric. As before, half of the probability mass would then be to the right of the mean of c_0, $(1-0.156)c_1$, and the other half to the left. The density function *might* happen to be such that the expected utility is just the average of utility at the 5th percentile and utility at the 95th percentile, that is, as if the two halves of the probability mass were placed at the 5th and 95th percentiles of 0.03 and 0.34, respectively; if so, the estimated certainty-equivalent loss is the $\hat{x}_0$ shown in Figure 8.3.

This would just be a peculiar coincidence, however. The probability mass of the symmetric density function could be more, or less, concentrated close to $EX = 0.156$. In case it is more concentrated, it is as if the two halves of the probability mass are placed at the consumption levels $(1 - 0.156 + a)c_1$ and $(1 - 0.156 - a)c_1$ for some "small" positive a, cf. Figure 8.3. The corresponding estimate of the certainty-equivalent loss is denoted $\hat{x}_0'$ in the figure and is smaller than $\hat{x}_0$ so that the associated c_0 is larger than before.

Finally, we may conjecture that allowing for the actual right-skewness of the density function will generally tend to *diminish* the estimate of the certainty-equivalent loss.

The conclusion seems to be that Arrow's procedure, as it stands, is questionable. Or the procedure is based on assumptions about the properties of the density function not spelled out in the article. Anyway, sensitivity analysis is called for. This could be part of an interesting master's thesis by someone better equipped in mathematical statistics than the present author is.

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