

Growth and non-renewable resources: The different roles of capital and resource taxes

Christian Groth^{a,*}, Poul Schou^b

^a*Department of Economics and EPRU, University of Copenhagen, Studiestraede 6, DK-1455, Copenhagen K, Denmark*

^b*Danish Rational Economic Agents Model (DREAM), Denmark*

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Abstract

We contrast effects of taxing non-renewable resources with the effects of traditional capital taxes and investment subsidies in an endogenous growth model. In a simple framework we demonstrate that when non-renewable resources are a necessary input in the sector where growth is ultimately generated, interest income taxes and investment subsidies can no longer affect the long-run growth rate, whereas resource tax instruments are decisive for growth.

The results stand out both against observations in the literature from the 1970's on non-renewable resources and taxation—observations which were not based on general equilibrium considerations—and against the general view in the newer literature on taxes and endogenous growth which ignores the role of non-renewable resources in the “growth engine”.

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1. Introduction

How does taxation of non-renewable natural resources affect growth possibilities in the long run? This question is little examined in the literature, even though it may be of great importance for consumption in the long run. The comprehensive literature on non-renewable resources following the oil crisis of the 1970's contains some observations on the effects of taxation. Examples include Stiglitz [34] and Dasgupta and Heal [8, Chapter 12]. However, essentially these observations are concerned only with taxation in a partial equilibrium framework. The rate of interest is taken as given, and this crucially affects the conclusions.

With the rise of the so-called New Growth literature around 1990, a large and growing number of contributions have examined how various environmental tax policies may affect economic growth. Examples are Bovenberg and Smulders [6], Stokey [35] and Fullerton and Kim [10]. These papers describe effects of

*Corresponding author. Fax: +45 35 32 30 00.

E-mail addresses: Chr.Groth@econ.ku.dk (C. Groth), psu@dreammodel.dk (P. Schou).

taxing pollution in various set-ups, but generally ignore the specific scarcity problems following from the use of *non-renewable* resources. Some contributions [23,38,16,1, pp. 163–164, 26–28,11] do examine endogenous growth in a framework where non-renewable resources are present, and some of these also consider effects of taxation. However, these models all share the feature that no non-renewable resource appears in the sector constituting the “growth engine” of the model. This is a crucial trait and at the same time it is unrealistic. Most production sectors, including educational institutions and research labs, use fossil fuels for heating and transportation purposes, or minerals and oil products for machinery, computers, etc.

In a simple endogenous growth model where the resource enters the “growth engine” the present paper studies the effects of different forms of taxation on capital and resources. Unlike results from (partial equilibrium) analyses as in Stiglitz [34] and Dasgupta and Heal [8], a tax on capital gains on a non-renewable resource stock (which under certain conditions is equivalent to a tax on resource companies’ profits) is here shown to be far from neutral, but rather of importance for long-run growth in the economy. The same is true for a time-varying tax on resource use. Our results also contrast with the general tenet within the endogenous growth literature that interest income taxes impede economic growth and investment subsidies promote economic growth. There may be disagreement as to the size of the effects, but not their sign [19,17,22,16,5,36,20,2]. These results rest on growth models that ignore non-renewable natural resources. In the simple framework of the present paper, we show that this conventional view does not go through when the non-renewable resource is a necessary input in the sector generating long-run growth.¹ Indeed, the framework allows a rich set of determinants of long-run growth, but interest income taxes and investment subsidies are not among these.²

Our analysis is based on a straightforward extension of the one-sector model of Stiglitz [32–34]. In contrast to Stiglitz we focus on the case of increasing returns at the aggregate level with respect to labor, “broad capital” and the resource taken together. This case may arise as a result of the “learning-by-investing” effect hypothesized by Arrow [4] or the non-rivalness of technical knowledge as emphasized by Romer [24]. In fact, constant or even increasing returns to capital alone are not excluded. Further, in order to face an interesting optimal taxation problem we allow for a negative externality of resource use (like the “greenhouse effect”, say). This externality is modeled in the simplest possible way, in line with Suzuki [37] and Sinclair [29,30]. While the focus of these authors was on sustainability of the *level* of consumption and on emission abatement by a specific tax (a carbon tax), respectively, the present paper focuses more generally on growth effects of different kinds of taxes and subsidies.

Conventional endogenous growth models rely on exactly constant returns to produced inputs in the sector that drives growth. Slightly increasing returns lead to explosive growth (infinite output in finite time!),³ whereas slightly decreasing returns eventually lead to a zero growth rate. As shown by Groth and Schou [12], if an essential natural resource enters the growth generating sector this knife-edge problem is alleviated: the need to save increasingly on resource use counteracts the potentially explosive effects of capital accumulation implied by increasing returns.

Following this lead the present paper demonstrates that conventional fiscal policy instruments like an interest income tax and a capital investment subsidy affect neither consumption growth nor the speed of resource depletion in the long run. The reason is that households can switch cost free between capital and resource assets, and the returns on savings are ultimately determined by returns on postponing resource extraction. Consequently, a policy directed at influencing long-run consumption growth has to affect returns to conservation of the resource. In this way, resource taxation becomes decisive for the influence of economic policy on growth. Two instruments that influence long-run growth in opposite directions are a tax on capital gains on the resource stock and a credibly announced declining tax on resource use.

¹In the present one-sector model, to the extent that growth is endogenous, the manufacturing sector itself is the growth-generating sector, i.e., the “growth engine”. Similarly in the AK model of standard endogenous growth theory.

²Uhlig and Yanagawa [39] also present an analysis arguing against the conventional view on interest income taxation and growth. They use an OLG framework to demonstrate that a rise in the tax on interest income may raise growth when the interest elasticity of savings is sufficiently low and the revenue is used to lower labor taxes (leaving the young generation with a larger disposable income from which they can make savings). By taking into consideration the—realistic—presence of natural resources in the production function, we show that also in a representative agent model, interest taxes need not affect growth negatively in the long run.

³Cf. Solow [31].

An alternative way to understand the basic result of this paper is to note that any balanced growth path of the economy has to comply with the linear differential equation describing resource depletion, $\dot{S} = -uS$, where S is the stock of the resource and u is the (proportionate) depletion rate. As noticed by Romer [25], for policy to affect long-run growth, it must affect a linear differential equation in the model. The depletion equation is such an equation. When the resource is an essential input in the growth engine, only policies that affect the depletion rate (directly or indirectly) can be important for long-run growth, that is, for future consumption possibilities. It turns out that an interest income tax and a capital investment subsidy are not such policies.

The organization of the paper is as follows. The next section describes the elements of the model. In Section 3 we study existence and stability of balanced growth paths and the effects of an investment subsidy and of taxes on interest income, capital gains and resource use in balanced growth. Section 4 describes a first-best policy. It turns out, in the setup we consider, that a declining tax on resource use and a capital investment subsidy are necessary ingredients of a first-best policy, while capital gains and interest income should not be taxed. A summary of the conclusions is given in the final section.

2. The model

2.1. Technology and firms' behavior

We study an economy where the non-renewable resource is necessary for production, but does not in advance rule out sustainable (non-decreasing) consumption. Thus, following Stiglitz [32,33], we assume a production function of Cobb–Douglas form. Let $Y_i(t)$ be firm i 's output, $K_i(t)$ its capital input, $N_i(t)$ its labor input and $R_i(t)$ its input of the non-renewable resource, all at time t . Then firm i produces according to

$$Y_i(t) = A(t)K_i(t)^{\alpha_1}N_i(t)^{\beta}R_i(t)^{\gamma}, \quad \alpha_1, \beta, \gamma > 0, \quad \alpha_1 + \beta + \gamma = 1, \quad (1)$$

where $A(t)$ is total factor productivity, given by

$$A(t) = e^{\theta t}K(t)^{\alpha_2}S(t)^{\lambda}, \quad \theta, \alpha_2, \lambda \geq 0.$$

As in Suzuki [37] and Sinclair [29,30], other things equal, total factor productivity is gradually decreased in line with the extraction and use of the resource. This is interpreted as the result of degradation of environmental quality associated with pollution from the use of fossil fuels etc., the stock of pollution being proxied inversely by the remaining resource stock $S(t)$ (think of the greenhouse effect).⁴ There are CRS with respect to the three inputs that the firm can control. But if $\alpha_2 > 0$, aggregate capital, $K(t)$, has a positive external effect on productivity (the “learning-by-investing” effect hypothesized by Arrow [4]). This gives rise to increasing returns at the aggregate level and possibly “endogenous growth”; this is where we depart from the model in Sinclair [30].⁵ Increasing returns to scale at the aggregate level or at least in the sector(s) constituting the “growth engine” of the economy is standard in models of semi-endogenous growth (as Jones [15]) as well as models of strictly endogenous growth (as [24]).⁶ Though we name K just ‘capital’, one may interpret K as ‘broad capital’ including technical knowledge and human capital.

⁴This interpretation ignores the regeneration ability of the atmosphere and is (at best) applicable only for a limited (though long) span of time, namely as long as emissions from resource use is much higher than the regeneration ability. In this sense we go to one extreme, whereas the standard literature on the greenhouse effect go to the other extreme, by ignoring the non-renewable nature of fossil fuel altogether. This makes it possible to see the differences between the two approaches more clearly.

⁵Empirical evidence furnished by, e.g., Hall [14] and Caballero and Lyons [7] suggests that there are quantitatively significant increasing returns to scale or external effects in U.S. and European manufacturing. Antweiler and Treffer [3] examine trade data for goods-producing sectors and find evidence for increasing returns to scale.

⁶The term “semi-endogenous growth models” refers to models where, first, per capita growth is driven by some internal mechanism (in contrast to exogenous technology growth). Second, unlike “strictly endogenous growth”, sustained per capita growth requires the support of some growing exogenous factor, typically the labor force. While strictly endogenous growth requires $\alpha > 1$ (when $\gamma > 0$), semi-endogenous growth may be an attractive alternative, requiring only $\alpha + \beta > 1$. For most of the results of the present paper the distinction is not important.

There may also be an irreducibly exogenous element in technology growth, represented by the parameter θ . By allowing both α_2 and θ to be positive, we are able to demonstrate whether and when the *source* of growth—exogenous or endogenous—matters for the results.

There is a large number of similar firms and each of them takes the aggregate capital and resource stocks as given. Assuming perfect competition and using current output as the *numeraire*, profit maximization leads to:

$$\alpha_1 \frac{Y_i}{K_i} = (1 - \sigma)(r + \delta), \quad 0 \leq \sigma < 1, \quad \delta \geq 0, \quad (2)$$

$$\beta \frac{Y_i}{N_i} = w, \quad (3)$$

$$\gamma \frac{Y_i}{R_i} = (1 + \tau_u)p, \quad (4)$$

where $r + \delta$ is the capital cost (rate of interest plus rate of capital depreciation), w is the real wage, and p is the real price of a unit flow of the resource (we ignore the time argument of the variables when not needed for clarity). The parameter σ in (2) represents a subsidy to absorb part of the cost of buying capital services (for brevity an “investment subsidy”), and τ_u in (4) is a (possibly time-dependent) tax on resource use (like a carbon tax). An alternative interpretation of (4) would be that it reflects a royalty to the government paid by the extractive agent as a percentage of the sales value. If the (possibly time-dependent) royalty rate is called ω and the price faced by the user is q , we have $q = (1 + \tau_u)p$, where $p = (1 - \omega)q$ is the revenue left to the extractive agent per unit of the resource supplied. This gives $\omega = 1 - 1/(1 + \tau_u)$. Focusing on ω instead of τ_u would not change anything of substance. To fix ideas, we shall stick to the first interpretation.

Since all firms hire factors in the same proportions, aggregate output can be written

$$Y \equiv \sum_i Y_i = e^{\theta t} K^\alpha N^\beta R^\gamma S^\lambda, \quad \alpha \equiv \alpha_1 + \alpha_2, \quad (5)$$

where $K \equiv \sum_i K_i$, $N \equiv \sum_i N_i$, and $R \equiv \sum_i R_i$. Assuming market clearing, K , N , and R can also be interpreted as the aggregate supplies, and (2)–(4) imply

$$r = \frac{\alpha_1}{1 - \sigma} \frac{Y}{K} - \delta, \quad (6)$$

$$w = \beta \frac{Y}{N}, \quad (7)$$

$$p = \frac{\gamma}{1 + \tau_u} \frac{Y}{R}. \quad (8)$$

2.2. Households

There is a fixed number of infinitely-lived households (families), all alike. For notational convenience we let the number of households be one, the representative household. It has N members, each supplying one unit of labor inelastically at a competitive labor market. We let household size grow at a constant exogenous rate $n \geq 0$, i.e., $N = N(0)e^{nt}$, $N(0) > 0$. The household consumes and saves, and savings can be either in loans, physical capital or the resource stock. At the aggregate level loans and deposits sum to zero (closed economy, no government debt). Let \hat{p} denote the market price of a unit of stock of the (not yet extracted) resource. Then, assuming no extraction costs, we have in equilibrium $\hat{p} = p$, and financial wealth, V , satisfies

$$V = K + pS, \quad (9)$$

where S is the resource stock owned by the household.

We assume iso-elastic instantaneous utility and a constant rate of time preference ρ . The intertemporal utility function then is

$$U_0 = \int_0^\infty \frac{c^{1-\varepsilon} - 1}{1 - \varepsilon} N e^{-\rho t} dt, \quad \varepsilon > 0, \quad \rho > n \geq 0, \quad (10)$$

where ε is the (constant) absolute value of the elasticity of marginal utility.⁷ The assumption $\rho > n$ is introduced to ease convergence of the integral.

The household has perfect foresight and chooses a path $(c, S)_{t=0}^{\infty}$ to maximize U_0 subject to $c \geq 0, S \geq 0$, the dynamic budget constraint,

$$\dot{V} = (1 - \tau_r)r(V - pS) + (1 - \tau_{cg})\dot{p}S + wN - T - cN, \quad V(0) \text{ given} \quad (11)$$

and the no-Ponzi-game condition,

$$\lim_{t \rightarrow \infty} V e^{-(1-\tau_r)t} \int_0^t r(s) ds \geq 0. \quad (12)$$

Here, $\tau_r < 1$ represents a constant rate of tax on interest income (if $\tau_r < 0$ one should think of a subsidy to interest income), $\tau_{cg} < 1$ is a constant rate of tax (subsidy if negative) on capital gains, and T is a lump-sum tax (amounting to a transfer, if negative). Notice that r and \dot{p} refer to the *real* interest rate and *real* capital gains, respectively. We assume a fully real income-based tax system and hence do not consider the extra distortions implied by the presence of nominal income-based ingredients in the actual tax systems of many countries; in a low-inflation world this limitation may not be serious. We abstract from wage income taxes and consumption taxes because their role is trivial in a model without utility of leisure.

On the face of it, taxation of capital gains on resource reserves are perhaps rarely seen in the real world. However, τ_{cg} together with τ_r can be seen as particular representations of a global tax on all types of capital income. In the present model where the household can invest in two different kinds of physical assets (the resource and physical capital), a comprehensive capital income tax corresponds to the special case $\tau_r = \tau_{cg}$. But in order to understand fully the differential roles of taxes on the two different assets, we allow τ_{cg} and τ_r to differ (and include the limiting case $\tau_{cg} = 0$).

Moreover, the capital gains tax is in fact equivalent to a profits tax on resource extracting companies if the depletion allowance equals the true economic depreciation on the remaining resource reserves. Instead of households directly owning the resource stock, imagine households have shares in resource companies. These companies satisfy the flow demand R from manufacturing firms at the given market price p . Gross revenue is pR . The only cost is the true economic depreciation $\mathcal{D} = -d(pS)/dt = -(p\dot{S} + \dot{p}S) = pR - \dot{p}S$. Profits are $\pi = pR - \mathcal{D} = \dot{p}S$. Let the profits (or rents as it really is) of these companies be taxed at rate τ_π . The after-tax profits, $(1 - \tau_\pi)\dot{p}S$, are paid out to share owners. Assuming no double taxation, the dynamic budget constraint of the representative household is now

$$\dot{V} = (1 - \tau_r)r(V - pS) + (1 - \tau_\pi)\dot{p}S + wN - T - cN.$$

With $\tau_\pi = \tau_{cg}$ this is the same as (11) above. The equivalence of a capital gains tax and a profits tax (under true economic depreciation) would also hold if extraction costs were present.⁸

Returning to the decision problem of the household, existence of an interior solution implies, first, the Keynes–Ramsey rule

$$\frac{\dot{c}}{c} = \frac{1}{\varepsilon} [(1 - \tau_r)r - \rho], \quad (13)$$

second, the (tax-adjusted) Hotelling rule

$$\frac{(1 - \tau_{cg})\dot{p}}{p} = (1 - \tau_r)r \quad (14)$$

and, third, a transversality condition implying that (12) holds with equality. The Hotelling rule is a no-arbitrage condition between investing in the resource (leaving it in the ground) and investing in ordinary financial assets. In case of a comprehensive tax on capital income, i.e., $\tau_{cg} = \tau_r$, the condition reduces to the well-known, simple Hotelling rule, $\dot{p}/p = r$.

⁷In case $\varepsilon = 1$, the expression $\frac{c^{1-\varepsilon}-1}{1-\varepsilon}$ should be interpreted as $\ln c$.

⁸Dasgupta and Heal [8, Chapter 12] discuss existence and partial equilibrium effects of resource taxation, including profits taxes, capital gains taxes and different kinds of depletion allowance.

2.3. Government

At any time the government balances its budget by adjusting T so that

$$\tau_u pR + \tau_r rK + \tau_{cg} \dot{p}S + T = \sigma(r + \delta)K. \tag{15}$$

The only public expense is the subsidy σ paid out to firms to reduce their capital costs. On the revenue side we have the tax τ_u on resource use (the “carbon tax”) imposed on firms, while the interest income tax τ_r , the capital gains tax τ_{cg} , and the lump-sum tax T (or transfer) are imposed on households.

There is a given finite resource stock to extract from. Thus what matters for resource extraction is not the level of the *ad valorem* tax τ_u , but its rate of change, as pointed out by Dasgupta and Heal [8]. Following Sinclair [29,30] we assume that the government credibly announces

$$\dot{\tau}_u = -(1 + \tau_u)\psi, \quad \lim_{t \rightarrow \infty} \psi = \bar{\psi} \geq 0, \tag{16}$$

where ψ may generally be time-dependent (in a smooth way), but is constant in the limit to ensure compatibility with balanced growth in the long run. We shall call ψ the conservation stimulus since, by resulting in a declining tax rate, $\psi > 0$ stimulates postponement of extraction.⁹

3. Economic development

Output is used for consumption and for investment in capital goods, so that

$$\dot{K} = Y - C - \delta K, \quad K(0) > 0, \tag{17}$$

where $C \equiv cN$ is total consumption. The resource stock S diminishes with resource extraction:

$$\dot{S} = -R, \quad S(0) > 0. \tag{18}$$

The definitional non-negativity condition on S implies, from (18), the restriction

$$\int_0^\infty R(t) dt \leq S(0), \tag{19}$$

showing the finite upper bound on cumulative extraction of the resource over the infinite future. Obviously, from this restriction it follows that resource use must approach zero for $t \rightarrow \infty$.

The transversality condition of the household says that (12) holds with equality; this implies, by (9) and (14),

$$\lim_{t \rightarrow \infty} K(t) e^{-(1-\tau_r) \int_0^t r(s) ds} = 0, \tag{20}$$

and

$$\lim_{t \rightarrow \infty} S(t) e^{\frac{\tau_{cg}}{1-\tau_{cg}}(1-\tau_r) \int_0^t r(s) ds} = 0. \tag{21}$$

The last condition requires (when $\tau_{cg} > 0$ and $\lim_{t \rightarrow \infty} r(t) > 0$) not only that no finite part of the resource stock will be left unused forever, but also that the resource stock diminishes at a sufficient speed.

The system characterized by the technology (1), the intertemporal utility function (10), and the dynamic resource conditions (17) and (18), will be called an *economic system*. A *viable* economic system is a system where C, Y, K, R , and S are (strictly) positive for all $t \geq 0$ (“no collapse”).

The quadruple $(\tau_r, \tau_{cg}, \sigma, (\psi)_{t=0}^\infty)$ of tax and subsidy instruments will be called a *policy*. Given the policy $(\tau_r, \tau_{cg}, \sigma, (\psi)_{t=0}^\infty)$, an *equilibrium* of a viable economic system is a path for prices and quantities such that: (i) households maximize discounted utility, taking the time paths of the interest rate, the resource price, and the wage rate as given; (ii) firms maximize profits choosing inputs of capital, labor, and the resource, taking the prices of these inputs as given; (iii) the government adjusts lump sum taxes T so that the budget constraint (15) is satisfied at any t ; and (iv) in each market, the supply is equal to the demand.

⁹Solving the differential equation (16) gives $\tau_u(t) = (1 + \tau_u(0))e^{-\int_0^t \psi(s) ds} - 1$, where we assume $\tau_u(0) > -1$. Hence, we allow $\tau_u(t)$ to be (or become) negative.

Let the output-capital ratio, the consumption-capital ratio, and the resource depletion rate be denoted z , x and u , respectively, i.e.,

$$z \equiv \frac{Y}{K}, \quad x \equiv \frac{C}{K}, \quad u \equiv \frac{R}{S}.$$

These ratios turn out to be central to the analysis. Let g_m denote the growth rate of a variable m (>0), that is $g_m \equiv \dot{m}/m$. Then, we may write (17) as

$$g_K = z - x - \delta. \quad (22)$$

Similarly, by (18),

$$g_S = -u. \quad (23)$$

In an equilibrium, (5) holds, implying, by logarithmic differentiation, using (23),

$$g_Y = \alpha g_K + \beta n + \gamma g_R + \theta - \lambda u. \quad (24)$$

Similarly, (6), (8), (14) and (16) lead to the equilibrium version of the Hotelling rule:

$$(1 - \tau_{cg})(g_Y - g_R + \psi) = (1 - \tau_r) \left(\frac{\alpha_1}{1 - \sigma} z - \delta \right) \quad (25)$$

and (6) together with (13) yields the equilibrium version of the Keynes–Ramsey rule:

$$g_C = g_c + n = \frac{1}{\varepsilon} \left[(1 - \tau_r) \left(\frac{\alpha_1}{1 - \sigma} z - \delta \right) - \rho \right] + n. \quad (26)$$

Note that any equilibrium satisfies (22)–(26).

3.1. Balanced paths

A path $(C, Y, K, R, S)_{t=0}^{\infty}$ generated by a viable economic system will be called a *balanced growth path* (henceforth abbreviated BGP) if C, Y, K, R and S change with constant relative rates for all $t > 0$ (some or all these rates may be negative). An equilibrium path $(C, Y, K, R, S)_{t=0}^{\infty}$ which is a BGP will be called a *balanced growth equilibrium* (abbreviated BGE).¹⁰

Lemma 1. For any BGE $(C, Y, K, R, S)_{t=0}^{\infty}$ the following holds:

- (i) $g_C = g_Y = g_K \equiv g^*$, a constant;
- (ii) $g_R = g_S = -u = -u^*$, where u^* is some positive constant;
- (iii) z and x are positive constants, and $\psi = \bar{\psi}$;
- (iv) g^* and u^* satisfy

$$(1 - \alpha)g^* + (\gamma + \lambda)u^* = \beta n + \theta, \quad (27)$$

$$[(1 - \tau_{cg}) - \varepsilon]g^* + (1 - \tau_{cg})u^* = \rho - \varepsilon n - (1 - \tau_{cg})\bar{\psi}; \quad (28)$$

- (v) $\tau_{cg}(g^* + \bar{\psi}) < (1 - \tau_{cg})u^*$.

Proof. See Appendix.

The necessity of (v) is due to the transversality conditions (20) and (21).

Let D be the determinant of the linear system (27) and (28):

$$D \equiv (1 - \tau_{cg})(1 - \alpha - \gamma - \lambda) + (\gamma + \lambda)\varepsilon. \quad (29)$$

From this system immediately follows:

¹⁰The values taken by the variables along a BGE are marked by *.

Proposition 1. Assume $D \neq 0$. Then the capital gains tax τ_{cg} and the conservation stimulus $\bar{\psi}$ can affect growth in a BGE, but the interest income tax τ_r and the investment subsidy σ cannot.

This result can be explained in the following way. Eq. (27), linking g^* and u^* independently of policy parameters, is dictated by mere technical feasibility, given the aggregate production function (5). In contrast, (28) represents the effects of the market mechanism through the Hotelling rule and the Keynes–Ramsey rule. These effects are such that τ_r and σ are de-coupled from the determination of growth. Indeed, the household can save in different assets (physical capital and the resource stock) and arbitrage equalizes the after-tax returns on these assets (uncertainty is ignored). Profit maximization implies (8), hence, in a BGE, $\dot{p}/p = g^* + u^* + \bar{\psi}$, and then after-tax returns on saving in capital, $(1 - \tau_r)r$, are equal to after-tax returns on saving in resource conservation, $(1 - \tau_{cg})(g^* + u^* + \bar{\psi})$, whatever the size of τ_r . Given the capital gains tax τ_{cg} and the conservation stimulus $\bar{\psi}$, the before-tax return on resource conservation depends, through the Hotelling rule, only on the two rates of change, g_Y and g_R ; and in a BGE these two rates are tied down by (27) and (28). In contrast, the before-tax return on capital depends on the productivity of capital, Y/K , which is free to adjust. In this way, ultimately, $(1 - \tau_r)r$ is replaced by $(1 - \tau_{cg})(g^* + u^* + \bar{\psi})$ in the Keynes–Ramsey rule. As a consequence both τ_r and σ (as well as α_1 , cf. (6)) become excluded from the subsystem of zero order in the causal structure.

To get the economic intuition consider a lowering of the tax, τ_r , on capital income. This leads to a portfolio adjustment, implying more demand for capital so that capital intensity goes up and r goes down until $(1 - \tau_r)r$ is as before. In the same way, an increase in the investment subsidy σ increases demand for capital so that, again, capital intensity goes up and $z \equiv Y/K$ goes down until $r = \alpha_1 z / (1 - \sigma) - \delta$ is as before. In both cases the feedback restores the previous after-tax rate of return as well as the rates of capital accumulation and resource depletion. It is otherwise with an increase in the capital-gains tax τ_{cg} . This reduces the after-tax return on conservation. Since the stock of the resource is inelastic, the resource price immediately falls. This incites firms to demand more of the resource as input, that is, u is raised. The resulting lift in the marginal productivity of capital means that the required rate of return is raised. Therefore, the unattractiveness of conservation tends to be reinforced rather than counteracted. The faster depletion implies a larger drag on growth and the economy settles down in the long run along a path with lower growth and a lower rate of return. Similarly with a decrease in the conservation stimulus $\bar{\psi}$.

3.2. Existence and stability

Given $D \neq 0$, by solving the system (27) and (28) we find the growth rate of output and the depletion rate, respectively, along a BGE:

$$g^* = \frac{(1 - \tau_{cg})(\beta n + \theta) + [\epsilon n - \rho + (1 - \tau_{cg})\bar{\psi}](\gamma + \lambda)}{D}, \tag{30}$$

$$u^* = -g_R^* = \frac{[(\alpha + \beta - 1)n + \theta]\epsilon - (1 - \tau_{cg})(\beta n + \theta) + [\rho - (1 - \tau_{cg})\bar{\psi}](1 - \alpha)}{D}. \tag{31}$$

Now, by (25), (22), and (27) we find

$$z^* = \frac{1 - \sigma}{\alpha_1} \left[\frac{1 - \tau_{cg}}{1 - \tau_r} (g^* + u^* + \bar{\psi}) + \delta \right] \tag{32}$$

and

$$x^* = \frac{\left[(1 - \sigma) \frac{1 - \tau_{cg}}{1 - \tau_r} (1 - \alpha - \gamma - \lambda) + (\gamma + \lambda)\alpha_1 \right] u^* + \left[(1 - \sigma) \frac{1 - \tau_{cg}}{1 - \tau_r} - \alpha_1 \right] (\beta n + \theta)}{(1 - \alpha)\alpha_1} + (1 - \sigma) \frac{1 - \tau_{cg}}{1 - \tau_r} \frac{\bar{\psi}}{\alpha_1} + \frac{1 - \sigma - \alpha_1}{\alpha_1} \delta, \tag{33}$$

where the formulas for g^* and u^* can be inserted.¹¹

Let π denote the 15-tuple $(\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \alpha, \delta, n, \epsilon, \rho, \sigma, \bar{\psi}, \tau_r, \tau_{cg})$, and let P be the set of $\pi \in R^{15}$ such that $\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \delta, n, \epsilon, \rho, \sigma, \bar{\psi} \in R_+$, $\tau_r < 1$, $\tau_{cg} < 1$, $\alpha = \alpha_1 + \alpha_2$, and the parameter inequalities stated in (1), (2),

¹¹The resulting formulas, given in the working paper version [13] of this paper, are voluminous and not particularly illuminating.

and (10) are satisfied. Let $P^* \subset P$ be the subset satisfying the requirements that $D \neq 0$, u^* , z^* , and x^* are strictly positive and (v) of Lemma 1 is satisfied. This subset P^* will be called the *BGE supporting set*. We have:

Lemma 2. *The set $P^* \subset P$ has a non-empty interior.*

Proof. The claim is evident from the example in footnote 14 below. \square

Proposition 2. *Let the parameter tuple $\pi \in P$. Assume $D \neq 0$ and let $\psi(t) = \bar{\psi}$ for all $t \geq 0$. Then:*

- (i) *There exists a BGE if and only if $\pi \in P^*$.*
- (ii) *For any given $\pi \in P^*$, the associated BGE has a unique quadruple (g^*, u^*, z^*, x^*) .*

Proof. See Appendix.

Remark 1. In the Stiglitz case ($\alpha + \beta + \gamma = 1$, $\lambda = 0$, and $\tau_{cg} = \tau_r = \sigma = \bar{\psi} = 0$) the existence requirements $z^* > 0$ and $x^* > 0$ are automatically satisfied and the only thing to check is whether the parameters are consistent with $u^* > 0$.

Remark 2. If $D = 0$, a BGE can exist only in case $\beta n + \theta = [(\rho - \epsilon n)/(1 - \tau_{cg}) - \bar{\psi}](\gamma + \lambda)$. Only in this knife-edge case is there scope for τ_r and σ to affect long-run growth, namely by selecting one BGE from the continuum of BGEs allowed by (27) and (28) in this case.

From now we consider π as given and belonging to the BGE supporting set P^* . Generally, we allow ψ to be time-dependent,¹² except in the limit. Therefore, a BGE (g^*, u^*, z^*, x^*) may be realizable only asymptotically. We shall use the phrase “there exists a BGE...” as a shorthand for “there exists, at least asymptotically, a BGE...”.

The dynamics of the model can be reduced to a three-dimensional system in u, z and x . In view of the assumption $\psi(t) \rightarrow \bar{\psi}$ for $t \rightarrow \infty$, the system is asymptotically autonomous. The associated steady state (u^*, z^*, x^*) is a BGE with $g^* = z^* - x^* - \delta$. We call the BGE saddle-point stable if there exists a unique solution converging to the steady state for $t \rightarrow \infty$. And we call the BGE totally unstable if all three eigenvalues of the associated Jacobian have positive real part. In order not to endanger saddle-point stability the assumption $\lambda/2 < 1 - \gamma$ is convenient (and quite innocent since, empirically, γ is likely to be quite low, say less than .05).

Proposition 3. *If $D > 0$ and $\lambda/2 < 1 - \gamma$, then a BGE is saddle-point stable. On the other hand, if $D < 0$ and in addition*

$$(A.1) \quad \sigma \leq 1 - \alpha_1/\alpha \quad \text{and} \quad \tau_{cg} \approx \tau_r,$$

then (at least for λ “small”) a BGE is totally unstable.

Proof. See Appendix.

The policy assumption (A.1) is invoked in order to have a clear-cut instability implication of $D < 0$. The assumption is “natural” in the sense that its first part says that the investment subsidy does not overcompensate the positive external effect of investment, and its second part says that the capital gains tax is not very different from other capital income taxes.¹³

3.3. Economic growth

We shall concentrate on the case $D > 0$ which seems also the most realistic case since $D \leq 0$ requires a considerable amount of increasing returns (i.e., $\alpha + \gamma \geq 1 - \lambda + \frac{(\gamma + \lambda)\epsilon}{1 - \tau_{cg}} > 1$ for λ “small”). The *per capita* growth

¹²This is because, as we shall see (Section 4), in the first-best solution ψ is only asymptotically constant.

¹³The qualifier referring to “smallness” of the externality λ is explained in Appendix.

rate in a BGE is

$$g_c^* = g^* - n = \frac{(1 - \tau_{cg})[(\alpha + \beta + \gamma + \lambda - 1)n + \theta] - [\rho - (1 - \tau_{cg})\bar{\psi}](\gamma + \lambda)}{D}. \quad (34)$$

Now, when is stable *positive* per capita growth possible in spite of the diminishing input of the resource? To answer this, we first introduce a purely technological condition that is necessary for $g_c^* > 0$.

Lemma 3. Assume $D > 0$. A BGE with $g_c^* > 0$ can exist only if $(\alpha + \beta - 1)n + \theta > 0$.

Proof. Consider a BGE. From Lemma 1, $u^* > 0$. Hence, by (27), $(1 - \alpha)g^* < \beta n + \theta$, and since $g^* = g_c^* + n$, this implies $(1 - \alpha)g_c^* < (\alpha + \beta - 1)n + \theta$. \square

Proposition 4. Assume $D > 0$. Then a BGE has $g_c^* > 0$ if and only if the parameters satisfy

$$(\alpha + \beta - 1)n + \theta > \max \left[\left(\frac{\rho}{1 - \tau_{cg}} - \bar{\psi} - n \right) (\gamma + \lambda), 0 \right]. \quad (35)$$

When $\theta = 0$ (the case of no exogenous technical progress), $g_c^* > 0$ if and only if

$$\alpha + \beta > 1 \quad \text{and} \quad n > \max \left[\left(\frac{\rho}{1 - \tau_{cg}} - \bar{\psi} \right) \frac{\gamma + \lambda}{\alpha + \beta + \gamma + \lambda - 1}, 0 \right]. \quad (36)$$

Proof. Assume $D > 0$. Then (35) follows immediately from (34) and Lemma 3. If $\theta = 0$, then (35) is equivalent to (36) since, by Lemma 3, $g_c^* > 0 \wedge \theta = 0 \Rightarrow \alpha + \beta > 1 \wedge n > 0$. \square

When $\alpha + \beta = 1$ or $n = 0$, the right hand side of (35) gives a lower bound for the rate of *exogenous* technical progress required to compensate for the growth drag resulting from non-renewable resources. The capital gains tax τ_{cg} tends to increase this bound, while the conservation stimulus $\bar{\psi}$ (acting similarly to a decrease in the rate of time preference) tends to decrease it. If there are increasing returns with respect to capital and labor, *endogenous* growth may occur. We define endogenous growth to be present if $g_c^* > 0$ even when $\theta = 0$. As the last part of the proposition shows, endogenous per capita growth requires not only increasing returns with respect to capital and labor, but also a *sufficient* amount of population growth to let the increasing returns come into action, given the preferences of the representative household. These features are needed to offset the effects of the inevitable decline in resource use.¹⁴ If on the other hand the opposite of (35) is true, the long-run perspective is famine and a Malthusian check on population—if not doomsday.

Based on the assumption $D > 0$, Proposition 4 presupposes stability and its last part thus shows that *stable endogenous* growth requires a growing population. This warrants a remark on the precise role of population growth in this context. If θ is large enough to generate growth without the assistance of increasing returns, then a BGE with $g_c > 0$ may be stable even if there is no population growth. But endogenous and stable growth can only occur, under *laissez-faire*, when there is population growth.¹⁵ Indeed, if $\theta = n = \bar{\psi} = \tau_{cg} = 0$, then $g_c^* = g^* > 0$ requires $D < 0$, by (30), and this implies instability. The interpretation is *not* that population growth stabilizes an otherwise unstable BGE; stability–instability is governed by the sign of D , independently of n . Rather, given $D > 0$, letting n decrease from a level above the critical value in (36) to a level below, changes g_c^* from positive to negative, i.e., growth comes to an end. This is in contrast to standard endogenous growth models with non-renewable resources (such as Schou [27]) where population growth is not necessary for stable growth. The difference is explained by the fact that the resource does not enter the “growth engine” of these models.

¹⁴A numerical example is: $\alpha_1 = .60$, $\alpha = .90$, $\beta = .30$, $\gamma = .015$, $\lambda = .005$, $n = .01$, $\theta = 0$, $\delta = .07$, $\varepsilon = 2.00$, $\rho = .02$, $\sigma = .33$, $\bar{\psi} = .007$, and $\tau_{cg} = \tau_r = 0$; then $D > 0$ (stability), $g_c^* = .016$ and $u^* = .02$.

¹⁵Groth and Schou [12] further explore this fact and its relation to the knife-edge property of conventional endogenous growth models without non-renewable resources.

The *source* of growth matters for the question whether population growth is good or bad for growth. Eq. (34) shows that in the Stiglitz case where $\theta > 0$ is combined with constant returns to scale with respect to K, N and R and no externalities, population growth either does not affect g_c^* (if $\lambda = 0$) or affects g_c^* negatively (if $\lambda > 0$). But when there are increasing returns, population growth affects g_c^* positively.¹⁶ This is the net result of three different effects: there is a direct positive effect on output growth because a growing population means a growing labor force, magnifying the effects of increasing returns to scale. In addition, there is an indirect effect because population growth also affects resource extraction as seen from (31), although the sign of this effect is ambiguous without further specification. Thirdly, higher population growth naturally means more mouths to feed, and this implies a drag on growth in per capita consumption possibilities.

3.4. Effects of taxes and subsidies

To prepare the ground for the analysis of tax and subsidy effects we make some observations on the rate of interest. If $g_c^* > 0$, then, by the Keynes–Ramsey rule, $r > \rho / (1 - \tau_r)$, a positive number. More generally, in view of (6) and (32),

$$r^* = \frac{1 - \tau_{cg}}{1 - \tau_r} (g^* + u^* + \bar{\psi}) \quad (37)$$

from which follows that, as a rule, $r^* > 0$. To be precise, let \bar{P} be the set of all parameter tuples $\pi = (\alpha_1, \beta, \gamma, \alpha_2, \lambda, \theta, \alpha, \delta, n, \varepsilon, \rho, \sigma, \bar{\psi}, \tau_r, \tau_{cg})$ in the BGE supporting set P^* such that if δ is substituted by zero, then the new tuple is also in P^* . Clearly, \bar{P} is non-empty.¹⁷

Lemma 4. For all $\pi \in \bar{P}$, $r^* > 0$.

Proof. Consider a given $\pi \in \bar{P}$. Fix all coordinates of π except δ . Consider π as a function $\pi(\delta)$ of δ . Let r^* in the BGE corresponding to $\pi(0) \in P^*$ be called r_0^* . Since $z^* > 0$ in a BGE, $r_0^* > 0$, by (6). Then for any $\delta > 0$ such that $\pi(\delta) \in P^*$, $r^* = r_0^* > 0$, by (37), since g^* and u^* are independent of δ . \square

The only policies capable of affecting long-run growth are the capital gains tax τ_{cg} and the conservation stimulus $\bar{\psi}$. Assuming $D > 0$ and $r^* > 0$, we have, from (34),

$$\frac{\partial g_c^*}{\partial \tau_{cg}} = - \frac{(1 - \tau_r)(\gamma + \lambda)}{(1 - \tau_{cg})D} r^* < 0, \quad (38)$$

$$\frac{\partial g_c^*}{\partial \bar{\psi}} = \frac{(1 - \tau_{cg})(\gamma + \lambda)}{D} > 0. \quad (39)$$

Thus, an increase in the capital gains tax impedes long-run growth. The explanation is that taxing capital gains on leaving the resource in the ground fuels resource extraction. This creates a tendency to faster exhaustion of the resource stock, hence faster decline in resource use, implying that mere sustainability of per capita consumption takes up a larger share of the ongoing capital accumulation, leaving less aside for growth. An announced declining tax on resource use ($\bar{\psi} > 0$) has the opposite effects. Indeed, the declining tax implies a lower required before-tax return on leaving the marginal resource in the ground. This defers resource extraction and thereby growth is enhanced.¹⁸

The exact effects on the depletion rate of the two policy instruments are, from (31),

$$\begin{aligned} \frac{\partial u^*}{\partial \tau_{cg}} &= (1 - \alpha) \frac{1 - \tau_r}{(1 - \tau_{cg})D} r^* > 0, \quad \text{when } \alpha < 1, \\ \frac{\partial u^*}{\partial \bar{\psi}} &= -(1 - \alpha) \frac{1 - \tau_{cg}}{D} < 0, \quad \text{when } \alpha < 1, \end{aligned} \quad (40)$$

¹⁶This trait should not be seen as a prediction about individual countries in an internationalized world, but rather as pertaining to larger regions, perhaps the global economy.

¹⁷The example in footnote 14 shows this.

¹⁸Introducing a tax rate on capital gains of .3, with the parameters from footnote 14, g_c^* in a BGE will fall with almost one fifth to about .013. Removal of the conservation stimulus reduces g_c^* further to .012.

respectively.¹⁹ It is only the rate of change, $\dot{\bar{\psi}}$, of the resource use tax, τ_u , that matters, not its initial level. This is so in balanced growth as well as during the transitional dynamics. Indeed, with a constant *ad valorem* tax τ_u the present discounted value of the tax liability of a unit used is independent of the date of the purchase—this follows from the Hotelling rule. And because of firm's profit maximization the same is true for the present discounted value of the marginal product of the resource in manufacturing. Hence, if the government introduces a constant τ_u , the time profile of resource use is not affected, the only effect being that the sales price p to the resource owner is reduced to a fraction, $1 - \tau_u$, of its original value.²⁰

As already commented on above, the interest income tax rate τ_r and the capital subsidy rate σ affect neither growth nor resource extraction in a BGE. These policy instruments affect only *levels*. Assuming $D > 0$, and $r^* > 0$, the level effects of all four policy instruments are, using (26), (22), (38), (39) and (13)

$$\frac{\partial z^*}{\partial \tau_{cg}} = -\frac{(1 - \sigma)\varepsilon(\gamma + \lambda)}{(1 - \tau_{cg})\alpha_1 D} r^* < 0, \quad (41)$$

$$\frac{\partial x^*}{\partial \tau_{cg}} = -\frac{(1 - \sigma)\varepsilon - (1 - \tau_r)\alpha_1 \gamma + \lambda}{(1 - \tau_{cg})\alpha_1 D} r^* \begin{cases} \leq 0 \\ \geq 0 \end{cases} \text{ for } \varepsilon \begin{cases} \geq \\ \leq \end{cases} \frac{(1 - \tau_r)\alpha_1}{1 - \sigma}, \quad (42)$$

$$\frac{\partial z^*}{\partial \bar{\psi}} = \frac{(1 - \sigma)(1 - \tau_{cg})\varepsilon(\gamma + \lambda)}{(1 - \tau_r)\alpha_1 D} > 0 \quad (43)$$

$$\frac{\partial x^*}{\partial \bar{\psi}} = \left(\frac{(1 - \sigma)\varepsilon}{(1 - \tau_r)\alpha_1} - 1 \right) \frac{(1 - \tau_{cg})(\gamma + \lambda)}{D} \begin{cases} \leq 0 \\ \geq 0 \end{cases} \text{ for } \varepsilon \begin{cases} \geq \\ \leq \end{cases} \frac{(1 - \tau_r)\alpha_1}{1 - \sigma}, \quad (44)$$

$$\frac{\partial z^*}{\partial \tau_r} = \frac{\partial x^*}{\partial \tau_r} = \frac{1 - \sigma}{(1 - \tau_r)\alpha_1} r^* > 0, \quad (45)$$

$$\frac{\partial z^*}{\partial \sigma} = \frac{\partial x^*}{\partial \sigma} = -\frac{1}{1 - \sigma} z^* < 0. \quad (46)$$

The counterpart of the dampening effect on growth of a higher capital gains tax is a lower rate of interest, that is, a lower marginal and average product of capital as seen by (41). For reasonable values of the desire for consumption smoothing, ε , also the consumption-capital ratio is diminished, cf. (42). The effects of a higher conservation stimulus go the opposite way. As to the effect of a change in τ_r , whatever the size of τ_r , net returns on saving, $(1 - \tau_r)r^*$, equals, by the Hotelling rule, $(1 - \tau_{cg})(g^* + u^* + \bar{\psi})$, where g^* and u^* are independent of τ_r . That is, if τ_r goes up, r^* goes up also, leaving $(1 - \tau_r)r^*$ unchanged. The increase in r^* is reflected in the increase in the marginal and average product of capital shown in (45). Finally, a higher capital subsidy rate, σ , stimulates demand for capital services and twists the capital intensity upwards and the output-capital ratio downwards, as seen by (46).²¹

The results here are in contrast to traditional endogenous growth models without natural resources (e.g., [19,17,16,5,36,20,2]) where taxation of interest income as well as investment subsidies influence long-run growth. In these models, what stimulates saving and investment, stimulates growth. However, when the sector that drives long-run growth (in the present one-sector case, the aggregate production sector) depends on a non-renewable resource, this alternative stock variable causes policy instruments directed towards capital accumulation as such to be unimportant for the growth rate.²² But policies affecting resource extraction

¹⁹We have just seen that the *growth* effects of τ_{cg} and $\bar{\psi}$ are qualitatively the same, whether $\alpha < 1$ (exogenous growth or semi-endogenous growth) or $\alpha > 1$ (as required for strictly endogenous growth). The effects on the depletion rate, however, depends on this. Indeed, if $\alpha > 1$, an increase in τ_{cg} as well as a decrease in $\bar{\psi}$ *diminishes* the depletion rate in the BGE. These counter-intuitive effects are due to the fact that if $\alpha > 1$, then changes in capital accumulation are self-enforcing and creates divergence if not counterbalanced by changes in the depletion rate in the *same* direction.

²⁰The formal proof that this is of no consequence for the transitional dynamics of the economy lies in the fact that the three-dimensional dynamic system shown in Appendix does not contain the level of τ_u , only its rate of change.

²¹It may be added that the level effects of τ_r and σ can endanger *existence* of a BGE and thereby indirectly affect growth. For example with $\sigma > .53$, keeping the other parameter values in footnote 14 unchanged, we get $x^* < 0$, implying non-existence of a BGE.

²²It might be argued that since the just listed traditional endogenous growth literature features *strictly* endogenous growth, not semi-endogenous growth, for comparison we should stick to the strictly endogenous growth case ($\alpha > 1$) in our model. Yet, in view of the growth neutrality of τ_r and σ coming to light independently of whether $\alpha > 1$ or $\alpha \leq 1$, this changes nothing.

(the bottleneck of the economy in the sense that the cumulative extraction ultimately cannot be increased by prices and incentives) are assigned a central role.

In general the key to having policy impinging on long-run growth is the presence of a linear differential equation linked to the basic goods sector in the model. In the present framework the resource depletion relation, $\dot{S} = -uS$, is such an equation. In balanced growth $g_S = -R/S \equiv -u$ is constant so that the proportionate rate of decline in R must comply with, indeed be equal to, that of S . Through the growth accounting relation (24), given u , this fixes g_Y and g_K (equal in balanced growth), hence also $g_c = g_Y - n$. Inspection of the growth accounting relation under balanced growth, (27), confirms that generally (i.e., when $\alpha \neq 1$) for policy to matter for long-run growth it must affect u^* .²³ The existing literature dealing explicitly with non-renewable resources and endogenous growth, mentioned in the introduction, extends the traditional two-sector endogenous growth models by including a non-renewable resource as an essential input only in the manufacturing sector, not in the R&D (or educational) sector. Therefore, it is possible for policy to affect the growth rate of knowledge without affecting the depletion rate. Consequently, unlike our results, policy effects are in this literature pretty much in conformity with results from the standard endogenous growth models without non-renewable resources.

3.5. A special case: constant returns to capital

In the case $\alpha = 1$ we have a kind of AK model augmented with an explicit role for both the labor force and natural resources in production. For $D \neq 0$ the growth rate and the depletion rate simplify to

$$g_c^* = \frac{(1 - \tau_{cg})[(\beta + \gamma + \lambda)n + \theta] - [\rho - (1 - \tau_{cg})\bar{\psi}](\gamma + \lambda)}{D},$$

$$u^* = \frac{\beta n + \theta}{\gamma + \lambda},$$

where in this case $D = (\varepsilon - 1 + \tau_{cg})(\gamma + \lambda)$. It is interesting that under laissez-faire ($\tau_{cg} = \bar{\psi} = 0$), only an elasticity of intertemporal substitution ($1/\varepsilon$) below 1 is compatible with stability ($D > 0$). But a positive τ_{cg} stabilizes, by making the condition $D > 0$ more likely to occur.

This is the only case where, given $D > 0$, a decrease in τ_{cg} or an increase in $\bar{\psi}$ promotes growth *without* affecting resource extraction in the BGE. This counter-intuitive feature is due to the “growth accounting” relation (24) saying that, when $\alpha = 1$, the balance between g_Y and g_K requires unchanged u .

For the case of no exogenous technical progress ($\theta = 0$) we see that $\alpha = 1$ combined with population growth may generate stable endogenous per capita growth. This contrasts with the model without non-renewable resources ($\gamma = 0$), where a situation with $\alpha = 1$, $\beta > 0$ and population growth is not compatible with a steady state, but implies a forever increasing growth rate. In the present model, population growth is not only compatible with, but necessary for positive stable growth. One of the models examined in Aghion and Howitt [1, pp. 162–163] is an AK model with a non-renewable resource. In that model, however, labor does not appear in the production function. Because of this specification, long-run growth is not possible in that model.

4. First-best solution

For the case of constant returns to scale and exogenous technical progress, Sinclair [30] showed that the negative resource externality leads to too slow growth in the long run and that its correction requires a *declining* tax on resource use. In this section we show that this and related results come true also if growth is endogenous and when alternative tax and subsidy instruments are available.

Consider the problem of maximizing discounted utility (10) subject to the technology and resource constraints (5), (17) and (18), including the usual non-negativity constraints. We call the solution to this

²³These circumstances are independent of whether the external effect from resource depletion is present or not ($\lambda = 0$). Note that the conclusion would not change by the addition of extraction costs to the model. Such costs leave the fundamental linearity, $\dot{S} = -uS$, unaffected.

problem (when it exists) an *optimal allocation* or the *social planner's rule*, indexed by “SP”. Given $D^{\text{SP}} \equiv 1 - \alpha + (\varepsilon - 1)\gamma \neq 0$, the GDP growth rate and the depletion rate in a steady state of the optimal allocation are,²⁴ respectively,

$$g^{*\text{SP}} = \frac{\beta n + \theta + (\varepsilon n - \rho)\gamma}{D^{\text{SP}}} \quad (47)$$

and

$$u^{*\text{SP}} = \frac{\gamma}{\gamma + \lambda} \frac{[(\alpha + \beta - 1)n + \theta]\varepsilon - (\beta n + \theta) + (1 - \alpha)\rho}{D^{\text{SP}}}. \quad (48)$$

As also Sinclair [30] observed, the resource externality λ has no effect on the optimal growth rate, but only on the optimal rate of resource depletion, which is smaller the larger is λ . The explanation is that a higher cost in terms of lower productivity in the future implies a lower required return on leaving the marginal resource in the ground. Thereby resource extraction is stretched out, offsetting a higher λ such that the drag on output growth stemming from the exhaustible resource, i.e., $(\gamma + \lambda)u$ by (27), is unaffected. Then a larger λ shows up only as a smaller resource depletion rate.²⁵

We want to compare the optimal allocation with the market equilibrium. There are two distortions, the negative externality from resource use and the positive externality from aggregate capital. Define “laissez-faire” as the policy $(\tau_r, \tau_{\text{cg}}, \sigma, (\psi)_{t=0}^{\infty}) = (0, 0, 0, 0)$ and let $D(0)$ denote the value of D for $\tau_{\text{cg}} = 0$, i.e., $D(0) \equiv 1 - \alpha - \gamma - \lambda + (\gamma + \lambda)\varepsilon$, cf. (29). We have:

Proposition 5. *Assume the parameters $\alpha_1, \beta, \gamma, \alpha, \lambda, \theta, \delta, n, \varepsilon, \rho$ are such that a BGP exists under both laissez-faire and the social planner's rule. Assume $D(0) > 0$.*

- (i) *If $\lambda > 0$, then laissez-faire implies $g^* < g^{*\text{SP}}$.*
- (ii) *If $\lambda = 0$, but $\alpha_1 < \alpha$, then laissez-faire implies $g^* = g^{*\text{SP}}$, but $z^* > z^{*\text{SP}}$.*

Proof. See Appendix.

Hence, irrespective of whether growth is exogenous or endogenous, under laissez-faire the negative resource externality leads to too slow growth in the long run; in the “normal” case where $\alpha < 1$ this reflects a too fast resource depletion, cf. (27). Further, the positive capital externality entails too little capital investment so that z^* and r^* become too high; this level effect implies too little scope for consumption in the long run even if the growth rate is appropriate.²⁶

Are the available tax and subsidy instruments adequate for correcting these distortions? It seems almost trivial that the capital externality can be compensated by subsidizing investment by $\sigma = 1 - \alpha_1/\alpha$. But what about the negative resource externality and the resulting drag on growth? On the face of it, both a declining tax on resource use and a *negative* capital gains tax could deliver the required premium on delaying extraction and making scope for optimal growth. It turns out, however, that only the first policy is adequate. Indeed:

Proposition 6. *Whenever an optimal allocation exists, it can be implemented as an equilibrium allocation if and only if the policy: $\sigma = 1 - \alpha_1/\alpha$, $\psi = \lambda u/\gamma$, and $\tau_{\text{cg}} = \tau_r = 0$, is applied.*

Proof. See Appendix.

²⁴For derivation, see Groth and Schou [13].

²⁵Also, the steady state values of z^{SP} and x^{SP} are independent of λ .

²⁶Existence of an optimal allocation is guaranteed for any parameter vector $(\alpha, \beta, \gamma, \lambda, \delta, n, \theta, \varepsilon, \rho)$ such that D^{SP} as well as $u^{*\text{SP}}$, $z^{*\text{SP}}$, and $x^{*\text{SP}}$ are strictly positive, provided $\alpha + \gamma + \lambda \leq 1$ (footnote 14 shows an example). For the case $\alpha + \gamma + \lambda > 1$, which violates concavity of the maximized Hamiltonian, we have not been able to prove existence.

The social planner's Hotelling rule

$$\frac{d\left(\frac{\partial Y}{\partial R}\right)}{dt} = \left(\frac{\partial Y}{\partial K} - \delta\right) \frac{\partial Y}{\partial R} - \frac{\partial Y}{\partial S} \quad (49)$$

is helpful in understanding this result. The rule states that along an interior optimal path the return (capital gain) on leaving the marginal unit of the resource in the ground must equal the marginal return on the alternative asset, capital, minus the marginal extraction cost in terms of lower productivity in the future. Under laissez-faire ($\psi \equiv 0$) this cost is not internalized, resulting in a too high required private return on delaying extraction, thereby speeding up extraction. The proper correction comes into sight when we divide through by $\partial Y/\partial R$ in (49) and use the Cobb–Douglas specification to get

$$g_Y - g_R = \alpha z - \delta - \frac{\lambda}{\gamma} u.$$

The corresponding relation for the market economy is the no-arbitrage condition (25). Inserting $\sigma = 1 - \alpha_1/\alpha$ and $\tau_{cg} = \tau_r = 0$ into this condition we see that the conservation stimulus should be set at $\psi = \lambda u/\gamma$, i.e., proportional to the current depletion rate, whether the system is in steady state or outside.²⁷

As to the roles of τ_{cg} and τ_r our results are in contrast to Dasgupta and Heal [8] who conclude (on p. 368) that a capital gains tax when accompanied by an equally high interest income tax does not distort resource extraction at all. Similarly, Stiglitz [34, pp. 77–78] maintains, from inspecting the Hotelling rule, that an economy will pursue an excessively conservationist resource extraction policy if and only if the tax on capital gains is lower (as it usually is) than the tax on interest income. However, though these observations may be true in partial equilibrium, they do not hold in general equilibrium where the rate of interest is endogenous. Whatever the value of τ_r , a positive capital gains tax tends to incite too little conservation and impede growth. In view of the limited use in practice of direct taxation of capital gains on resources, this result may seem not so relevant in relation to real-world taxation. However, we showed in Section 2.2 that the capital gains tax is in fact equivalent to a profits tax on resource-extracting companies if the depletion allowance equals the true economic depreciation on the remaining resource reserves.

5. Conclusion

Based on a Cobb–Douglas one-sector model, allowing for increasing returns to scale and an essential non-renewable resource, this paper has studied the influence of various policy instruments on long-run growth. Contrary to the predictions of standard endogenous growth theory neither a tax on interest income nor a subsidy to capital accumulation affect the long-run growth rate. However, policies directed towards the returns to resource conservation do influence growth. For example, taxing the capital gains, due to the rising price of the resource as its scarcity grows, makes extraction too favorable—to the detriment of long-run growth possibilities. A tax on resource use (like a carbon tax) matters if it is time-varying. When it keeps declining over time, it favors conservation and growth, and this is desirable if externalities like a greenhouse effect are present. Hence, the conclusion is that resource taxes are decisive for long-run growth whereas traditional capital taxes and subsidies only influence *levels*.

A problem left for future research is whether and how the results are modified when resource extraction depends on capital and labor as inputs. Productivity increases in the extractive industries may (besides discoveries of new deposits) be one of the reasons why, contrary to the prediction of the model, the data for the last century do not indicate a rising resource price trend [21]; Krautkraemer [18] additionally discusses other reasons for the empirical failure of the Hotelling rule prediction. This also invites considering more specific tax issues of a real-world character. Yet another problem to be studied is how far the conclusions

²⁷Notice also that Proposition 6 implies that the initial *level* of the tax τ_u on resource use does not matter. Anyway, there is no doubt that our formalization of the resource externality is very simplistic and far from accurate if one has, for example, the climate change problem in mind. There seems to be no generally accepted economic model of that complex phenomenon. Farzin and Tahvonen [9] point out, in a partial equilibrium model, that the time path of an optimal carbon tax may be rather sensitive to various specifications of CO₂ accumulation.

generalize to a model with a separate sector for research or education, where a non-renewable resource is still essential either directly or indirectly (in the sense of being necessary in the manufacturing sector, which then delivers necessary inputs to the research or educational sector). A two-sector framework may also be more suitable for investigating consequences of an elasticity of substitution between capital and resources *less* than one, which is by most ecological economists considered to be the more realistic case.

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Appendix

Proof of Lemma 1. Let the path $(C, Y, K, R, S)_{t=0}^{\infty}$ be a BGE. (i) By definition of a BGE, g_C, g_Y , and g_K are constant. By constancy of g_C, z is a constant, in view of (26). This implies, first, that $g_Y = g_K$, and, second, that x is constant, in view of (22). Therefore, $g_C = g_K$. The common constant value of g_C, g_Y , and g_K is called g^* . (ii) By definition of a BGE, g_S is constant. Hence, by (23), u is a constant, say u^* , implying $g_R = g_S = -u^*$, and therefore $R(t) = R(0)e^{-u^*t}$. In view of $R(0) > 0$ we have $u^* > 0$ since otherwise (19) would be violated. (iii) and (iv) That z and x are constants has already been proved; by definition of a BGE they must also be positive. From (24), with $g_Y = g_K = g^*$ and $g_S = -u^*$, follows (27). Inserting (25) in (26) gives $g^* = \frac{1}{\epsilon}[(1 - \tau_{cg})(g^* - g_R^* + \psi) - \rho] + n$, which is a contradiction unless ψ is constant, and by (16) this constant must be $\bar{\psi}$. Inserting $g_R = -u^*$ and reordering gives (28). (v) In view of (25), $(1 - \tau_{cg})(g^* - g_R^* + \psi) = (1 - \tau_r)r$, since, by (6), $r = \alpha_1 z / (1 - \sigma) - \delta$, where z is constant in the BGE. Inserting $u^* = -g_R^*$ and $\psi = \bar{\psi}$ gives

$$(1 - \tau_{cg})(g^* + u^* + \bar{\psi}) = (1 - \tau_r)r. \quad (50)$$

Now, the desired conclusion follows from the transversality conditions (20) and (21), by Lemma 5 below. \square

Lemma 5. Let $\lim_{t \rightarrow \infty} g_K = g^*$, $\lim_{t \rightarrow \infty} g_S = -u^*$, and $\lim_{t \rightarrow \infty} r(t) = r$, where r satisfies (50). Then the transversality conditions (20) and (21) taken together are equivalent to the inequality in (v) of Lemma 1.

Proof. We have

$$(20) \Leftrightarrow g^* - (1 - \tau_r)r < 0 \Leftrightarrow g^* < (1 - \tau_{cg})(g^* + u^* + \bar{\psi}) \quad (\text{from (50)})$$

$$\Leftrightarrow \tau_{cg}g^* < (1 - \tau_{cg})(u^* + \bar{\psi}); \quad (51)$$

$$(21) \Leftrightarrow -u^* + \frac{\tau_{cg}(1 - \tau_r)}{1 - \tau_{cg}}r < 0 \Leftrightarrow -(1 - \tau_{cg})u^* + \tau_{cg}(1 - \tau_r)r < 0$$

$$\Leftrightarrow \tau_{cg}(1 - \tau_{cg})(g^* + u^* + \bar{\psi}) < (1 - \tau_{cg})u^* \quad (\text{from (50)})$$

$$\Leftrightarrow \tau_{cg}(g^* + \bar{\psi}) < (1 - \tau_{cg})u^*.$$

The last inequality ensures (51) since, by (16), $\bar{\psi} \geq 0$. \square

Proof of Proposition 2. Let the parameter tuple $\pi \in P$ and assume $D \neq 0$. (i) As to the “if” part we can construct a BGE in the following way. Given $\pi \in P^*$, let g^*, u^*, z^* and x^* be defined as in (30)–(33), respectively. By definition of P^* we know u^*, z^* and x^* are strictly positive, and (v) of Lemma 1 is satisfied. Let $Q = (C, Y, K, R, S)_{t=0}^{\infty}$ be a path satisfying $u \equiv R/S = u^*$, $z \equiv Y/K = z^*$, and $x \equiv C/K = x^*$ for all t . This path has $g_R = g_S = -u^*$, from (23). By construction, u^* and g^* satisfy (27) and (28). By (27) and (24), g^* is the common value of g_Y and g_K along the path Q , given $g_R = -u^*$. Further, given g^* and u^* , z^* satisfies (32), which implies the Hotelling rule (25). Now, combining (28) and (25) shows that g^* satisfies (26); hence, with $r = \frac{\alpha_1}{1 - \sigma}z^* - \delta$ and $g_c = g^* - n$, the Keynes–Ramsey rule (13) is satisfied. Finally, with $r = \frac{\alpha_1}{1 - \sigma}z^* - \delta$, (25) implies (50); hence, in view of (v) of Lemma 1 the transversality conditions are satisfied, by Lemma 5, and the

path Q is a BGE. On the other hand, since $D \neq 0$, $\pi \notin P^*$ implies, by definition of P^* , that at least one of z^* , x^* and u^* is non-positive or (v) of Lemma 1 is not satisfied. Hence, no BGE exists. (ii) Given $\pi \in P^*$, uniqueness of the BGE (g^*, u^*, z^*, x^*) follows from (30)–(33). \square

Proof of Proposition 3. Let the path $(C, Y, K, R, S)_{t=0}^\infty$ be generated by a viable economic system. Using (22), the identities $z \equiv Y/K$ and $x \equiv C/K$ imply

$$\dot{z} = (g_Y - z + x + \delta)z, \tag{52}$$

$$\dot{x} = (g_C - z + x + \delta)x. \tag{53}$$

Define the relative taxation index $\xi \equiv (1 - \tau_r)/(1 - \tau_{cg}) > 0$. Inserting (22) and (25) into (24) yields

$$g_Y = \frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma}z - \frac{\alpha}{1-\gamma}x - \frac{\lambda}{1-\gamma}u + \frac{\gamma}{1-\gamma}\psi + \frac{\beta n + \theta + (\xi\gamma - \alpha)\delta}{1-\gamma}. \tag{54}$$

Substituting this expression into (52), we find

$$\dot{z} = \left[\left(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1 \right) z + \frac{1-\alpha-\gamma}{1-\gamma}x - \frac{\lambda}{1-\gamma}u + \frac{\gamma}{1-\gamma}\psi + \frac{\beta n + \theta + (1-\alpha-\gamma + \xi\gamma)\delta}{1-\gamma} \right] z. \tag{55}$$

Inserting $g_C = g_c + n$ and (26) into (53) gives

$$\dot{x} = \left[\left(\frac{(1-\tau_r)\alpha_1}{(1-\sigma)\varepsilon} - 1 \right) z + x + n - \frac{\rho}{\varepsilon} + \left(1 - \frac{1-\tau_r}{\varepsilon} \right) \delta \right] x. \tag{56}$$

Differentiating the identity $u \equiv R/S$ with respect to time and using (18), we get

$$\dot{u} = (g_R + u)u. \tag{57}$$

By (25) and (54), this gives

$$\dot{u} = \left(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1}{1-\gamma}z - \frac{\alpha}{1-\gamma}x + \frac{1-\gamma-\lambda}{1-\gamma}u + \frac{1}{1-\gamma}\psi + \frac{\beta n + \theta + (\xi - \alpha)\delta}{1-\gamma} \right) u. \tag{58}$$

The dynamics of z , x , and u are completely described by the system (55), (56), and (58). Along a BGE this system is in a steady state (z^*, x^*, u^*) , which, by Proposition 2, is unique, given the parameter tuple $\pi \in P^*$. We form the Jacobian evaluated in the steady state:

$$J = \begin{bmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial u} \\ \frac{\partial \dot{x}}{\partial z} & \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial u} \\ \frac{\partial \dot{u}}{\partial z} & \frac{\partial \dot{u}}{\partial x} & \frac{\partial \dot{u}}{\partial u} \end{bmatrix} = \begin{bmatrix} \left(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1 \right) z^* & \frac{1-\alpha-\gamma}{1-\gamma} z^* & -\frac{\lambda}{1-\gamma} z^* \\ \left(\frac{(1-\tau_r)\alpha_1}{(1-\sigma)\varepsilon} - 1 \right) x^* & x^* & 0 \\ \frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1}{1-\gamma} u^* & -\frac{\alpha}{1-\gamma} u^* & \frac{1-\gamma-\lambda}{1-\gamma} u^* \end{bmatrix}. \tag{59}$$

The determinant of J is

$$\det J = -\frac{\xi D}{(1-\sigma)(1-\gamma)\varepsilon} a_1 z^* x^* u^* \tag{60}$$

which is negative if and only if $D > 0$. The trace of J is

$$\begin{aligned} \text{tr } J &= \left(\frac{\alpha - \frac{\xi}{1-\sigma}\alpha_1\gamma}{1-\gamma} - 1 \right) z^* + x^* + \frac{1-\gamma-\lambda}{1-\gamma} u^* \\ &\geq \frac{1-\xi\gamma}{1-\gamma} \frac{\alpha_1}{1-\sigma} z^* - z^* + x^* + \frac{1-\gamma-\lambda}{1-\gamma} u^* \quad \left(\text{since } \frac{\alpha_1}{1-\sigma} \leq \alpha, \text{ by (A.1)} \right) \\ &= \frac{1-\xi\gamma}{1-\gamma} \left(\frac{g^* + u^* + \bar{\psi}}{\xi} + \delta \right) - g^* - \delta + \frac{1-\gamma-\lambda}{1-\gamma} u^* \quad \text{(by (32) and (22))} \\ &= \frac{(1-\xi)g^* + (1-\xi\gamma)(u^* + \bar{\psi} + \xi\delta)}{\xi(1-\gamma)} + \frac{1-\gamma-\lambda}{1-\gamma} u^* - \delta \\ &\approx \left(2 - \frac{\lambda}{1-\gamma} \right) u^* + \bar{\psi} > 0 \quad \left(\text{since } \xi \approx 1, \text{ by (A.1), } \lambda/2 < 1-\gamma, \text{ and } \bar{\psi} \geq 0 \right). \end{aligned}$$

This shows that the three eigenvalues cannot all be negative (or have negative real part). Hence, if $D > 0$, i.e., $\det J < 0$, there is one negative eigenvalue and two eigenvalues with positive real part. Even though z, x , and u are all ‘jump variables’, by substituting uS for R in the production function (5) we get $z = e^{\theta t} K^{\alpha-1} N^{\beta} u^{\gamma} S^{\gamma+\lambda}$, showing that, given K, N , and S , the values of z and u are not independent. In view of the implied boundary value condition, one negative eigenvalue and two eigenvalues with positive real part implies saddle-point stability.

Now, assume on the contrary $D < 0$. Then $\det J > 0$, and on the face of it there are two possible cases: either all three eigenvalues are non-negative or there is one positive eigenvalue and two eigenvalues with non-positive real part. However, there exists a number $\bar{\lambda} > 0$ such that for $\lambda < \bar{\lambda}$ this last-mentioned case can be excluded. Indeed, if $\lambda = 0$, then J is block-triangular, and $u^* > 0$ is an eigenvalue. And since both the determinant and the trace of the upper left 2×2 sub-matrix of J are positive (at least given the policy assumption (A1)), the other two eigenvalues are also positive. By continuity, all three eigenvalues will be positive also if $\lambda \in (0, \bar{\lambda})$ for some sufficiently small positive number $\bar{\lambda}$. \square

Proof of Proposition 5. Let the parameter tuple $(\alpha_1, \beta, \gamma, \alpha, \lambda, \theta, \delta, n, \varepsilon, \rho)$ be such that a BGP exists both under laissez-faire and the social planner’s rule. Assume $D(0) > 0$ and consider the laissez-faire BGE. By Proposition 6, a first-best BGE has $\bar{\psi} = \lambda u^{*SP} / \gamma > 0$ and $\sigma = 1 - \alpha_1 / \alpha$. (i) Assume $\lambda > 0$. Then, a shift from first-best to laissez-faire affects g^* only through the decrease of $\bar{\psi}$ to zero, and this effect is negative, since $\partial g^* / \partial \bar{\psi} = \partial g_c^* / \partial \bar{\psi} > 0$, by (39). (ii) Assume $\lambda = 0$, $\alpha_1 < \alpha$. Then, a shift from first-best to laissez-faire changes only σ , which decreases from $1 - \alpha_1 / \alpha > 0$ to zero. This decrease in σ does not affect g^* , but increases z^* in view of (46). \square

Proof of Proposition 6. The dynamic system (55), (56), and (58) for the market economy is identical to that of the optimal allocation (see [13, pp. 26–27]), if and only if $\sigma = 1 - \alpha_1 / \alpha$, $\psi = \lambda u / \gamma$ and $\tau_{cg} = \tau_r = 0$. This proves the proposition. \square

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