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Too little or too much R&D?

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Abstract

According to the first generation models of endogenous growth based on expanding product variety, the market economy unambiguously generates too little R&D. Later, by disentangling returns to specialization from the market power parameter, it was shown that with sufficiently low returns to specialization too much R&D can occur. The present paper takes a step further, disentangling the market power parameter from the capital share in final output. At a theoretical level this helps finding too much R&D as well. On the other hand, in view of the empirically realistic order of magnitude between the parameters, disentangling market power and capital share tends to *diminish* the scope for excess R&D. Finally, by differentiating between net and gross returns to specialization we demonstrate what drives the differing inefficiency results in this literature.

JEL classification: O33; O41

Keywords: Endogenous growth; R&D; Expanding product variety; Creative destruction; Optimal growth

1. Introduction

The purpose of this paper is to differentiate a few central parameters that were rigidly linked in the Romer model of endogenous growth. Through this differentiation the possibility arises that the market economy has too much research, as in the "quality ladder" type of models of endogenous growth.

The original Romer (1990) article had implicit links between the three parameters: Returns to specialization, market power, and the capital share in output. As a result, the model had the particular feature that the amount of research is always insufficient.

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In later articles Benassy (1998) and Groot and Nahuis (1998) showed that if returns to specialization and market power are chosen independently, then too much research can occur when returns to specialization are sufficiently low. The present paper takes a step further, differentiating also between market power and the capital share. This helps finding too much research as well, at least at a theoretical level. From an empirical point of view, taking account of the realistic order of magnitude between the parameters, disentangling market power and capital share implies a *diminished* scope for excess R&D to occur. In any case, an advantage of the more general framework is better agreement with the observed level of markups and the observed falling tendency of the patent/R&D ratio. Finally, by differentiating between net and gross returns to specialization we are able to demonstrate what drives the differing inefficiency results in the literature.

On one point our model has similarity with the Chapter 5 version of the increasing variety model in Grossman and Helpman (1991b). That model also implies a separation of the substitution parameter from the capital share in manufacturing. But the expanding product variety feature is limited to non-durable intermediate goods, and returns to specialization are implicitly given as a function of the share of these intermediate goods and the markup parameter. This leads to the reappearance of the Romer result that there is always too little R&D generated under laissez-faire.

Our paper is related to Jones and Williams (2000) who also, in a model of semi-endogenous growth, among other things loosen the usual parameter links. But this is done only halfway, and the three central entities – returns to specialization, monopolist markup, and capital share – are still linked in an arbitrary way. The focus is not on the analytical relationships opened up by parameter separations, but on calibrating the model for the US economy. The conclusion is that the decentralized economy vastly underinvests in R&D relative to what is socially optimal. Stokey (1995), which is a paper in the "quality ladder" tradition, is less firm about this matter.

The remainder of the present paper is organized as follows. Section 2 presents the elements of our extended Romer model. Section 3 considers the control problem of the social planner and characterizes its solution. Section 4 embeds the economic system into a market economy. In addition, this section shows how different contributions to the literature can be seen as special cases of the model. In Section 5 we compare the balanced growth properties of the market economy with those of the social optimum. Section 6 relates the model to the empirics of markups and the trend of the patent/R&D ratio. Finally, Section 7 concludes.

2. Elements of the economy

The economy is populated by a constant number, L, of infinitely lived identical households with constant size. Each household supplies one unit of labor inelastically. Their preferences can be represented by a discounted utility function,

$$U_0 = \int_0^\infty \mathrm{e}^{-\rho t} \frac{c(t)^{1-\theta} - 1}{1-\theta}\, \mathrm{d}t, \qquad \rho > 0, \theta > 0, \tag{2.1}$$

where c is consumption at time t, θ is the elasticity of marginal utility, and ρ is the rate of time preference.[1] From now the time index will be suppressed when not needed for clarity.

The economy has two production sectors: The basic-goods sector and the specialized capital-goods sector. In the *basic-goods sector*, labor, N_Y, and a composite of specialized capital goods, X, are the inputs to produce the aggregate output:

$$Y = A^{\eta} X^{\alpha} N_Y^{1-\alpha}, \quad 0 < \alpha < 1, \;\; \eta > 0. \tag{2.2}$$

There is a continuum of specialized capital goods, measured on the interval $[0, A]$, where A indicates the level of technical knowledge in society or the stock of engineering principles that grows through research. The parameter η captures the "returns to specialization", i.e., the degree to which society benefits from specializing production in an increasing number of branches.[2] The composite factor X is a CES aggregate of quantities, x_i, of specialized capital goods:

$$X = A\left(\frac{1}{A}\int_0^A x_i^{\varepsilon}\mathrm{d}i\right)^{1/\varepsilon}, \quad 0 < \varepsilon < 1. \tag{2.3}$$

Thus the existing specialized capital goods exhibit a constant (direct) elasticity of substitution given by $1/(1-\varepsilon) > 1$ implying that no specialized capital good is essential. A higher ε indicates larger substitutability between the specialized capital goods. Hence, we call ε the "substitution parameter", and in the market economy this parameter is inversely related to monopoly power. Notice that (2.3) inserted into (2.2) shows the specialized capital goods to be complements in the production of basic goods ($\partial^2 Y/\partial x_i \partial x_j > 0$) if $\alpha > \varepsilon$ and to be substitutes ($\partial^2 Y/\partial x_i \partial x_j < 0$) if $\alpha < \varepsilon$.

The original Romer (1990) article had implicitly the three parameters linked by $\eta = 1 - \alpha$ and $\alpha = \varepsilon$. As a result, the model had the particular feature that the amount of research is always insufficient. In later articles Benassy (1998) and Groot and Nahuis (1998), henceforth BGN, showed that if η and ε are chosen independently, then too much research can occur if η is sufficiently low. But the parameter link $\varepsilon = \alpha$ is implicit in the BGN analysis.[3] We are going to study how differentiating between ε and α affects the scope for excessive research.

The output of basic goods is used for consumption, $C \equiv cL$, and investment in "raw" capital. The stock of raw capital K changes according to

$$\dot{K} = Y - C - \delta K, \quad \delta \geqslant 0, \;\; K(0) = K_0 > 0 \text{ given}, \tag{2.4}$$

where δ is the capital depreciation rate.

In the *specialized capital-goods sector*, which is also the *innovative sector*, a unit of raw capital can immediately be transformed to a specialized capital good on the basis of a given technical design. The number of new designs created per time unit is

[1] In case $\theta = 1$, the expression $(c^{1-\theta} - 1)/(1-\theta)$ should be interpreted as $\ln c$.

[2] More precisely, η is *net* returns to specialization, cf. Section 5.2.

[3] On the face of it, neither the Benassy or the Groot and Nahuis paper might seem to fit into this framework since they are based on the Grossman and Helpman (1991a) expanding consumer goods variety model without physical capital. As we shall see, however, the reduced form of their (essentially identical) models corresponds exactly to the case $\varepsilon = \alpha$.

assumed proportional to the existing stock of knowledge, measured by A (this is the standing-on-shoulders effect, also present in Romer (1990); Benassy (1998); and Groot and Nahuis (1998)),

$$\dot{A} = \gamma N_A A, \quad \gamma > 0, \; A(0) = A_0 > 0 \text{ given,} \tag{2.5}$$

where γ is a productivity parameter, and N_A is aggregate research work. Finally, with full employment,

$$N_Y + N_A = L. \tag{2.6}$$

Because of the strict concavity of X in x_i and the symmetric cost structure, static efficiency requires $x_i = x$ for all $i \in [0, A]$.[4] Hence, assuming static efficiency, $X = Ax$ from (2.3), and when demand for raw capital equals supply we have

$$X = Ax = K. \tag{2.7}$$

Inserting into (2.2) gives output of basic goods as

$$Y = A^{\eta} K^{\alpha} N_Y^{1-\alpha}. \tag{2.8}$$

A feasible path $(K, A, C, Y, N_Y, N_A)_{t=0}^{\infty}$ is called a *steady state* if K, A, C, and Y are strictly positive and grow at constant (though not necessarily equal or positive) rates. Let the rate of growth of a strictly positive variable x be denoted g_x, i.e., $g_x \equiv \dot{x}/x$. Let u be the fraction of total labor supply employed in the basic-goods sector, i.e., $N_Y \equiv uL, 0 \leqslant u \leqslant 1$. By (2.5), $g_A \geqslant 0$ always.

Lemma 1. (i) *In a steady state with* $g_A = \bar{g}_A$, $u = 1 - \bar{g}_A/\gamma L$, *a constant. Moreover,* $0 < u \leqslant 1$ *and* $0 \leqslant \bar{g}_A < \gamma L$. (ii) *If, in addition,* Y/K *is constant, then* $g_c = g_Y = g_K \equiv \bar{g} = [\eta/(1-\alpha)]\bar{g}_A$.

Proof. See Appendix. □

3. The social optimum

The social planner will of course ensure static efficiency. Therefore, in the social optimum, output of basic goods is given by (2.8). The social planner chooses a path $(c, N_Y)_{t=0}^{\infty}$ to maximize U_0 subject to (2.4)–(2.6), (2.8), and the non-negativity requirements: $A, K \geqslant 0$ for all $t \geqslant 0$. Necessary conditions for an interior solution are that for all $t \geqslant 0$:

$$c^{-\theta} = \lambda_1 L, \tag{3.1}$$

$$\lambda_1 (1-\alpha) \frac{Y}{N_Y} = \lambda_2 \gamma A, \tag{3.2}$$

$$\dot{\lambda}_1 = \rho \lambda_1 - \lambda_1 \left(\frac{\partial Y}{\partial K} - \delta \right), \tag{3.3}$$

[4] Therefore, obsolescence of old capital goods never occurs.

$$\dot{\lambda}_2 = \rho\lambda_2 - \lambda_1 \frac{\partial Y}{\partial A} - \lambda_2 \gamma (L - N_Y), \tag{3.4}$$

where λ_1 and λ_2 are the shadow prices of the state variables K and A, respectively. Log-differentiating (3.1) w.r.t. t and using (3.3) gives

$$g_c = \frac{1}{\theta}\left(\frac{\partial Y}{\partial K} - \delta - \rho\right) = \frac{1}{\theta}(\alpha \tilde{k}^{\alpha-1} - \delta - \rho), \tag{3.5}$$

where $\tilde{k} \equiv k/A^{\eta/(1-\alpha)}$ (the *effective* capital–labor ratio in the basic-goods sector) and $k \equiv K/N_Y$. Since in a steady state, by definition, g_c is constant, $\tilde{k}$ is also constant in view of (3.5).

Define $q \equiv \lambda_2/\lambda_1$. From (3.2), $q = (1-\alpha)Y/[\gamma A N_Y] = (1-\alpha)\tilde{k}^{\alpha} A^{\eta/(1-\alpha)-1}/\gamma$, hence, in steady state

$$\frac{\dot{q}}{q} = \left(\frac{\eta}{1-\alpha} - 1\right) g_A.$$

But, by definition of q, $\dot{q}/q = (\dot{\lambda}_2/\lambda_2) - (\dot{\lambda}_1/\lambda_1)$. Therefore, inserting (3.3), (3.4), and (2.5) we get

$$\frac{\partial Y}{\partial K} = \alpha \frac{Y}{K} = \frac{\eta}{1-\alpha}\gamma L + \delta. \tag{3.6}$$

To permit existence of an optimal solution we need the assumption that

$$\rho > (1-\theta)\frac{\eta}{1-\alpha}\gamma L, \tag{A1}$$

where $\eta(1-\alpha)^{-1}\gamma L$ is the supremum of g_c in a steady state (from Lemma 1 combined with (2.5)). If (A1) is violated, the rate of time preference is so small that the system cannot avoid the temptation to specialize in R&D activity forever (thus postponing production of consumption goods forever); in this case no optimal solution exists.

Steady-state values of the variables in the social planner's solution are marked by $*$. We have

Proposition 1. *Assume* (A1). *If*

$$\rho < \frac{\eta}{1-\alpha}\gamma L, \tag{A2}$$

there exits a unique optimal solution and it converges to a steady state with

$$g_c^* = \frac{1}{\theta}\left(\frac{\eta}{1-\alpha}\gamma L - \rho\right) > 0. \tag{3.7}$$

If (A2) *is violated, an optimal steady state has* $g_c^* = 0$.

Proof. By (3.5) and (3.6), an interior steady state (i.e., one with $0 < u < 1$) satisfying the necessary first-order conditions (3.1) through (3.4) has g_c^* determined by (3.7). In Alvarez-Pelaez and Groth (2002) it is shown that the steady state is saddle-point stable, satisfies the necessary transversality conditions, and that these, together with the first-order conditions, are also sufficient for an optimal solution. As to existence, from Lemma 1, $0 < u^* < 1$ if and only if $0 < g_c^* < \eta(1-\alpha)^{-1}\gamma L$, where, in view of (3.7), the second inequality holds if and only if (A1) holds, while the first inequality is valid if and only if (A2) is valid. □

We see that the optimum rate of growth does not depend on the substitution parameter ε; this parameter gets a role only through the market forces considered in the next section. However, when (A2) holds, an increase in the returns to specialization parameter, η, raises the degree to which society benefits from new inventions, which leads to an increase in g_c^*. In addition, the inverse of the labor share in output of basic goods acts as a "multiplier" transforming returns to specialization into an elasticity of labor efficiency with respect to technical knowledge. Given η, this elasticity decreases when the labor share, $1-\alpha$, increases, thereby lowering the growth rate in a steady state. Further, as expected, the higher is the desire for smoothing consumption (higher θ) and the higher is the rate of impatience (larger ρ), the smaller is g_c^*. In addition, the growth rate increases with the size of population; this is the well-known, though controversial,[5] "scale effect" of R&D-based endogenous growth models.

On the other hand, if (A2) is violated, then impatience is so large that there will be no R&D activity and no growth in an optimal steady state. Formula (3.7) is no longer valid; instead, the steady state will be like that of a standard one-sector Ramsey model without technical progress.

Now, we will embed the economic system in a market economy. Apart from the more general specifications of technology, the set-up is similar to Romer (1990).

4. The market economy

4.1. Agents

The representative firm in the basic goods sector rents labor at the wage w and specialized capital goods at the rental rate R_i, $i\in[0,A]$. Using basic goods as our *numeraire*, profit maximization under perfect competition yields

$$\frac{\partial Y}{\partial N_Y}=(1-\alpha)\frac{Y}{N_Y}=w, \tag{4.1}$$

$$\frac{\partial Y}{\partial x_i}=\alpha\frac{Y}{X}\frac{\partial X}{\partial x_i}=R_i. \tag{4.2}$$

From (4.2) we can express the demand for the specialized capital good i conditional on a given X as

$$x_i=\frac{X}{A}\left(\frac{R_i}{R}\right)^{-1/(1-\varepsilon)}, \quad i=1,2,\ldots,A, \tag{4.3}$$

where $R=(A^{-1}\int_0^A R_i^{\varepsilon/(\varepsilon-1)})^{(\varepsilon-1)/\varepsilon}$ is the "ideal" price index for X (the minimum cost per unit of X).

The supply of each specialized capital good is decided by the firm that invented the design for the capital good in question, i.e., firm i supplies capital good variety i.

[5] Cf. Jones (1995).

The firms get compensated for the sunk research cost through retention of monopoly power over the commercial use of the invention and this monopoly power is supported by patents of infinite duration. Given design i, to deliver $x(i)$ units of capital good i, it takes $x(i)$ units of raw capital. At each instant of time, firm i, facing the demand curve (4.3) and taking X and R as given, sets the rental rate R_i so that current profit $\pi_i \equiv R_i x_i - (r+\delta)x_i$ is maximized, i.e.,

$$R_i = \frac{1}{\varepsilon}(r+\delta), \tag{4.4}$$

where r is the real rate of interest. Smaller ε (indicating less substitutability between specialized capital goods) gives larger monopoly power to the suppliers of specialized capital goods.

Since, by (4.4), all firms in the specialized capital-goods sector set the same rental price, they supply the same quantity, x, and they earn the same profit

$$\pi = \left(\frac{1}{\varepsilon} - 1\right)(r+\delta)x. \tag{4.5}$$

The equilibrium value, p, of a patent (the present discounted value of the revenue that the patent generates) satisfies the no-arbitrage condition

$$\frac{\pi + \dot{p}}{p} = r, \tag{4.6}$$

i.e., the return on a patent must be equal to the return on capital.

There is free entry to research activity. Research is done by new firms wanting to enter the specialized capital-goods sector. Given the invention production function (2.5), the value of the marginal product of labor in research is $p\gamma A$. Hence, profit maximization subject to (2.5) entails, in equilibrium,

$$w \geqq p\gamma A, \quad \text{with ‘} = \text{’, if } N_A > 0. \tag{4.7}$$

Once a new technical design has been invented, a patent is taken out and the new firm starts supplying the corresponding new specialized capital good. By increasing A, research activity has a positive external effect on the productivity of future research activity.[6] In addition, research activity has a positive overall effect on total factor productivity in manufacturing (through the term A^{η} in (2.8)).

Households consume and save, and savings can be either in capital or in shares of the monopoly firms. Financial wealth of the representative household is $v \equiv (K + pA)/L$. The household makes a plan $(c)_{t=0}^{\infty}$ to maximize U_0 subject to $\dot{v} = w + rv - c$, $v(0) = v_0$, and the standard no-Ponzi-game condition. Necessary and sufficient conditions for a solution are that the Keynes–Ramsey rule,

$$g_c = \frac{1}{\theta}(r - \rho), \tag{4.8}$$

and the transversality condition, $\lim_{\tau\to\infty} v e^{-\int_t^{\tau} r\,\mathrm{d}s} = 0$, hold for all $t \geqslant 0$.

[6] Each research firm is small and therefore perceives, correctly, its contribution to aggregate $\dot{A}$ to be practically negligible.

4.2. General equilibrium and steady state

Given the clearing conditions, $K = X = xA$, $L = N_A + N_Y$, and the definitions $\tilde{k} \equiv K/(N_Y A^{\eta/(1-\alpha)})$ and $u \equiv N_Y/L$ we have

$$x = \frac{K}{A} = uL\tilde{k}A^{(\eta+\alpha-1)/(1-\alpha)}. \tag{4.9}$$

Output per unit of *effective* labor in the basic-goods sector is $\tilde{y} \equiv Y/(A^{\eta/(1-\alpha)}uL) = \tilde{k}^{\alpha}$. Combining with (4.1),(4.2), and (4.4) gives

$$w = (1-\alpha)\frac{Y}{uL} = (1-\alpha)\tilde{k}^{\alpha}A^{\eta/(1-\alpha)}, \tag{4.10}$$

$$\frac{1}{\varepsilon}(r+\delta) = \frac{\partial Y}{\partial K} = \alpha\frac{Y}{K} = \alpha\tilde{k}^{\alpha-1}. \tag{4.11}$$

If $u < 1$, i.e., $N_A > 0$, then w also equals the value of the marginal product of labor in research so that (4.7) reduces to $w = p\gamma A$. This together with (4.10) gives the market value of a patent as

$$p = \frac{1-\alpha}{\gamma}\tilde{k}^{\alpha}A^{(\eta+\alpha-1)/(1-\alpha)}. \tag{4.12}$$

An *interior equilibrium* is an equilibrium such that for all $t \geqslant 0$, $0 < u < 1$ (there is positive employment in both sectors). In an interior equilibrium, by (4.12), the market value of a patent grows according to $g_p = \alpha g_{\tilde{k}} + [(\eta+\alpha-1)/(1-\alpha)]g_A$. Inserting this together with (4.10), (4.12), (4.5), and (4.11) into (4.6), using the fact that $\gamma uL = \gamma L - g_A$, from (2.5), gives the market interest rate as

$$r = (1-\varepsilon)(1-\alpha)^{-1}\alpha(\gamma L - g_A) + [\eta(1-\alpha)^{-1} - 1]g_A + \alpha g_{\tilde{k}}.$$

Thus, in steady state (where $g_{\tilde{k}} = 0$ [7]) we have

$$(1-\alpha)r + (1-\varepsilon\alpha-\eta)g_A = (1-\varepsilon)\alpha\gamma L. \tag{4.13}$$

Further, since Y/K is constant in steady state, (ii) of Lemma 1 together with (4.8) gives

$$(1-\alpha)r - \eta\theta g_A = (1-\alpha)\rho. \tag{4.14}$$

To avoid endangering existence and stability we shall concentrate on the case where the determinant of the system (4.13)–(4.14),

$$D \equiv 1 - \varepsilon\alpha - (1-\theta)\eta,$$

is positive. This is always satisfied when $\theta \geqslant 1$ as well as when η is not *much* larger than the Romerian value, $1-\alpha$.

It can be shown [8] that the transversality condition of the household is satisfied in a steady state if and only if

$$\rho > \frac{1-\theta}{1-\varepsilon\alpha}\eta\frac{1-\varepsilon}{1-\alpha}\alpha\gamma L. \tag{A3}$$

[7] In a steady state, by definition, c grows at a constant rate, hence, from (4.8), r is constant. Therefore, in view of (4.11), Y/K is constant, and $g_{\tilde{k}} = 0$.

[8] Alvarez-Pelaez and Groth (2002).

If this does not hold, then the rate of time preference is so small that the market economy tends to grow at a rate above the interest rate, and human wealth of the household tends to infinity, thus violating the equilibrium assumption.

Proposition 2. *Assume* (A3) *and* $D > 0$. *If*

$$\rho < \frac{1-\varepsilon}{1-\alpha}\alpha\gamma L, \tag{A4}$$

there exits a market equilibrium and it has a steady state with

$$g_c = \frac{(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho}{1-\varepsilon\alpha-(1-\theta)\eta}\left(\frac{\eta}{1-\alpha}\right) > 0. \tag{4.15}$$

If (A4) *is violated, a steady state of a market equilibrium has* $g_c = 0$.

Proof. The implication of (i) of Lemma 2 below is that, taken together, (A3) and (A4) are consistent with $D > 0$. Solving (4.13)–(4.14) and using (ii) of Lemma 1 gives (4.15). If (A4) is violated, there can be no steady state with $u < 1$ in view of (ii) of Lemma 2; hence, a steady state has $u = 1$, i.e., $g_c = g_A = 0$. □

Lemma 2. (i) (A3) *and* (A4) *imply* $D > 0$; (ii) *given* $D > 0$, *then* (A3) *is equivalent to* $u > (1-\alpha)/(1-\varepsilon\alpha)$, *and* (A4) *is equivalent to* $u < 1$.

Proof. See Appendix. □

According to (ii) of the Lemma, (A3) and (A4) ensure interiority of the steady state. If (A4) is violated, then impatience is so large that R&D activity and growth cannot be supported in a steady-state equilibrium. In this case the formula (4.15) is no longer valid; the steady-state solution of the model is like that of a one-sector model without technical progress.[9]

Let us consider the comparative statics of the interior steady state.[10] Assume (A3) and (A4). Then $D > 0$, and from (4.15) we get, after some manipulations,

$$\frac{\partial g_c}{\partial \eta} = \frac{(1-\varepsilon\alpha)[(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho]}{[1-\varepsilon\alpha-(1-\theta)\eta]^2(1-\alpha)} > 0, \tag{4.16}$$

$$\frac{\partial g_c}{\partial \varepsilon} = \frac{[\gamma L((1-\theta)\eta - (1-\alpha)) - (1-\alpha)\rho]}{[1-\varepsilon\alpha-(1-\theta)\eta]^2(1-\alpha)}\alpha\eta < 0, \tag{4.17}$$

$$\frac{\partial g_c}{\partial \alpha} = \frac{\frac{1-\varepsilon}{1-\alpha}\gamma L[1-\varepsilon\alpha-(1-\theta)\eta] + \varepsilon[(1-\varepsilon)\alpha\gamma L - (1-\alpha)\rho]}{[1-\varepsilon\alpha-(1-\theta)\eta]^2(1-\alpha)}\eta > 0, \tag{4.18}$$

[9] In spite of (i) of Lemma 2 we need $D > 0$, separately, to substantiate the last claim in Proposition 2.

[10] At least within the empirically relevant domain of the parameter space, the steady state can be shown to be saddle-point stable (see Alvarez-Pelaez and Groth, 2002).

where the second inequality is proved in the appendix. It follows that returns to specialization, η, affect the growth rate in a much more complex way than in the social optimum, though the sign of the effect is the same. Similarly, the capital share parameter α affects growth through an additional channel compared with the social optimum. Indeed, the higher is α, the lower is the wage share, $1-\alpha$, which, given the factor prices, implies less room for profitable employment in the basic-goods sector.[11] Thereby more of the fixed labor force is available for employment in research, and growth is enhanced. A completely new feature is that the growth rate of the market economy depends (negatively) on the substitution parameter ε while that of the social optimum did not. When specialized capital-goods are close substitutes (ε high), the markup over marginal cost in the specialized capital-goods sector becomes low, making inventions of new designs less profitable, thereby reducing growth. These features come from effects on private incentives.

4.3. Earlier contributions as special cases

The Romer (1990) model is simply the case $\eta = 1-\alpha$, and $\varepsilon = \alpha$.[12] The restriction $\eta = 1-\alpha$ implies two benchmark features. First, the value p of a patent and the size x of the market for a specific capital good stay constant in a steady state, cf. (4.12) and (4.9). Second, along a steady-state path every new invention leaves profits π of the single monopoly firm unchanged as shown by (4.5) and (4.9). The restriction $\varepsilon = \alpha$ is more serious, since it blurs the positive effect on growth of an increase in the monopolist markup. The restriction implies that a high markup, $1/\varepsilon$, goes with a low capital share and this overturns the markup effect on growth. Indeed, when $\varepsilon = \alpha$, (4.15) reduces to $g_c = [(\alpha\gamma L - \rho)/(1-\alpha^2-(1-\theta)\eta)]\eta$ so that $\partial g_c/(\partial(1/\alpha)) < 0$, given (A4).

As mentioned earlier the contributions by Benassy (1998) and Groot and Nahuis (1998) have much similarity between them. They are based on essentially the same model (here called the BGN model), that is, an extension of the Grossman and Helpman (1991a) expanding consumer goods variety model without capital accumulation. The only input in production is labor. Hence, on the face of it the BGN model does not fit into our framework. Nevertheless, as far as the reduced form (4.15) is concerned, the BGN model corresponds to the case $\varepsilon = \alpha$ (and $\theta = 1$ since only logarithmic utility is considered). Indeed, the technology specification of the BGN model leads to the formula $g_c = [((1-\omega)\gamma L - \omega\rho)/(1-\omega(1-\theta)v)]v$, where ω is a substitution parameter such that the monopolist markup becomes $1/\omega$, while v is the elasticity of labor efficiency with respect to technical knowledge (corresponding to our $\eta/(1-\alpha)$).[13]

[11] (4.4), (4.10), and (4.11) give $uL = [(1-\alpha)/\alpha]K(R/w)$.

[12] Strictly speaking, this refers to the textbook version (Aghion and Howitt, 1998, p. 35 ff.). In the original version of the Romer model, the basic-goods sector also employs an exogenous amount of a second type of labor called human capital, but this is of secondary importance in our context.
It should be recognized that Romer (1990, p. S81) actually encouraged an investigation of cases where $\varepsilon \neq \alpha$.

[13] The just mentioned Grossman and Helpman (1991a) model is the special case $v = (1-\omega)/\omega$, and this implies the Romer result that there is always too little R&D.

With $\varepsilon = \alpha$, our (4.15) gives the same formula for g_c when we put $\omega = 1/(1+\alpha)$. In this sense the BGN model is nested in our more general framework, and absence of capital accumulation can be interpreted as the special case $\varepsilon = \alpha$. As to the social optimum the BGN model has $g_c^* = [1/\theta](v\gamma L - \rho)$ which is the same as our result in Section 3.

In the next section we study the necessary and sufficient conditions for the market economy to do too much R&D. We shall see how allowing for $\varepsilon \neq \alpha$ modifies these conditions compared with the Romer and BGN results. Since the BGN model contains the Grossman and Helpman (1991a) contribution as a special case, our comparison with BGN will cover this contribution as well.[14]

5. Comparing market outcome and social optimum

5.1. Growth rates compared

Observe that the parameter restriction (A4), containing ε, but not η, is of a quite different nature compared to (A2) from the social optimum. On the other hand, if (A1) from the social optimum is assumed, we do not have to worry about (A3). Indeed:

Lemma 3. (i) (A1) *implies* (A3). (ii) (A1) *and* (A4) *imply* $D > 0$. (iii) (A1) *and* (A3) *are satisfied automatically when* $\theta \geqslant 1$.

Proof. (i) $(1-\varepsilon)\alpha/(1-\varepsilon\alpha) = (\alpha - \varepsilon\alpha)/(1-\varepsilon\alpha) < 1$; hence, (A1) implies (A3). (ii) Assume (A1) and (A4); then, since (A1) implies (A3), $D > 0$ follows from Lemma 2. (iii) This is obvious since $\rho > 0$. □

To clear the ground it is useful to initiate the analysis by varying the returns to specialization parameter, keeping the other parameters constant. We claim that there exists a value, $\tilde{\eta}$, such that if and only if returns to specialization are equal to this value, then $g_c = g_c^*$. Indeed:

Proposition 3. *Assume* (A4). *Then*:

(i) *Given* $(\alpha, \varepsilon, \theta, \rho, \gamma, L)$ *there is a unique value* $\tilde{\eta} > 0$ *such that if* $\eta = \tilde{\eta}$, *then both a social optimum and a market equilibrium exist and have a steady state*,

[14] On the other hand Grossman and Helpman consider also an increasing variety model *with* capital accumulation (Grossman and Helpman, 1991b). Physical capital is a homogenous good while the specialized intermediate goods, sold under conditions of monopolistic competition, are non-durable. By including a capital share parameter α, this Grossman and Helpman contribution leads to the formula $g_c = [((1-\alpha-\omega)\gamma L - \omega\rho)/(1-\alpha-\omega(1-\theta)v)]v$, where BGN has $\alpha = 0$. But here ω depends not only on the markup, but also on the share of capital in final output and the share of non-durable intermediate goods in final output. Also, the knowledge elasticity parameter v is a function of the markup and these two share parameters. The implied parameter links lead to the reappearance of the Romer result that there is always too little R&D generated under laissez-faire.

and $g_c = g_c^*$. *This value is*

$$\tilde{\eta} = \begin{cases} (1-\varepsilon\alpha)\dfrac{\rho}{\rho+\gamma L} & \text{if } \theta = 1, \\ \dfrac{B - \sqrt{B^2 - 4(1-\theta)(1-\alpha)(1-\varepsilon\alpha)(\rho/\gamma L)}}{2(1-\theta)} & \text{if } \theta \neq 1, \end{cases}$$

where $B \equiv \frac{(1-\alpha)(\rho+\gamma L)}{\gamma L} + (1-\theta)(1-\varepsilon)\alpha$.

(ii) *In any case* $(1-\alpha)(\rho/\gamma L) < \tilde{\eta} < (1-\varepsilon)\alpha$.

(iii) *For any* η *satisfying* (A1), *both the social optimum and the market equilibrium exist and have a steady state, and the following rule applies*: $g_c \gtreqless g_c^*$ *for* $\eta \lesseqgtr \tilde{\eta}$, *respectively*.

Proof. See Appendix. □

By (ii) and (iii) of the proposition follows that the excess growth phenomenon, $g_c > g_c^*$, can arise only when the returns to specialization parameter, η, is below $(1-\varepsilon)\alpha < \alpha$. In the specific Romer case, in addition to $\alpha = \varepsilon$, $\eta = 1-\alpha$; hence, $\tilde{\eta} < 1-\alpha = \eta$, and we get the Romer conclusion that $g_c < g_c^*$ unambiguously. As long as $\alpha = \varepsilon$, the opposite inequality, $g_c > g_c^*$, arises, if and only if returns to specialization, η, is considerably below the Romerian value $1-\alpha$ (in accordance with the Benassy and Groot and Nahuis conclusion).

However, disentangling the substitution parameter, ε, and the share of capital, α, it turns out that $\tilde{\eta} \geqslant 1-\alpha$ is possible. Indeed:

Corollary 1. *Assume* (A4). *A simple example where* $\tilde{\eta}$ *is above* $1-\alpha$ *is*: $\theta = 1$, $\varepsilon < 1 - [(1-\alpha)/\alpha]\max(\gamma L/\rho, \rho/(\gamma L))$.

Proof. Assume (A4). Let $\theta = 1$. Then, from (i) of Proposition 3 $\tilde{\eta} = (1-\varepsilon\alpha)\rho/(\rho+\gamma L) \gtreqless 1-\alpha$ for $\varepsilon \lesseqgtr 1-(1-\alpha)\gamma L/(\alpha\rho)$, respectively. But (A4) holds if and only if $\varepsilon < 1-(1-\alpha)\rho/(\alpha\gamma L)$. □

This shows (for the case $\theta = 1$) that, given α, for a sufficiently low ε we have $\tilde{\eta} > 1-\alpha$ so that g_c can end up larger than g_c^* even when η equals, or is above, $1-\alpha$. This cannot happen, as just argued, if $\varepsilon = \alpha$ since in this case a high capital share α goes with a *low* degree of market power, $1/\varepsilon$. More generally:

Proposition 4. *Assume* (A4). *For any* $\eta \in (0, (1-\varepsilon)\alpha)$ *such that the social optimum exists and has a steady state, the market equilibrium exists and has a steady state with* $g_c > g_c^*$ *if and only if the substitution parameter* $\varepsilon < 1 - \max(\eta/\alpha + (1-\alpha)[\eta\gamma L - (1-\alpha)\rho]/\{\alpha[(1-\alpha)\rho - (1-\theta)\eta\gamma L]\}, (1-\alpha)\rho/(\alpha\gamma L))$.

Proof. See Appendix. □

Hence, given the returns to specialization parameter η, by choosing a sufficiently low value of ε (high degree of market power), excess growth can be generated for a larger range of the capital share parameter α than in the BGN case $\varepsilon = \alpha$. This is

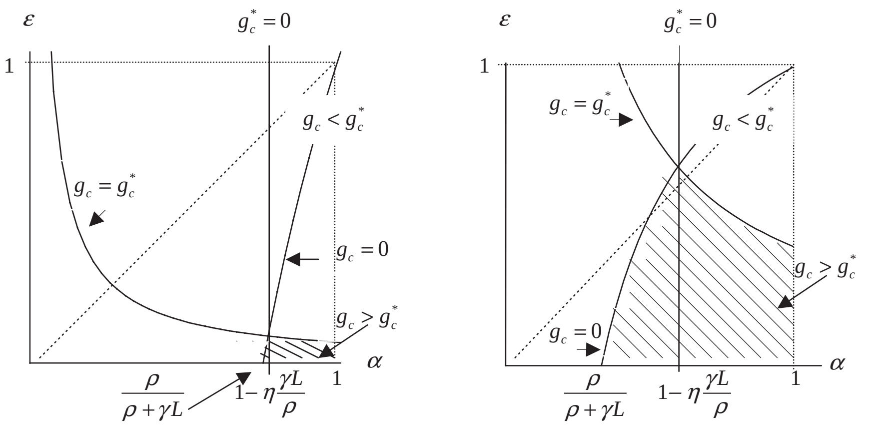

Fig. 1. Excess growth in terms of parameters α and ε.

illustrated in Fig. 1. In both panels the hatched area displays combinations of α and ε leading to $g_c > g_c^*$. A case where this area does not intersect the 45° degree line is shown in the panel to the left (Romer's case in a sense), while a case where it does is shown in the panel to the right (η relatively small).[15] In both cases, at the theoretical level disentangling ε from α opens up more scope for excessive R&D in the market economy.

On the other hand, from an empirical point of view, as argued in Section 6 below, the relevant alternative to $\varepsilon = \alpha$ is $\varepsilon > \alpha$ rather than $\varepsilon < \alpha$. Since, by (4.17), g_c is a decreasing function of ε while g_c^* is independent of ε, we have $\partial(g_c - g_c^*)/\partial\varepsilon < 0$. Hence, going from the assumption $\varepsilon = \alpha$ to the assumption $\varepsilon > \alpha$ implies *diminishing* the scope for excess R&D to occur.

5.2. *Excess growth and externalities of specialization*

While Benassy makes no attempt at explaining the excess growth phenomenon, Groot and Nahuis offer some intuition, but only for the special case they consider and only within the framework of the expanding consumer goods variety model. We shall see that differentiating between net and gross returns to specialization lays bare the negative externality needed for excess growth to occur. This provides the *general* principle behind the occurrence of deficient or excessive growth.

The point is to recognize the conceptual distinction between A, the number of *existing* different varieties at time t, and A_0, the number of these varieties being *in use* at time t (that is, the number of varieties for which $x_i > 0$); since we are aiming at intuitive interpretation we treat A and A_0 as integer variables. Consider the situation just before the invention of a new variety. Static efficiency requires $A_0 = A$ and

[15] The left-hand panel is based on $\eta = 0.7$, $\rho/(\gamma L) = 3$, and $\theta = 1$, the right-hand panel on $\eta = 0.2$, $\rho/(\gamma L) = 0.5$, and $\theta = 1$.

$x_i = x = \bar{K}/A_0$, where $\bar{K}$ is a given aggregate amount of raw capital. We may write (2.2) as

$$Y = A^{\eta+(1-1/\varepsilon)\alpha}\left(\sum_{i=1}^{A_0} x_i^{\varepsilon}\right)^{\alpha/\varepsilon} N_Y^{1-\alpha}$$

$$= A^{\eta+(1-1/\varepsilon)\alpha} A_0^{\alpha/\varepsilon} x^{\alpha} N_Y^{1-\alpha} \equiv F(A, A_0, x, N_Y). \tag{5.1}$$

Now, *after* the invention of the new variety, static efficiency requires a *redistribution* of the given amount of raw capital to an enlarged spectrum of varieties. We call the effect on aggregate output of this redistribution the *direct effect* of increased specialization. This direct effect is

$$\frac{\partial F}{\partial A_0} + \frac{\partial F}{\partial x}\frac{\partial x}{\partial A_0} = \frac{\alpha}{\varepsilon}\frac{Y}{A_0} + \alpha\frac{Y}{x}\left(-\frac{\bar{K}}{A_0^2}\right) = \left(\frac{1}{\varepsilon} - 1\right)\alpha\frac{Y}{A}, \tag{5.2}$$

since $x = \bar{K}/A_0$, and $A_0 = A$ after the redistribution. In view of the assumption $0 < \varepsilon < 1$, this effect is always positive.

In general there is also an *indirect effect* of increased specialization, that is, an effect on the productivity of the already existing specialized capital goods. To calculate this effect, we freeze A_0 and x at the level they have before redistribution. From (5.1) the indirect effect is

$$\frac{\partial F}{\partial A} = \left[\eta + \left(1 - \frac{1}{\varepsilon}\right)\alpha\right]\frac{Y}{A} \equiv \xi\frac{Y}{A}. \tag{5.3}$$

This effect may be positive, zero, or negative depending on the circumstances. We shall speak of "creative synergy" when it is positive, and of "creative down-weighing" (a mild form of the "creative destruction" inherent in quality ladder models) when it is negative. An interpretation of the first case is that the direct contribution of the invention is complemented by the positive indirect effect due to the other capital goods becoming more productive when "assisted" by a more complete network of intermediate goods. An interpretation of the second case is that though the overall effect of an invention on capital productivity is positive (since $\eta > 0$, by assumption), the direct contribution of the invention is partly offset by a negative indirect effect due to, say, coordination difficulties in a more specialized and complex world.[16] The indirect effect, whether positive or negative, appears in the market economy as an externality.

Expressing the direct and indirect productivity effects as elasticities, the overall effect of inventions can be written

$$\frac{A}{Y}\frac{\partial Y}{\partial A} = \text{direct effect} + \text{indirect effect} = \left(\frac{1}{\varepsilon} - 1\right)\alpha + \xi = \eta.$$

Thus, we may interpret returns to specialization η as a *derived* parameter, given α, ε, and the indirect effect ξ. When the indirect effect, ξ, is positive, we may speak

[16] Note that "creative down-weighing" refers to a reduction in the *productivity* of old capital goods. This need not imply reduced profits. Indeed, along a steady-state path, profits per firm are reduced by the arrival of new inventions if and only if $\eta < 1 - \alpha$, cf. (4.5) and (4.9).

of η as *total* returns to specialization. When ξ is non-positive, η measures *net* returns to specialization while the direct effect, $(\varepsilon^{-1}-1)\alpha$, measures *gross* returns to specialization. [17]

When both parameter links, $\eta = 1-\alpha$ and $\varepsilon = \alpha$, occur, we have the benchmark case where the indirect effect vanishes, so that $g_c < g_c^*$, definitely. But allowing $\varepsilon < \alpha$ or $\eta < 1-\alpha$ makes room for a negative indirect effect. If this effect is large enough, growth is excessive. To be more precise:

Proposition 5. *Assume* (A4). *Let the indirect effect of inventions*, ξ, *on the productivity of old capital goods be given such that* $\xi > -(1/\varepsilon - 1)\alpha$, *let* (A1) *hold with* $\eta = (1/\varepsilon - 1)\alpha + \xi$, *and let* $\tilde{\eta}$ *be defined as in Proposition* 3. *Then the case* $g_c > g_c^*$ *arises if and only if* $\xi < \tilde{\eta} - (1/\varepsilon - 1)\alpha$, *a negative number.*

Proof. See Appendix. □

To understand this result, notice that there are three potential market failures that may cause private and social incentives to diverge in the model. (i) The *intertemporal spillover*: By adding to the stock of technical knowledge, research increases the productivity of future research, cf. (2.5), but this effect is not compensated in the market. (ii) The *surplus appropriability problem*: Innovator's monopoly profits capture only a fraction of their (direct) contribution to output. Indeed, in view of (4.5) and (4.11), profits per capital good can be written

$$\pi = (1-\varepsilon)\,\alpha\,\frac{Y}{A} = \varepsilon \cdot \text{direct effect}, \tag{5.4}$$

by (5.2). [18] Market failures (i) and (ii) are well-known from the increasing variety literature as well as the quality ladder literature (Grossman and Helpman, 1991a, Chapter 4; Aghion and Howitt, 1992, 1998). What is less well-known is that a reminiscence of the famous additional market failure appearing in the quality ladder models, that of "creative destruction", may come up in the increasing variety framework, namely as a negative ξ in (5.3). The externality represented by ξ, whether negative or positive, is our market failure (iii). The market failures (i) and (ii) tend to generate insufficient research under laissez-faire, while market failure (iii) works in the same or the opposite direction, depending on the sign of ξ.

Now, Proposition 5 says that unless there is *enough* "creative down-weighing", the market leads to underinvestment in R&D. To avoid this underinvestment, it is not sufficient that the indirect effect, ξ, is negative and not even enough that it equals $-(\varepsilon^{-1}-1)\eta$ so that, by (5.3), $(1-\varepsilon)\alpha = \eta$, hence, by (5.4) and (2.8), $\pi = \partial Y/\partial A$,

[17] Consistency with our assumption that $\eta > 0$ requires $\xi > -(1/\varepsilon - 1)\alpha$.

[18] The surplus appropriability problem reflects that, from (4.11), capital costs are $r+\delta = \varepsilon\partial Y/\partial K < \partial Y/\partial K$. This inequality is a result of *monopoly pricing*: The markup implies a wedge between the price of the services of specialized capital goods and the marginal cost of providing them so that the demand for capital services is reduced. This also entails a wedge between social returns to saving, $\partial Y/\partial K - \delta$, and the private return, r, and therefore a tendency to too little saving. Hence, even in cases where $g_c = g_c^*$, the market economy underinvests in capital, implying too low consumption because of too low K/Y.

i.e., net surplus *is* appropriated. Indeed, also the intertemporal spillover should be overcome, thus requiring a numerically larger ξ.

6. Remarks on the empirics

In addition to disclosing a richer set of theoretical possibilities, the parameter separations made in this paper allow better accordance with the empirical evidence on markups and the trend of the patent–R&D ratio than does the more rigid Romer framework. Consider first the patent–R&D ratio, that is, the number of new patents per year divided by R&D expenditures. Since the late fifties, in the US a systematic decline in this ratio has taken place (on average a fall at 3.5% per year, see Griliches, 1989; Kortum, 1993). In our model, the patent–R&D ratio is given by $\dot{A}/(wN_A) = \gamma A/w$, and in a steady state this ratio will be decreasing over time if and only if (net) returns to specialization, η, is larger than $1 - \alpha$ (this is so because, in a steady state, by (4.10), $g_w = \eta/(1-\alpha)g_A$).[19] But Romer–style models have $\eta = 1 - \alpha$ and are therefore inconsistent with the observed fall in the patent–R&D ratio.

The markup is in Romer–style models implicitly given by the inverse of the capital share. If the capital share in output, α, is around 0.4 (as estimated for the US by Cooley and Prescott, 1995), then Romer-style models predict markups around 2.5. According to Norrbin (1993) and Basu (1996), however, markup estimates are between 1.05 and 1.40 in US industry. The present framework allows accordance with this, since we can choose a value for the substitution parameter ε in the interval $[0.70, 0.95]$.

These observations indicate the following size relation between the critical ("equal-growth") returns to specialization parameter value, $\tilde{\eta}$, the direct effect of innovations, $(1/\varepsilon - 1)\alpha$, and the actual value, η, of the returns to specialization parameter in US industry: $\eta > 1-\alpha > (1/\varepsilon - 1)\alpha > \tilde{\eta}$, where the last two inequalities follow from $\varepsilon > \alpha$ and (ii) of Proposition 3, respectively. In particular, $\eta > (1/\varepsilon - 1)\alpha$ is an indication that there is "creative synergy", and $\eta > \tilde{\eta}$ indicates that there is too little R&D so that the growth rate is inefficiently low. This result is consistent with the empirical evidence presented by Jones and Williams (1998). These authors find that the optimal R&D investment is at least four times larger than the actual spending.[20] Therefore, in this respect the prediction from the simple Romer framework with $\eta = 1 - \alpha$ seems to point in the right direction. Our hint that $\eta > (1/\varepsilon - 1)\alpha$ is likely, strengthens the confidence in that prediction.[21]

[19] To put it differently, the patent-R&D ratio falls when productivity in manufacturing increases faster than in R&D activity. This need not be a sign of exhaustion of technological opportunities. Rather than being something to worry about, it may, according to the present model, be a sign of high potentiality of new technical knowledge. (Of course, in reality the level of patenting lacks a lot in precision as an indicator of aggregate R&D successes inasmuch as many firms, at least outside the chemical and pharmaceutical industries, rely on other ways of protecting their innovations.)

[20] Stokey (1995), a paper in the "quality ladder" tradition, is less firm about this vast underinvestment.

[21] It can be shown that an active government needs two instruments to establish the optimal allocation. These instruments could be, first, a subsidy to buyers of capital services in order to eliminate distorting demand effects of monopoly pricing, and, second, a tax on – or subsidy to – monopoly profits, depending on the parameters (if the above parameter values are accepted, a subsidy is indeed called for).

7. Conclusion

According to the first generation of models of endogenous growth based on expanding product variety, the market economy unambiguously yields a too low level of R&D. However, disentangling returns to specialization from the market power parameter, later studies found that this result arises due to the implicit choice of a relatively high value for the returns to specialization.

The present paper takes a step further, analyzing an extended Romer-style model where also the monopolistic markup and the capital share in final output are given by independent parameters. At the theoretical level this opens up more scope for excessive R&D in the market economy. From an empirical point of view, however, taking account of the realistic order of magnitude between the parameters, disentangling market power and capital share implies a *diminished* scope for excess R&D to occur. In any case, the decisive factor behind excessive growth is the implicit presence of enough negative externalities of increased specialization. These externalities are a reminiscence of the creative destruction effect in the quality ladder models. In this way, there seems to be less asymmetry than hitherto recognized between the expanding variety models and the quality ladder models.

A rudimentary calibration of the model suggests that the actual outcome for the US economy is not that of too much R&D, but that of too little R&D. In any case, an advantage of the more general framework is better agreement with the observed level of markups and the observed falling tendency of the patent/R&D ratio.

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Appendix A

Proof of Lemma 1. (i) Consider a steady state with $g_A = \bar{g}_A$. Then, from (2.5), N_Y is constant. By definition of a steady state, $Y > 0$; hence, from (2.8), $N_Y > 0$. Therefore $u \equiv N_Y/L$ is constant, $u \in (0, 1 - \bar{g}_A/(\gamma L)]$, and $0 \leqslant \bar{g}_A = \gamma(1-u)L < \gamma L$. (ii) Y/K constant implies $g_Y = g_K$ which is constant in a steady state. Then, by (2.4), cL/K is constant, hence $g_c = g_K$. From (2.8), the common growth rate, $\bar{g}$, of c, K, and Y is $\eta(1-\alpha)^{-1}\bar{g}_A$. □

Proof of Lemma 2. (i) Clearly, (A3) and (A4) imply $(1-\theta)\eta/(1-\varepsilon\alpha) < 1$, hence

$$(1-\theta)\eta < 1 - \varepsilon\alpha, \tag{A.1}$$

which is equivalent to $D>0$. (ii) From (4.15), (2.5) with $N_A=(1-u)L$, and (ii) of Lemma 1,

$$u=\frac{[1-\alpha-(1-\theta)\eta]\gamma L+(1-\alpha)\rho}{[1-\varepsilon\alpha-(1-\theta)\eta]\gamma L}. \tag{A.2}$$

Now, in view of (A.1), by straightforward derivation we get $u>(1-\alpha)(1-\varepsilon\alpha)^{-1} \Leftrightarrow$ (A3), and $u<1 \Leftrightarrow$ (A4). □

Proof of (4.17). Assume (A3) and (A4). The formula for $\partial g_c/\partial\varepsilon$ follows from (4.15). By Lemma 2, (A.1) is valid, and $u>0$, implying, by (A.2), $\rho>[(1-\theta)\eta/(1-\alpha)-1]\gamma L$. It follows that $\partial g_c/\partial\varepsilon>0$. □

Proof of Proposition 3. Assume (A4) and let $\beta\equiv\rho/(\gamma L)$. (i) The η we look for must satisfy, first, (A1) (if not, we know from Section 3 that there would not exist a social optimum); hence, in view of Lemma 3, it satisfies (A3) and the condition $D>0$ so that both a social optimum and a market equilibrium exist and have a steady state. Second, the required η must satisfy the condition $g_c^*/g_c=1$. By (3.7) and (4.15),

$$\frac{g_c^*}{g_c}=\frac{[1-\varepsilon\alpha-(1-\theta)\eta]\,[\eta-(1-\alpha)\beta]}{[(1-\varepsilon)\alpha-(1-\alpha)\beta]\theta\eta}\equiv\varsigma(\eta;\theta). \tag{A.3}$$

The condition $\varsigma(\eta;\theta)=1$ implies the quadratic equation

$$Q(\eta)\equiv(1-\theta)\eta^2-[(1-\alpha)(1+\beta)+(1-\theta)(1-\varepsilon)\alpha]\eta+(1-\varepsilon\alpha)(1-\alpha)\beta=0.$$

For $\theta\neq1$ the roots are

$$\begin{pmatrix}\eta_2\\ \eta_1\end{pmatrix}=\frac{(1-\alpha)(1+\beta)+(1-\theta)(1-\varepsilon)\alpha\pm\sqrt{\Delta}}{2(1-\theta)}, \tag{A.4}$$

where

$$\begin{aligned}\Delta&\equiv[(1-\alpha)(1+\beta)+(1-\theta)(1-\varepsilon)\alpha]^2-4(1-\theta)(1-\varepsilon\alpha)(1-\alpha)\beta\\ &=[(1-\alpha)(1-\beta)+(1-\theta)(1-\varepsilon)\alpha]^2+4(1-\alpha)^2\theta\beta>4(1-\alpha)^2\theta\beta>0.\end{aligned}$$

Case 1: $\theta<1$. From (A.4) follows $0<\eta_1<\eta_2$; we have $Q(\eta)<0$ for $\eta_1<\eta<\eta_2$ and $Q(\eta)>0$ for $\eta<\eta_1$ and $\eta>\eta_2$. Since $Q((1-\alpha)\beta/(1-\theta))=-(1-\alpha)^2\theta\beta/(1-\theta)<0$, $\eta_1<(1-\alpha)\beta/(1-\theta)<\eta_2$. Therefore, η_1 satisfies (A1), but η_2 does not and can be discarded. Hence, $\tilde{\eta}$ is unique and equal to $\eta_1\in(0,(1-\alpha)\beta/(1-\theta))$.

Case 2: $\theta>1$. Now $\Delta>[(1-\alpha)(1+\beta)+(1-\theta)(1-\varepsilon)\alpha]^2$ so that

$$(1-\alpha)(1+\beta)+(1-\theta)(1-\varepsilon)\alpha\pm\sqrt{\Delta}\gtrless0,$$

respectively. Hence, $\eta_2<0<\eta_1$, and again η_2 can be discarded. Since $\theta>1$, we have $Q(\eta)>0$ for $\eta_1<\eta<\eta_2$ and $Q(\eta)<0$ for $\eta<\eta_2$ and $\eta>\eta_1$.

Case 3: $\theta=1$. In this case $Q(\eta)=0$ has one root $\eta_1(=\eta_2)=[\beta/(1+\beta)](1-\varepsilon\alpha)>0$, and $Q(\eta) \gtrless 0$ for $\eta \lessgtr \eta_1$, respectively.

In all three cases $\tilde{\eta}>0$ exists, is equal to η_1, and satisfies (A1). This proves (i).

(ii) That $\tilde{\eta}>(1-\alpha)\beta$ can be seen in the following way. By (A4), $g_c>0$; hence $\varsigma(\tilde{\eta};\theta)=1 \Rightarrow g_c^*=g_c>0$, i.e., $\tilde{\eta}>(1-\alpha)\beta$ from Proposition 1. To show that $\tilde{\eta}<(1-\varepsilon)\alpha$, consider

$$
\begin{aligned}
&Q((1-\varepsilon)\alpha)\\
&\quad =(1-\theta)(1-\varepsilon)^2\alpha^2-(1-\alpha)(1+\beta)(1-\varepsilon)\alpha\\
&\qquad -(1-\theta)(1-\varepsilon)^2\alpha^2+(1-\varepsilon\alpha)(1-\alpha)\beta\\
&\quad =(1-\alpha)[(1-\alpha)\beta-(1-\varepsilon)\alpha]<0, \quad \text{by (A4)}.
\end{aligned}
$$

Hence, whether case 1, case 2, or case 3 above is true, we get $\tilde{\eta}=\eta_1<(1-\varepsilon)\alpha$.

(iii) Consider an η such that (A1) holds. In view of Lemma 3, (A1) implies (A3) and thereby (together with (A4)) $D>0$ so that for this η both a social optimum and a market equilibrium exist and have a steady state, by Propositions 1 and 2. Since also, from these propositions, g_c and g_c^* are increasing in η, we have $\eta \gtreqless \tilde{\eta} \Rightarrow g_c \gtreqless g_c^*$, respectively. $\square$

Proof of Proposition 4. Assume (A4). Consider an $\eta \in (0,(1-\varepsilon)\alpha)$ such that the social optimum exists and has a steady state. Then, from Section 3 we know that (A1) is satisfied. In view of Lemma 3 also (A3) holds and this together with (A4) ensures $D>0$ so that also the market equilibrium exists and has a steady state, by Proposition 2. Now, suppose our η satisfies (A2), i.e., $\eta>(1-\alpha)\rho/(\gamma L)$. Then, straightforward calculation from (4.15) and (3.7) yields

$$g_c>g_c^* \Leftrightarrow 1-\varepsilon>\frac{\eta}{\alpha}+\frac{1-\alpha}{\alpha}\cdot\frac{\eta\gamma L-(1-\alpha)\rho}{(1-\alpha)\rho-(1-\theta)\eta\gamma L}. \tag{A.5}$$

If instead, $\eta \leqslant (1-\alpha)\rho/(\gamma L)$, then, by Proposition 1, $g_c^*=0$ whatever the value of ε; but, by Proposition 2,

$$g_c>0 \Leftrightarrow 1-\varepsilon>\frac{(1-\alpha)\rho}{\alpha\gamma L}. \tag{A.6}$$

Now, when (A2) holds, the RHS of (A.5) is larger than η/α, which, by (A2), is larger than the RHS of (A.6). But when (A2) does not hold, we have instead

$$\frac{\eta}{\alpha}+\frac{1-\alpha}{\alpha}\cdot\frac{\eta\gamma L-(1-\alpha)\rho}{(1-\alpha)\rho-(1-\theta)\eta\gamma L} \leqslant \frac{\eta}{\alpha} \leqslant \frac{(1-\alpha)\rho}{\alpha\gamma L}.$$

This completes the proof of (iii). $\square$

Proof of Proposition 5. From (5.3) we have $\xi=\eta-(1/\varepsilon-1)\alpha$. By (iii) of Proposition 3, $g_c>g_c^*$ if and only if $\eta<\tilde{\eta}$. Hence, $g_c>g_c^*$ if and only if $\xi<\tilde{\eta}-(1/\varepsilon-1)\alpha$, where, since $0<\varepsilon<1$, $\tilde{\eta}-(1/\varepsilon-1)\alpha<\tilde{\eta}-\varepsilon(1/\varepsilon-1)\alpha=\tilde{\eta}-(1-\varepsilon)\alpha<0$ by (ii) of Proposition 3. $\square$

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