

# Embodied learning by investing and speed of convergence Online Appendix

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## Simulations

The numerical results in this appendix refer to the benchmark model with learning based on gross investment. “Speed of convergence” refers to the common asymptotic speed of convergence of  $x$  and  $z$ , i.e.,  $\sigma_x = \sigma_z$ . By Proposition 4, in the *absence* of learning,  $\sigma_u \neq \sigma_i$ ,  $i \in \{x, z\}$ . In the tables, numbers in parentheses indicate the speed of convergence, in percentage points, of  $u$  in the *absence* of learning. Unless otherwise specified, values of the background parameters are the baseline values specified in Table 2 of the text. The range of the parameter appearing in the first column of the tables is limited to values not requiring the adjusting variable to take on a negative value to maintain  $g_c^* = 0.02$ .

TABLE A  
ASYMPTOTIC SPEED OF CONVERGENCE AS THE EMBODIED LEARNING  
PARAMETER,  $\lambda$ , RISES AND  $\gamma$  IS ADJUSTED SO AS TO MAINTAIN  $g_c^* = 0.02$ .

			SPEED OF CONVERGENCE IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	.....	$\theta = 1.75$	.....	.....
PANEL A. $n = 0.01$										
$\lambda$	$\gamma$	$\alpha$								
0.00	0.020	0.324	10.48 (3.00)	8.78 (3.00)	7.52 (3.00)	6.96 (3.00)	0.055	0.25	0.32	0.000
0.28	0.016	0.324	2.55	2.49	2.42	2.37	0.055	0.25	0.35	-0.008
0.56	0.012	0.324	2.10	2.01	1.91	1.85	0.055	0.26	0.38	-0.017
0.84	0.008	0.324	1.66	1.57	1.47	1.41	0.055	0.26	0.40	-0.025
1.11	0.004	0.324	1.25	1.17	1.08	1.03	0.055	0.27	0.43	-0.033
1.39	0.000	0.324	0.86	0.80	0.73	0.70	0.055	0.27	0.45	-0.042
PANEL B. $n = 0.005$										
$\lambda$	$\gamma$	$\alpha$								
0.00	0.020	0.324	10.38 (2.50)	8.67 (2.50)	7.40 (2.50)	6.85 (2.50)	0.055	0.23	0.32	0.000
0.32	0.016	0.324	2.10	2.05	2.00	1.97	0.055	0.24	0.32	-0.008
0.63	0.012	0.324	1.68	1.62	1.54	1.50	0.055	0.24	0.35	-0.016
0.95	0.009	0.324	1.27	1.21	1.14	1.13	0.055	0.25	0.37	-0.024
1.27	0.005	0.324	0.89	0.84	0.78	0.75	0.055	0.25	0.40	-0.032
1.58	0.000	0.324	0.53	0.49	0.45	0.43	0.055	0.26	0.45	-0.040
PANEL C. $n = 0.001$										
$\lambda$	$\gamma$	$\alpha$								
0.00	0.020	0.324	10.31 (2.10)	8.57 (2.10)	7.32 (2.10)	6.77 (2.10)	0.055	0.22	0.32	0.000
0.40	0.016	0.324	1.69	1.65	1.61	1.59	0.055	0.23	0.35	-0.008
0.79	0.012	0.324	1.26	1.21	1.16	1.13	0.055	0.23	0.38	-0.017
1.19	0.008	0.324	0.84	0.80	0.76	0.73	0.055	0.24	0.40	-0.025
1.59	0.004	0.324	0.46	0.43	0.40	0.38	0.055	0.24	0.43	-0.033
1.98	0.000	0.324	0.09	0.08	0.08	0.07	0.055	0.25	0.45	-0.042

NOTE:  $\beta = 0, \psi = 0$ . When  $\lambda = 0$ ,  $u$  converges with a lower speed than  $(x, z)$ . This lower speed is shown in brackets.

TABLE B  
ASYMPTOTIC SPEED OF CONVERGENCE AS THE EMBODIED LEARNING  
PARAMETER,  $\lambda$ , RISES AND  $\psi$  IS ADJUSTED SO AS TO MAINTAIN  $g_c^* = 0.02$

			SPEED OF CONVERGENCE IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	..... $\theta = 1.75$ .....			
$\lambda$	$\psi$	$\alpha$								
0.00	0.042	0.324	15.43 (3.00)	12.51 (3.00)	10.32 (3.00)	9.33 (3.00)	0.055	0.27	0.45	-0.042
0.28	0.033	0.324	2.54	2.49	2.42	2.38	0.055	0.27	0.45	-0.042
0.56	0.025	0.324	2.09	2.01	1.92	1.86	0.055	0.27	0.45	-0.042
0.84	0.017	0.324	1.66	1.58	1.48	1.42	0.055	0.27	0.45	-0.042
1.11	0.008	0.324	1.25	1.17	1.08	1.03	0.055	0.27	0.45	-0.042
1.39	0.000	0.324	0.86	0.80	0.73	0.70	0.055	0.27	0.45	-0.042

NOTE:  $\beta = 0, \gamma = 0$ .

TABLE C  
ASYMPTOTIC SPEED OF CONVERGENCE AS THE DISEMBODIED LEARNING  
PARAMETER,  $\beta$ , RISES AND  $\gamma$  IS ADJUSTED SO AS TO MAINTAIN  $g_c^* = 0.02$

			SPEED OF CONVERGENCE IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	..... $\theta = 1.75$ .....			
PANEL A.										
$\beta$	$\gamma$	$\alpha$								
0.00	0.020	0.324	10.48 (3.00)	8.78 (3.00)	7.52 (3.00)	6.96 (3.00)	0.055	0.25	0.32	0.0
0.13	0.016	0.324	2.59	2.51	2.42	2.36	0.055	0.25	0.32	0.0
0.27	0.012	0.324	2.17	2.05	1.90	1.82	0.055	0.25	0.32	0.0
0.40	0.008	0.324	1.76	1.61	1.45	1.37	0.055	0.25	0.32	0.0
0.53	0.004	0.324	1.35	1.20	1.06	0.99	0.055	0.25	0.32	0.0
0.67	0.000	0.324	0.95	0.82	0.71	0.66	0.055	0.25	0.32	0.0
PANEL B. $[\alpha + (1 - \alpha)\beta] = 0.5$										
$\beta$	$\gamma$	$\alpha$								
0.00	0.020	0.500	6.23 (3.00)	5.12 (3.00)	4.23 (3.00)	3.81 (3.00)	0.055	0.38	0.21	0.0
0.14	0.016	0.420	2.53	2.42	2.29	2.21	0.055	0.32	0.25	0.0
0.24	0.013	0.340	2.24	2.12	1.97	1.89	0.055	0.26	0.31	0.0
0.32	0.010	0.260	2.03	1.90	1.77	1.69	0.055	0.20	0.40	0.0
0.39	0.008	0.180	1.86	1.74	1.61	1.54	0.055	0.14	0.58	0.0
0.44	0.007	0.100	1.73	1.61	1.49	1.43	0.055	0.08	1.05	0.0

NOTE:  $\lambda = 0, \psi = 0$ .

TABLE D  
 ASYMPTOTIC SPEED OF CONVERGENCE AS THE DISEMBODIED LEARNING  
 PARAMETER,  $\beta$ , RISES AND  $\psi$  IS ADJUSTED SO AS TO MAINTAIN  $g_c^* = 0.02$

			SPEED OF CONVERGENCE IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	..... $\theta = 1.75$ .....			
PANEL A.										
$\beta$	$\psi$	$\alpha$								
0.00	0.042	0.324	15.4 (3.00)	12.5 (3.00)	10.3 (3.00)	9.33 (3.00)	0.055	0.27	0.45	-0.042
0.13	0.033	0.324	2.58	2.52	2.43	2.38	0.055	0.27	0.43	-0.033
0.27	0.025	0.324	2.16	2.05	1.92	1.85	0.055	0.26	0.40	-0.025
0.40	0.017	0.324	1.75	1.62	1.47	1.39	0.055	0.26	0.38	-0.017
0.53	0.008	0.324	1.34	1.20	1.06	1.00	0.055	0.25	0.35	-0.008
0.67	0.000	0.324	0.95	0.82	0.71	0.66	0.055	0.25	0.32	0.000
PANEL B. $[\alpha + (1 - \alpha)\beta] = 0.5$										
$\beta$	$\psi$	$\alpha$								
0.00	0.020	0.500	7.64 (3.00)	6.18 (3.00)	5.00 (3.00)	4.45 (3.00)	0.055	0.40	0.25	-0.020
0.14	0.022	0.420	2.54	2.45	2.33	2.25	0.055	0.34	0.30	-0.022
0.24	0.025	0.340	2.23	2.13	2.00	1.92	0.055	0.28	0.38	-0.025
0.32	0.029	0.260	2.01	1.90	1.78	1.71	0.055	0.21	0.52	-0.029
0.39	0.038	0.180	1.83	1.73	1.62	1.55	0.055	0.15	0.79	-0.038
0.44	0.060	0.100	1.68	1.59	1.49	1.43	0.055	0.09	0.65	-0.060

NOTE:  $\lambda = 0, \gamma = 0$ .

TABLE E  
 ASYMPTOTIC SPEED OF CONVERGENCE AS THE DISEMBODIED LEARNING  
 PARAMETER,  $\beta$ , RISES AND  $\lambda$  IS ADJUSTED SO AS TO MAINTAIN  $g_c^* = 0.02$

			SPEED OF CONVERGENCE IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	..... $\theta = 1.75$ .....			
$\beta$	$\lambda$	$\alpha$								
0.00	1.39	0.324	0.86	0.80	0.73	0.70	0.055	0.27	0.45	-0.042
0.13	1.11	0.324	0.87	0.80	0.73	0.69	0.055	0.27	0.43	-0.033
0.27	0.84	0.324	0.89	0.81	0.73	0.69	0.055	0.26	0.40	-0.025
0.40	0.56	0.324	0.91	0.81	0.72	0.68	0.055	0.26	0.38	-0.017
0.53	0.28	0.324	0.92	0.82	0.72	0.67	0.055	0.25	0.35	-0.008
0.67	0.00	0.324	0.95	0.82	0.71	0.66	0.055	0.25	0.32	0.000

NOTE:  $\gamma = 0, \psi = 0$ .

TABLE F  
ASYMPTOTIC SPEED OF CONVERGENCE AS THE EXOGENOUS EMBODIED  
CHANGE PARAMETER,  $\psi$ , RISES AND  $\gamma$  IS ADJUSTED SO AS TO MAINTAIN  
 $g_c^* = 0.02$

			SPEED OF CONVERGENCE OF $(x, z)$ IN %				$r^*$	$s^*$	$(Y/(pK))^*$	$g_p^*$
			$\theta = 1$	$\theta = 1.75$	$\theta = 3$	$\theta = 4$	.....	$\theta = 1.75$	.....	
$\psi$	$\gamma$	$\alpha$								
0.000	0.020	0.324	10.48	8.78	7.52	6.96	0.055	0.25	0.32	0.000
0.008	0.016	0.324	11.47	9.52	8.08	7.43	0.055	0.25	0.35	-0.008
0.017	0.012	0.324	12.46	10.27	8.63	7.90	0.055	0.26	0.38	-0.017
0.025	0.008	0.324	13.45	11.01	9.19	8.37	0.055	0.26	0.40	-0.025
0.033	0.004	0.324	14.44	11.76	9.75	8.85	0.055	0.27	0.43	-0.033
0.042	0.000	0.324	15.43	12.51	10.32	9.33	0.055	0.27	0.45	-0.042

NOTE:  $\beta = 0, \lambda = 0$ . In the decomposable case, the SOC of  $u$  equals the constant  $g_Y^* = 3.00\%$ .