Growth-essential non-renewable resources and limits to growth

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Abstract

The standard approach to modelling endogenous technical change in an economy with an essential non-renewable resource ignores that also R&D may need the resource (directly or indirectly). This biases the limits-to-growth discussion in an optimistic direction. Indeed, sustained per capita growth requires stronger parameter restrictions when the resource is directly or indirectly an input in R&D and thus "growth essential" than when it is not. When the resource is “growth essential”, a policy aiming at stimulating long-run growth generally has to reduce the long-run depletion rate. In this sense promoting long-run growth and "supporting the environment" go hand in hand.

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1 Introduction

A series of papers has analysed the role of non-renewable natural resources for endogenous growth (Robson 1980, Takayama 1980, Jones and Manuelli 1997, Aghion and Howitt 1998, Chapter 5, Scholz and Ziemes 1999, Schou 2000, Schou 2002, Grimaud and Rougé 2003). This literature typically analyses an economy with two sectors, a manufacturing sector and a "knowledge sector" where a fraction of the labour force is employed in R&D or education. The conclusions reached are pretty much in conformity with those of the conventional endogenous growth models without non-renewable resources. In particular, the cited papers associate sustained per capita growth with the usual (but problematic) knife-edge condition that the knowledge sector has exactly constant returns to scale with respect to the producible input(s) (at least asymptotically). And in the R&D-based models, the controversial scale effect on growth tends to pop up, although sometimes hidden by the labour force being normalized to one. The general impression is that limited non-renewable resources may be a drag on long-run growth, but never an impediment.

However, common to these papers is the assumption that labour is the only input in the growth-generating sector (the "growth engine"). Thus, natural resources do not appear in the growth-generating sector, not even indirectly in the sense of such resources being a necessary ingredient in the production of physical capital goods which are then used in the growth engine (e.g., a research sector). This is clearly an unrealistic feature. After all, most production sectors, including educational institutions and research labs, use fossil fuels for heating and transportation purposes or at least they use indirectly minerals and oil products via the machinery, computers, etc. they employ.

The purpose of this paper is to show that taking this fact into account affects the conclusions in crucial ways. Thus, we focus on the distinction between models where the non-renewable resource is growth-essential and models where it is not; by *growth-essential* we mean that the non-renewable resource is a necessary input to the growth-generating sector(s) in the economy, either directly or indirectly. We shall see that this distinction has important implications for the limits-to-growth question and economic policy issues.

Although we shall primarily be concerned with two-sector models with R&D, an expedient point of departure is the one-sector model by Suzuki (1976), which is introduced in the next section. This model provides the simplest conceivable framework for displaying the effects of a growth-essential non-renewable resource. Then Section 3 describes the conventional approach to a two-sector setup with R&D and resource depletion as exemplified by the above-mentioned literature. In contrast, Section 4 introduces two-sector models where the non-renewable resource is growth-essential, either directly or indirectly. In Section 5 we consider how limited substitutability in the R&D sector affects the conclusions. Section 6 concludes.

2 A one-sector model with R&D

It is not always recognised that the research of the 1970s on macro implications of essential non-renewable natural resources already laid the groundwork for a theory of endogenous and policy-dependent growth with natural resources. Actually, by extending the Dasgupta-Heal-Solow-Stiglitz (D-H-S-S) model from 1974,[1] Suzuki (1976) studied how endogenous innovation may affect the prospect of overcoming the finiteness of natural resources. Suzuki insisted that technical innovations are the costly result of intentional R&D. A part of aggregate output is used as R&D investment and results in additional technical knowledge and thereby higher productivity. The labour force, L, equals the population and grows according to $L = L_0 e^{nt}$, $n \geq 0$, constant. The dating of the variables is suppressed unless needed for clarity.

2.1 Elements of the Suzuki model

Aggregate manufacturing output is

$$Y = A^{\varepsilon} K^{\alpha} L^{\beta} R^{\gamma}, \qquad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, \tag{1}$$

where A is an index of the "stock of knowledge" and K, L and R are inputs of capital, labour and a non-renewable resource (say oil), respectively, at time t. The stock of knowledge increases through R&D investment, I_A :

$$\dot{A} = I_A - \delta_A A, \qquad \delta_A \geq 0. \tag{2}$$

The interpretation is that the technology for creating new knowledge uses the same inputs as manufacturing, in the same proportions. The parameter δ_A is the (exogenous) rate of depreciation (obsolescence) of knowledge. After consumption and R&D investment, the remainder of output is invested in physical capital:

$$\dot{K} = Y - cL - I_A - \delta_K K, \qquad \delta_K \geq 0, \tag{3}$$

[1] Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974).

where c is per capita consumption and δ_K is the (exogenous) rate of depreciation (decay) of capital. Finally, the resource stock, S, of the non-renewable resource (e.g., oil reserves) diminishes with resource extraction:

$$\dot{S} = -R \equiv -uS, \tag{4}$$

and u is the rate of depletion. Since we must have $S \geq 0$ for all t, there is a finite upper bound on cumulative resource extraction:

$$\int_0^\infty R(t)dt \leq S(0). \tag{5}$$

Uncertainty and costs of extraction are ignored.[2]

We shall limit our attention to *efficient paths*, i.e., paths such that consumption can not be increased in some time interval without being decreased in another time interval. Assuming, for simplicity, that $\delta_A = \delta_K = \delta$,[3] the net marginal productivities of A and K are equal if and only if $\varepsilon Y/A - \delta = \alpha Y/K - \delta$, i.e.,

$$A/K = \varepsilon/\alpha.$$

Initial stocks, A_0 and K_0 are historically given. Suppose $A_0/K_0 > \varepsilon/\alpha$. Then, initially, the net marginal product of capital is larger than that of knowledge, i.e., capital is relatively scarce. An investing efficient economy will therefore for a while invest only in capital, i.e., there will be a phase where $I_A = 0$. This phase of complete specialisation lasts until $A/K = \varepsilon/\alpha$, a state reached in finite time, say at time $\bar{t}$. Hereafter, there is investment in both assets so that their ratio remains equal to the efficient ratio ε/α forever. Similarly, if initially $A_0/K_0 < \varepsilon/\alpha$, then there will be a phase of complete specialisation in R&D, and after a finite time interval the efficient ratio $A/K = \varepsilon/\alpha$ is achieved and maintained forever. Thus, for $t > \bar{t}$ it is as if there were only one kind of capital, which we may call "broad capital" and define as $\tilde{K} = K + A = (\alpha + \varepsilon)K/\alpha$. Indeed, substitution of $A = \varepsilon K/\alpha$ and $K = \alpha\tilde{K}/(\varepsilon + \alpha)$ into (1) gives

$$Y = \frac{\varepsilon^\varepsilon \alpha^\alpha}{(\varepsilon + \alpha)^{\varepsilon+\alpha}} \tilde{K}^{\varepsilon+\alpha} L^\beta R^\gamma \equiv B\tilde{K}^{\tilde{\alpha}} L^\beta R^\gamma, \qquad \tilde{\alpha} \equiv \alpha + \varepsilon, \tag{6}$$

[2]Thus the model's description of resource extraction is trivial. That is why it is natural to classify the model as a *one-sector* model notwithstanding there are two activities in the economy, manufacturing and resource extraction.

[3]Suzuki (1976) has $\delta_A = \delta_K = 0$. But in order to comply with the general framework in this article, we allow $\delta_K > 0$, hence $\delta \geq 0$. Chiarella (1980) modifies (2) into $\dot{A} = I_A^\xi$, $\xi > 0$, and focuses on the resulting quite complicated transitional dynamics.

so that $\tilde{\alpha} + \beta + \gamma > 1$. Further, adding (2) and (3) gives

$$\dot{\tilde{K}} = \dot{A} + \dot{K} = Y - cL - \delta \tilde{K}. \tag{7}$$

Thus, we can proceed with a model based on broad capital, using (6), (7) and the resource depletion equation (4). Essentially, this model provides a theoretical basis for extending the D-H-S-S model to include increasing returns to scale, thereby offering a simple framework for studying *endogenous* growth with essential non-renewable resources. Groth and Schou (2006) study a similar configuration where the source of increasing returns to scale is not intentional creation of knowledge, but learning as a by-product of investing as in Arrow (1962) and Romer (1986). Empirically, the evidence furnished by, e.g., Hall (1990) and Caballero and Lyons (1992) suggests that there are quantitatively significant increasing returns to scale w.r.t. capital and labour or external effects in US and European manufacturing. Similarly, Antweiler and Trefler (2002) examine trade data for goods-producing sectors and find evidence for increasing returns to scale. Whatever the source of increasing returns to scale we shall call a D-H-S-S framework with $\tilde{\alpha} + \beta + \gamma > 1$ an *extended D-H-S-S model*.

For any positive variable x, let $g_x \equiv \dot{x}/x$ (the growth rate of x). Log-differentiating (6) w.r.t. t gives the "growth-accounting equation"

$$g_Y = \tilde{\alpha} g_{\tilde{K}} + \beta n + \gamma g_R. \tag{8}$$

A balanced growth path (BGP) is defined as a path along which the quantities Y, C and $\tilde{K}$ change at constant proportionate rates (some or all of which may be negative). It is easily shown that along a BGP $g_{\tilde{K}} = g_Y = g_C \equiv g_c + n$ and, if nothing of the resource is left un-utilized forever, $g_R = g_S = -R/S \equiv -u =$ constant. Then, along a BGP, by (8), we get

$$(1 - \tilde{\alpha}) g_c + \gamma u = (\tilde{\alpha} + \beta - 1) n. \tag{9}$$

Since $u > 0$, it follows immediately that:

Result (i) A BGP with $g_c > 0$ is technologically feasible only if

$$(\tilde{\alpha} + \beta - 1) n > 0 \qquad \text{or} \qquad \tilde{\alpha} > 1. \tag{10}$$

This result warrants some remarks from the perspective of new growth theory. We define *endogenous growth* to be present if sustained per capita growth ($g_c > 0$) is driven by some internal mechanism (in contrast to exogenous technology growth). Hence, Result (i) tells us that endogenous growth is theoretically possible, if there are either increasing returns to the capital-*cum*-labour input combined with population growth *or* increasing returns to capital (broad capital) itself. At least one of these conditions is required in order for capital accumulation to offset the effects of the inescapable waning of resource use over time. The reasoning of Mankiw (1995) suggests β to be in the neighbourhood of 0.25. And Barro and Sala-i-Martin (2004, p. 110) argue that, given the "broad capital" interpretation of capital, $\tilde{\alpha}$ being around 0.75 accords with the empirical evidence. In view of this, $\tilde{\alpha}$ and β summing to a value above 1 cannot be excluded (but it is, on the other hand, not assured). Hence, $(\tilde{\alpha} + \beta - 1)n > 0$ seems possible when $n > 0$.

We define *fully endogenous growth* to be present if the long-run growth rate in per capita output is positive without the support of growth in any exogenous factor. Result (i) shows that only if $\tilde{\alpha} > 1$, is *fully* endogenous growth possible. Although the case $\tilde{\alpha} > 1$ has potentially explosive effects on the economy, if $\tilde{\alpha}$ is not too much above 1, these effects can be held back by the strain on the economy imposed by the declining resource input.[4]

In some sense this is "good news": fully endogenous steady growth is theoretically possible and no knife-edge assumption is needed. As we saw in Section 2, in the conventional framework, without non-renewable resources, fully endogenous growth requires constant returns to the producible input(s) in the growth engine. In our one-sector model the growth engine is the manufacturing sector itself, and without the essential non-renewable resource, fully endogenous growth would require the knife-edge condition $\tilde{\alpha} = 1$ ($\tilde{\alpha}$ being above 1 is excluded in this case, because it would lead to explosive growth in a setting without some countervailing factor). When non-renewable resources are an essential input in the growth engine, they entail a drag on the growth potential. In order to offset this drag, fully endogenous growth requires *increasing* returns to capital.

However, the "bad news" is that even in combination with essential non-renewable resources, an assumption of increasing returns to capital seems too strong and too optimistic. A technology having $\tilde{\alpha}$ just slightly above 1 can sustain *any* per capita growth rate - there is no upper bound on g_c.[5] This appears overly optimistic.

[4] It is shown in Groth (2004) that "only if" in Result (i) can be replaced by the stronger "if and only if". Note also that if some irreducibly exogenous element in the technological development is allowed in the model by replacing the constant B in (6) by $e^{\tau t}$, where $\tau \geq 0$, then (10) is replaced by $\tau + (\tilde{\alpha} + \beta - 1)n > 0$ or $\tilde{\alpha} > 1$. Both Stiglitz (1974, p. 131) and Withagen (1990, p. 391) ignore implicitly the possibility $\tilde{\alpha} > 1$. Hence, from the outset they preclude fully endogenous growth.

[5] See Groth (2004).

Essentially, this leaves us with *semi-endogenous* growth as the only plausible form of endogenous growth (as long as n is not endogenous). We say there is semi-endogenous growth when 1) per capita growth is driven by some internal mechanism (as distinct from exogenous technology growth), but 2) sustained per capita growth requires support in the form of growth in some exogenous factor. In innovation-based growth theory, this factor is typically the size of population. Result (i) indicates that semi-endogenous growth corresponds to the case $1 - \beta < \tilde{\alpha} \leq 1$. In this case sustained per capita growth driven by some internal mechanism is possible, but only if supported by $n > 0$, that is, by growth in an exogenous factor, here population (the source of new ideas).

2.2 Growth policy and conservation

Result (i) is about as far as Suzuki's analysis takes us, since his focus is only on whether the technology as such allows the growth rate to be positive or not.[6] That is, he does not study the *size* of the growth rate. A key issue in new growth theory is to explain the size of the growth rate and how it can temporarily or perhaps permanently be affected by economic policy. The simple growth-accounting relation (9) immediately shows:

Result (ii) Along a BGP, policies that decrease (increase) the depletion rate u (and only such policies) will increase (decrease) the per capita growth rate (here we presuppose $\tilde{\alpha} < 1$, the plausible case).

This observation is of particular interest in view of the fact that changing the perspective from exogenous to endogenous technical progress implies bringing a source of numerous market failures to light. On the face of it, the result seems to run against common sense. Does high growth not imply *fast* depletion (high u)? Indeed, the answer is affirmative, but with the addition that exactly because of the fast depletion such high growth will only be temporary - it carries the seeds to its own obliteration. For faster sustained growth there must be sustained slower depletion. The reason for this is that with protracted depletion, the rate of decline in resource input becomes smaller; hence, so does the drag on growth caused by this decline.

As a statement about policy and long-run growth, (ii) is a surprisingly succinct conclusion. It can be clarified in the following way. For policy to affect long-run growth,

[6] Suzuki's (1976) article also contains another model, with a resource externality. That model is analyzed and extended in Groth and Schou (2007).

it must affect a linear differential equation linked to the basic-goods sector in the model (Romer 1995). In the present framework the resource depletion relation,

$$\dot{S} = -uS,$$

is such an equation. In balanced growth $g_S = -R/S \equiv -u$ is constant so that the proportionate rate of decline in R must comply with, indeed be equal to, that of S. Through the growth accounting relation (8), given u, this fixes g_Y and $g_{\tilde{K}}$ (equal in balanced growth) and thereby also $g_c = g_Y - n$. The conventional wisdom in the endogenous growth literature is that interest income taxes impede economic growth and investment subsidies promote economic growth. Interestingly, this is not so when non-renewable resources are an essential input in the growth engine (which is here the manufacturing sector itself). Then, generally, only those policies that interfere with the depletion rate u in the long run (like a profits tax on resource-extracting companies or a time-dependent tax on resource use) can affect long-run growth. This is further explored in Groth and Schou (2007). It is noteworthy that this long-run policy result holds whether $g_c > 0$ or not and whether growth is exogenous, semi-endogenous or fully endogenous.[7] The general conclusion is that with non-renewable resources entering the growth-generating sector in an essential way, conventional policy tools receive a different role and there is a role for new tools (affecting long-run growth through affecting the depletion rate).

2.3 Further implications

In order to be more specific we introduce household preferences and a "social planner". The resulting resource allocation will coincide with that of a decentralized economy with appropriate subsidies and taxes. As in Stiglitz (1974), let the utilitarian social planner optimize

$$U_0 = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta} L(t) e^{-\rho t} dt, \qquad \theta > 0, \rho \geq n \geq 0, \tag{11}$$

subject to the constraints given by technology ((6), (7) and (4)) and initial conditions. Here, θ is the (numerical) elasticity of marginal utility (desire for consumption smoothing) and ρ is a constant rate of time preference (impatience).[8]

[7] This is a reminder that the distinction between fully endogenous growth and semi-endogenous growth is not the same as the distinction between policy-dependent and policy-invariant growth.

[8] If $\rho = n$, the improper integral U_0 tends to be unbounded and then the optimization criterion is not maximization, but "overtaking" or "catching-up" (see Seierstad and Sydsaeter, 1987). For simplicity we have here ignored (as does Stiglitz) that also environmental quality should enter the utility function.

Using the Pontryagin Maximum Principle, the first order conditions for this problem lead to, first, the *Ramsey rule*,[9]

$$g_c = \frac{1}{\theta}(\frac{\partial Y}{\partial \tilde{K}} - \delta - \rho) = \frac{1}{\theta}(\tilde{\alpha}\frac{Y}{\tilde{K}} - \delta - \rho), \tag{12}$$

second, the *Hotelling rule,*[10]

$$\frac{d(\partial Y/\partial R)}{dt} = \frac{\partial Y}{\partial R}(\frac{\partial Y}{\partial \tilde{K}} - \delta) = \gamma\frac{Y}{R}(\tilde{\alpha}\frac{Y}{\tilde{K}} - \delta). \tag{13}$$

The first rule says: as long as the net return on investment in capital is higher than the rate of time preference, one should let current c be low enough to allow positive net saving (investment) and thereby higher consumption in the future. The second rule is a no-arbitrage condition saying that the return ("capital gain") on leaving the marginal unit of the resource in the ground must equal the return on extracting and using it in production and then investing the proceeds in the alternative asset (reproducible capital).[11]

Using the Cobb-Douglas specification, we may rewrite the Hotelling rule as $g_Y - g_R = \tilde{\alpha}Y/\tilde{K} - \delta$. Along a BGP $g_Y = g_C = g_c + n$ and $g_R = -u$, so that the Hotelling rule combined with the Ramsey rule gives

$$(\theta - 1)g_c - u = n - \rho. \tag{14}$$

This linear equation in g_c and u combined with the growth-accounting relationship (9) constitutes a linear two-equation system in the growth rate and the depletion rate. The determinant of this system is $D \equiv 1 - \tilde{\alpha} - \gamma + \theta\gamma$. We assume $D > 0$, which seems realistic and is in any case necessary (and sufficient) for stability.[12] Then

$$g_c = \frac{(\tilde{\alpha} + \beta + \gamma - 1)n - \gamma\rho}{D}, \quad \text{and} \tag{15}$$

$$u = \frac{[(\tilde{\alpha} + \beta - 1)\theta - \beta]\, n + (1 - \tilde{\alpha})\rho}{D}. \tag{16}$$

[9] After Ramsey (1928).

[10] After Hotelling (1931). Assuming perfect competition, the real resource price becomes $p_R = \partial Y/\partial R$ and the real rate of interest is $r = \partial Y/\partial K - \delta$. Then the rule takes the more familar form $\dot{p}_R/p_R = r$. If there are extraction costs at rate $C(R, S, t)$, then the rule takes the form $\dot{p}_S - \partial C/\partial S = rp_S$, where p_S is the price of the unextracted resource (whereas $p_R = p_S + \partial C/\partial R$).

It is another thing that the rise in resource prices and the predicted decline in resource use have not yet shown up in the data (Krautkraemer 1998, Smil 2003); this may be due to better extraction technology and discovery of new deposits. But in the long run, if non-renewable resources *are* essential, this tendency inevitably will be reversed.

[11] After the initial phase of complete specialization described in Section 2.1, we have, due to the proportionality between K, A and $\tilde{K}$, that $\partial Y/\partial K = \partial Y/\partial A = \partial Y/\partial \tilde{K} = \tilde{\alpha}Y/\tilde{K}$. Notice that the Hotelling rule is independent of preferences; *any* path that is *efficient* must satisfy the Hotelling rule (as well as the exhaustion condition $\lim_{t\to\infty} S(t) = 0$).

[12] As argued above, $\tilde{\alpha} < 1$ seems plausible. Generally, θ is estimated to be greater than one (see, e.g., Attanasio and Weber 1995); hence $D > 0$. The stability result as well as other findings reported here are documented in Groth and Schou (2002).

Interesting implications are:

Result (iii) If there is impatience ($\rho > 0$), then even when a non-negative g_c is technologically feasible ((10) satisfied), a negative g_c can be optimal and stable.

Result (iv) Population growth is *good* for economic growth. In its absence, when $\rho > 0$, we get $g_c < 0$ along an optimal BGP; if $\rho = 0$, $g_c = 0$ when $n = 0$.

Result (v) There is never a scale effect on the growth rate.

Result (iii) reflects that utility discounting and consumption smoothing weaken the "growth incentive". Result (iv) is completely contrary to the conventional (Malthusian) view and the learning from the D-H-S-S model. The point is that two offsetting forces are in play. On the one hand, higher n means more mouths to feed and thus implies a drag on per capita growth (Malthus). On the other hand, a growing labour force is exactly what is needed in order to exploit the benefits of increasing returns to scale (anti-Malthus).[13] And in the present framework this dominates the first effect.[14] This feature might seem to be contradicted by the empirical finding that there is no robust correlation between g_c and population growth in cross-country regressions (Barro and Sala-i-Martin 2004, Ch. 12). However, the proper unit of observation in this context is not the individual country. Indeed, a positive association between n and g_c as in (15) should not, in an internationalized world with technology diffusion, be seen as a prediction about individual countries, but rather as pertaining to larger regions, perhaps the global economy. In any event, the second part of Result (iv) is a dismal part - in view of the projected long-run stationarity of world population (United Nations 2005).

A somewhat surprising result appears if we imagine (unrealistically) that $\tilde{\alpha}$ is sufficiently above one to make D a negative number. If population growth is absent, $D < 0$ is in fact needed for $g_c > 0$ along a BGP. However, $D < 0$ implies instability. Hence this would be a case of an instable BGP with fully endogenous growth.[15]

[13] This aspect will become more lucid in the two-sector models of the next section, where the non-rival character of technical knowledge is more transparent.

[14] This as well as the other results go through if a fixed resource like land is included as a necessary production factor. Indeed, letting J denote a fixed amount of land and replacing (1) by $Y = A^{\varepsilon} K^{\alpha} L^{\beta} R^{\gamma} J^{1-\alpha-\beta-\gamma}$, where now $\alpha + \beta + \gamma < 1$, leave (8)-(10), (15) and (16) unchanged.

[15] Thus, if we do not require $D > 0$ in the first place, (iv) could be reformulated as: existence of a *stable* optimal BGP with $g_c > 0$ *requires* $n > 0$. This is not to say that reducing n from positive to zero renders an otherwise stable BGP instable. Stability-instability is governed solely by the sign of D. Given $D > 0$, letting n decrease from a level above the critical value, $\gamma\rho/(\tilde{\alpha} + \beta + \gamma - 1)$, given from (15), to a level below, changes g_c from positive to negative, i.e., growth comes to an end.

Result (v) is about the absence of the problematic scale effect (larger population, larger growth rate) that appears in many R&D-based endogenous growth models. It is noteworthy that this absence holds for *any* value of $\tilde{\alpha}$, including $\tilde{\alpha} = 1$.[16]

A pertinent question now is: are the above results just an artifact of the one-sector set-up? This leads us to consider two-sector models.

3 The standard two-sector framework with R&D and resource depletion

The conclusions (i), (ii), (iii) and (v) above (and partly also (iv)) differ from most of the new growth literature,[17] including most of the contributions that deal explicitly with non-renewable resources and endogenous growth (Jones and Manuelli 1997, Aghion and Howitt 1998 (Chapter 5), Scholz and Ziemes 1999, Schou 2000, Schou 2002, Grimaud and Rougé 2003). These contributions extend the first-generation two-sector endogenous growth models (like Romer 1990 and Aghion and Howitt 1992), by including a non-renewable resource as an essential input in the manufacturing sector. The non-renewable resource does not, however, enter the R&D or educational sector in these models (not even indirectly in the sense of physical capital produced in the manufacturing sector being used in the R&D sector). As we shall now see, this is the reason that these models give results quite similar to those from conventional endogenous models without non-renewable resources.

The following two-sector framework is a prototype of the afore-mentioned contributions:

$$
\begin{aligned}
Y &= A^{\varepsilon} K^{\alpha} L_Y^{\beta} R^{\gamma}, \qquad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, && (17)\\
\dot{K} &= Y - cL - \delta K, \qquad \delta \geq 0, &&\\
\dot{A} &= \bar{\mu} L_A, \qquad \bar{\mu} = \mu A, \qquad \mu > 0, && (18)\\
\dot{S} &= -R, &&\\
L_Y + L_A &= L, \text{ a constant.} &&
\end{aligned}
$$

Unlike in the previous model, additions to society's "stock of knowledge", A, are now

[16] More commonplace observations are that increased impatience leads to faster depletion and lower growth (in the plausible case $\tilde{a} < 1$). Further, in the log-utility case ($\theta = 1$) the depletion rate u equals the effective rate of impatience, $\rho - n$.

[17] Here we have in mind the fully endogenous growth literature. The results are more cognate with the results in semi-endogenous growth models without non-renewable resources, like Jones (1995).

produced in a separate sector, the R&D sector, with a technology different from that in manufacturing. The only input in the R&D sector is labour (thus taking to the extreme the feature that this sector is likely to be relatively intensive in human capital). The individual research lab, which is "small" in relation to the economy as a whole, takes R&D productivity, $\bar{\mu}$, as given. At the economy-wide level, however, this productivity depends positively on the stock of technical knowledge in society, A (this externality is one of several reasons that the existence of endogenous technical change implies market failures). Usually, there is no depreciation of knowledge, i.e., $\delta_A = 0$. Aggregate employment in the R&D sector is L_A. Total employment, L, in the economy is the sum of L_A and employment, L_Y, in the manufacturing sector. In that sector, the firms take A as given and the technology they face at the micro level may involve different capital-good varieties and qualities. There are many interesting details and disparities between the models concerning these aspects as well as the specifics of the market structure and the policy questions considered. Yet, whether we think of the "increasing variety" models (or Romer-style models to which Scholz and Ziemes 1999 and Schou 2002 belong) or the "increasing quality models" (or quality ladder models to which Aghion and Howitt 1998 and Grimaud and Rougé 2003 belong), at the aggregate level these models end up with a formal structure basically like that above.[18] The accumulation-based growth models by Jones and Manuelli (1997) and Schou (2000) are in one respect different - we shall return to this.

Two key features emphasised by new growth theory are immediately apparent. First, because technological ideas - sets of instructions - are *non-rival*, what enters both in the production function for Y and that for $\dot{A}$ is *total* A. This is in contrast to the *rival* goods: capital, labour and the resource flow. For example, a given unit of labour can be used no more than one place at a time. Hence, only a fraction of the labour force enters manufacturing, the remaining fraction entering R&D. Second, there is a tendency for *increasing returns to scale* to arise when knowledge is included in the total set of inputs. At least when we ignore externalities, the well-known replication argument gives reason to expect constant returns to scale w.r.t. the *rival* inputs (here K, L_Y and R in the manufacturing sector and L_A in R&D). Consequently, as we double these rival inputs and *also* double the amount of knowledge, we should expect more than a doubling of Y and $\dot{A}$. An additional key feature of new growth theory, apparent when the above technology description is combined with assumptions about preferences and market structure, is the

[18] Essentialy this structure also characterizes the two-sector models by Robson (1980) and Takayama (1980), although these contributions do not fully comprehend the non-rival character of knowledge, since they have L_A/L in (18) instead of L_A.

emphasis on incentives as driving R&D investment. When the resource becomes more scarce and its price rises, the value of resource-saving knowledge increases and R&D is stimulated.[19]

Using the principle of growth accounting on (17), taking $n = 0$ into account, we get, along a BGP,[20]

$$(1-\alpha)g_c = \varepsilon g_A - \gamma u, \tag{19}$$

where

$$g_A = \mu \ell_A L, \qquad \ell_A \equiv \frac{L_A}{L}, \text{ constant.}$$

We have $g_A > 0$ if $\ell_A > 0$. The essential non-renewable resource implies a drag on the growth of consumption. Yet, by sufficient conservation of the resource (implying a small $u \equiv R/S$) it is always possible to obtain $g_c > 0$. And it is possible to increase g_c without decreasing u, simply by increasing ℓ_A. These two last conclusions have a quite different flavour compared to the results (i) and (ii) from the extended D-H-S-S model.

The fraction, ℓ_A, of the labour force in R&D will depend on parameters such as α, ε, μ and those describing preferences and the allocation device, whether this is the market mechanism in a decentralized economy or the social planner in a centralized economy. To be specific, let us again consider a social planner and the criterion (11). Along a BGP we get once more (14) (from the Ramsey rule and the Hotelling rule). Further, efficient allocation of labour across the two sectors and across time leads to $\ell_A = 1 - \beta u/(\varepsilon \mu L)$. Combining this with (19) and (14) we find, along a BGP,

$$\begin{aligned}
\ell_A &= \frac{\varepsilon \mu L(\beta + \theta\gamma) - \beta(1-\alpha)\rho}{\varepsilon \mu L \theta (1-\alpha)}, \\
g_c &= \frac{\varepsilon \mu L - (1-\alpha)\rho}{\theta(1-\alpha)}, \qquad \text{and} \\
u &= \frac{(\theta - 1)\varepsilon \mu L + (1-\alpha)\rho}{\theta(1-\alpha)}.
\end{aligned}$$

This is an example of fully endogenous growth: given $(1-\theta)\varepsilon\mu L < (1-\alpha)\rho < \varepsilon \mu L$,[21] per capita growth is positive along a BGP without support of growth in any exogenous

[19] Using patent data, Popp (2002) finds a strong, positive impact of energy prices on energy-saving innovations.

[20] In this two-sector framework a BGP means a path along which Y, C, K *and* N grow at constant rates (not necessarily positive). It is understood that the path considered is *efficient* and thus leaves nothing of the resource unutilized forever.

[21] The first inequality ensures $u > 0$ (equivalent with the necessary transversality condition in the optimal control problem being satisfied), the second ensures $g_c > 0$.

factor. A caveat is that this result relies on the knife-edge assumption that the growth engine (the R&D sector) has exactly constant returns to the producible input(s), here A. The problematic (empirically unrealistic) scale effect on growth ($\partial g_c/\partial L > 0$) crops up (although often hidden by the labour force being normalized to one). Indeed, this is why these models assume a constant labour force; with $n > 0$ the growth rate will be forever rising. In any event, contrary to the implication of (15), sustained growth is conceivable without population growth and whether $\rho = 0$ or $\rho > 0$.

Overall, we have a more optimistic perspective than in the extended D-H-S-S model. Indeed, the conclusions are quite different from the results (i), (ii) and (v) above (and partly also different from (iv)). The conclusions are, however, pretty much in conformity with those of the fully endogenous growth models without non-renewable resources. With the exception of the scale effect on growth we get similar results in the models by Jones and Manuelli (1997) and Schou (2000). Jones and Manuelli consider an economy with a sector producing consumption goods with labour, capital and the non-renewable resource and a sector producing capital goods with only capital (not even labour). Schou develops a Lucas-style human-capital-based model extended with a non-renewable resource entering only the manufacturing sector (with the addition of pollution from this resource). Since in these models it is no longer the accumulation of a *non-rival* good that drives growth, the scale effect on growth disappears, but this is the only difference in relation to the questions considered here.

The explanation of the optimistic results in all these models is that the growth-generating sector is presumed not to depend on the non-renewable resource (neither directly nor indirectly). In reality, however, most sectors, including educational institutions and research laboratories, use fossil fuels for heating and transportation purposes, or at least they use indirectly minerals and oil products via the machinery, computers etc. they employ. The extended D-H-S-S model in the previous section *did* take this dependency of the growth engine (in that model the manufacturing sector itself) on the natural resource into account and therefore gave substantially different results. In the next section we shall see that a two-sector model with the resource entering (also) the R&D sector leads to results similar to those of the extended D-H-S-S model from Section 1, but quite different from those of the above two-sector model.

4 Growth-essential non-renewable resources

When a natural resource is an essential input (directly or indirectly) in the growth engine, we call the resource *growth-essential*.

4.1 The resource as input in both sectors

Extending the above two-sector framework as in Groth (2007a), we consider the setup:

$$\begin{aligned}
Y &= A^{\varepsilon}K^{\alpha}L_Y^{\beta}R_Y^{\gamma}, \qquad \varepsilon,\alpha,\beta,\gamma>0, \quad \alpha+\beta+\gamma=1, && (20)\\
\dot{K} &= Y-cL-\delta K, \qquad \delta\geq 0, && (21)\\
\dot{A} &= \bar{\mu}L_A^{\eta}R_A^{1-\eta}, \qquad \bar{\mu}=\mu A^{\varphi}, \qquad \mu>0, \quad 0<\eta<1, && (22)\\
\dot{S} &= -R, && (23)\\
L_Y+L_A &= L=L(0)e^{nt}, \quad n\geq 0, && (24)\\
R_Y+R_A &= R. && (25)
\end{aligned}$$

There are three new features. First, only a fraction of the resource flow R is used in manufacturing, the remainder being used as an essential input in R&D activity. Second, the knowledge elasticity, φ, of research productivity is allowed to differ from one; as argued in the section on the Jones critique, even $\varphi<0$ should not be excluded *a priori*. Third, population growth is not excluded.

Along a BGP, using the principle of growth accounting on (20) yields

$$(1-\alpha)g_c=\varepsilon g_A-\gamma(n+u). \tag{26}$$

Applying the same principle on the R&D equation (22) (after dividing by A and presupposing the R&D sector is active) and assuming balanced growth we get, after substituting into (26),

$$(1-\alpha)g_c=\left(\frac{\varepsilon\eta}{1-\varphi}-\gamma\right)n-\left(\frac{\varepsilon(1-\eta)}{1-\varphi}+\gamma\right)u. \tag{27}$$

Since $u>0$, from this[22] follows that a BGP with $g_c>0$ is technologically feasible only if

$$\varphi<1+\frac{\varepsilon(1-\eta)}{\gamma} \text{ and either } (n>0 \text{ and } \varepsilon\eta>(1-\varphi)\gamma) \text{ or } \varphi>1.$$

Naturally, the least upper bound for φ's that allow non-explosive growth is here higher than when the resource is not a necessary input in the R&D sector. We also see that

[22] For ease of interpretation we have written (27) on a form analogue to (26). In case $\varphi=1$, (27) should be interpreted as $(1-\varphi)(1-\alpha)g_c=[\varepsilon\eta-(1-\varphi)\gamma]\,n-[\varepsilon(1-\eta)+(1-\varphi)\gamma]\,u$.

for the technology to allow steady positive per capita growth, *either* φ must be above one *or* there must be population growth (to exploit increasing returns to scale) *and* an elasticity of Y w.r.t. knowledge large enough to overcome the drag on growth caused by the inevitable decline in resource use. Not surprisingly, in the absence of population growth, sustained per capita growth requires a higher elasticity of research productivity with respect to knowledge than when the growth engine does not need the resource as an input. The "standard" two-sector model of the previous section relied on the aggregate invention production function having exactly constant returns (at least asymptotically) to produced inputs, that is, $\varphi = 1$. Slightly increasing returns w.r.t. A would in that model lead to explosive growth, whereas slightly decreasing returns lead to growth petering out. Interestingly, when the resource is growth-essential, the case $\varphi = 1$ loses much of its distinctiveness. Yet, the "bad news" for fully endogenous growth is again that $\varphi > 1$ seems to be a too optimistic and strong assumption. The reason is similar to that given in Section 1.1 for doubting that $\tilde{\alpha} > 1$, namely that whenever a given technology has $\varphi > 1$, it can sustain *any* per capita growth rate no matter how high - a rather suspect implication. Thus, once more we are left with semi-endogenous growth ($\varphi \leq 1$) as the only appealing form of endogenous growth (as long as n is exogenous).

In parallel to Result (ii) above, (27) shows that when $\varphi < 1$, only policies that decrease the depletion rate u along a BGP, can increase the per capita growth rate g_c. For example, embedding the just described technology in a Romer (1990)-style market structure, Groth (2007b) shows that a research subsidy, an interest income tax and an investment subsidy do not affect long-run growth whereas taxes that impinge on resource extraction do. The point is that whatever market forms might embed the described technology and whatever policy instruments are considered, the growth-accounting relation (27) *must* hold (given the assumed Cobb-Douglas technologies).

Let us again consider a social planner and the criterion (11). Then, along a BGP we have once more (14) (from the Ramsey rule and the Hotelling rule). Combining this with (27) we find, along a BGP,

$$
\begin{aligned}
g_c &= \frac{\varepsilon n - [\varepsilon(1-\eta) + (1-\varphi)\gamma]\,\rho}{\tilde{D}}, \qquad \text{and} \\
u &= \frac{\left[(\theta-1)\varepsilon - \tilde{D}\right] n + (1-\varphi)(1-\alpha)\rho}{\tilde{D}},
\end{aligned}
$$

where $\tilde{D} \equiv (1-\varphi)(\beta+\theta\gamma) + (\theta-1)\varepsilon(1-\eta)$ is assumed positive (this seems to be the empirically relevant case and it is in any event necessary, though not sufficient, for stability).[23] We see that in the plausible case $\varphi < 1 + \varepsilon(1-\eta)\gamma$ the analogy of the results (iii), (iv) and (v) from the extended D-H-S-S model of Section 1 go through.[24]

The conclusion is that when a non-renewable resource is an essential input in the R&D sector, quite different and more pessimistic conclusions arise compared to those of the previous section. Sustained growth without increasing effort (i.e., without $n > 0$) now requires $\varphi > 1$ in contrast to $\varphi = 1$ in the previous section. Now policies aimed at stimulating long-run growth generally have to go via resource conservation.

4.2 Capital in the R&D sector

The results are essentially the same in the case where the resource is a direct input only in manufacturing, but the R&D sector uses capital goods (apparatus and instruments) produced in the manufacturing sector. Thus, indirectly the resource is an input also in the R&D sector, hence still growth-essential. The model is:

$$\begin{aligned}
Y &= A^{\varepsilon} K_Y^{\alpha} L_Y^{\beta} R^{\gamma}, \qquad \varepsilon, \alpha, \beta, \gamma > 0, \quad \alpha + \beta + \gamma = 1, && (28)\\
\dot{K} &= Y - cL - \delta K, \qquad \delta \geq 0, &&\\
\dot{A} &= \bar{\mu} K_A^{1-\eta} L_A^{\eta}, \qquad \bar{\mu} = \mu A^{\varphi}, \qquad \mu > 0, \qquad 0 < \eta < 1, && (29)\\
\dot{S} &= -R, &&\\
K_Y + K_A &= K, && (30)\\
L_Y + L_A &= L = L(0)e^{nt}, \quad n \geq 0. &&
\end{aligned}$$

Possibly, $1 - \eta < \alpha$ (since the R&D sector is likely to be relatively intensive in human capital), but for our purposes here this is not crucial.

Using the growth accounting principle on (28) again gives (26) along a BGP. Applying the same principle on the R&D equation (29) (presupposing the R&D sector is active) and assuming balanced growth, we find

$$(1-\varphi)g_A = (1-\eta)g_K + \eta n = (1-\eta)g_c + n, \tag{31}$$

in view of $g_K = g_C = g_c + n$. This shows that existence of a BGP with positive growth

[23] A possible reason for the popularity of the model of the previous section is that it has transitional dynamics that are less complicated than those of the present model (four-dimensional dynamics versus five-dimensional).

[24] Although a scale effect on growth is absent, a positive scale effect on levels remains, as shown in Groth (2007a). This is due to the non-rival character of technical knowledge.

requires $\varphi < 1$.[25] Both K and A are essential producible inputs in the two sectors; hence, the two sectors together make up the growth engine.

Substituting (31) into (26) yields

$$[(1-\varphi)(1-\alpha)-\varepsilon(1-\eta)]\, g_c = [\varepsilon-(1-\varphi)\gamma]\, n-(1-\varphi)\gamma u. \tag{32}$$

Since $u > 0$, we see that a BGP with $g_c > 0$ is technologically feasible only if, in addition to the requirement $\varphi < 1$,

$$\text{either } (\varepsilon > (1-\varphi)\gamma \text{ and } n > 0) \;\; \text{or } \varepsilon > \frac{(1-\varphi)(1-\alpha)}{1-\eta}.$$

That is, given $\varphi < 1$, the knowledge elasticity of manufacturing output should be high enough. These observations generalize Result (i) from the extended D-H-S-S model and also Result (ii), when we (plausibly) assume $\varepsilon < (1-\varphi)(1-\alpha)/(1-\eta)$, which corresponds to $\tilde{\alpha} < 1$ in the one-sector model. The combined accumulation of K and A drives growth, possibly with the help of population growth.

Again, let us consider a social planner and the criterion (11). Along a BGP we get once more (14) (from the Ramsey rule and the Hotelling rule). Combining this with (32) yields, along a BGP,

$$\begin{aligned} g_c &= \frac{\varepsilon n-(1-\varphi)\gamma\rho}{D^*}, \quad \text{and} \\ u &= \frac{[(\theta-1)\varepsilon-D^*]\, n+[(1-\varphi)(1-\alpha)-\varepsilon(1-\eta)]\, \rho}{D^*}, \end{aligned}$$

where $D^* \equiv (1-\varphi)(\beta+\theta\gamma)-\varepsilon(1-\eta)$ is assumed positive. The results (iii), (iv) and (v) from the extended D-H-S-S model immediately go through.

Thus, also when the non-renewable resource is only indirectly growth-essential, do we get conclusions in conformity with those in the previous subsection, but quite different from those of standard endogenous growth models with non-renewable resources entering only the manufacturing sector. This is somewhat at variance with the section on growth and non-renewable resources in Aghion and Howitt (1998). They compare their two-sector Schumpeterian approach (which in this context is equivalent to what was above called "the standard approach") with a one-sector AK model extended with an essential non-renewable resource and no population growth (which is equivalent to the extended D-H-S-S model with $\alpha = 1$ and $n = 0$). Having established that sustained growth is possible in the first approach, but not in the second, they ascribe this difference to "the ability

[25] As soon as $\varphi \geq 1$, growth becomes explosive.

of the Schumpeterian approach to take into account that the accumulation of intellectual capital is 'greener' (in this case, less resource intensive) than the accumulation of tangible capital" (p. 162). However, as the above example shows, even allowing the R&D sector to be "greener" than the manufacturing sector, we may easily end up with AK-style results. The crucial distinction is between models where the non-renewable resource is growth-essential - directly or indirectly - and models where it is not. To put it differently: by not letting the resource enter the growth engine (not even indirectly), Aghion and Howitt's "Schumpeterian approach" seems biased toward sustainability.

5 The case of limited substitutability in the R&D sector

One might argue that, at least in the R&D sector, the elasticity of substitution between labour (research) and other inputs must be low. Hence, let us consider the limiting case of zero substitutability in the models of the two previous subsections. First, we replace (22) in the model of Section 4.1 by

$$\dot{A} = \mu A^{\varphi} \min\left(L_A, A^{\psi} R_A\right), \qquad \psi > 0.$$

Then, along any efficient path with $g_A > 0$ we have $L_A = A^{\psi} R_A$ so that $g_A = \mu A^{\varphi-1} L_A = \mu A^{\varphi+\psi-1} R_A$. Log-differentiating this w.r.t. t and setting $\dot{g}_A = 0$ gives, along a BGP, $(\varphi - 1)g_A + n = 0 = (\varphi + \psi - 1)g_A - u$. Since $n \geq 0$ and $u > 0, 1 - \psi < \varphi \leq 1$ is required (if $\varphi > 1$, growth becomes explosive). In the generic case $\varphi < 1$, $g_A = n/(1-\varphi)$ so that $g_A > 0$ requires $n > 0$; we end up with

$$\begin{aligned} g_c &= \frac{\varepsilon - \gamma\psi}{(1-\alpha)(1-\varphi)} n, \\ u &= \frac{\varphi + \psi - 1}{1-\varphi} n. \end{aligned}$$

Thus, both the per capita consumption growth rate and the depletion rate u along a BGP are in this case technologically determined. As an implication, preferences and economic policy can have only level effects, not long-run growth effects. If $n = 0$, no BGP with $g_c > 0$ exists in this case.

The singular case $\varphi = 1$ is different. This is the only case where there is scope for preferences and policy to affect long-run growth. Indeed, in this case, where $n = 0$ is needed to avoid a forever increasing growth rate, along a BGP we get $g_c = (\varepsilon - \gamma\psi)\mu L_A$ and $u = \psi\mu L_A$.

We get similar results if in the model of Section 4.2 we replace (29) by

$$\dot{A} = \mu A^{\varphi} \min\left(K_A, A^{\psi} L_A\right), \qquad \psi > 0.$$

Along any efficient path with $g_A > 0$, now $K_A = A^{\psi} L_A$ so that $g_A = \mu A^{\varphi-1} K_A = \mu A^{\varphi+\psi-1} L_A$. Log-differentiating this w.r.t. t and setting $\dot{g}_A = 0$ gives, along a BGP, $(\varphi - 1) g_A + g_K = 0 = (\varphi + \psi - 1) g_A + n$. Since $n \geq 0, \varphi \leq 1 - \psi$ is required (if $\varphi > 1 - \psi$, growth becomes explosive). In the generic case $\varphi < 1 - \psi$, both the depletion rate u and the per capita consumption growth rate become technologically determined:

$$\begin{aligned} g_c &= \frac{\psi}{1 - \varphi - \psi} n, \\ u &= \frac{\varepsilon - \beta\psi - \gamma(1 - \varphi)}{(1 - \varphi - \psi)\gamma} n, \end{aligned}$$

where the inequalities $n > 0$ and $\varepsilon > \beta\psi + \gamma(1 - \varphi)$ are presupposed. If $n = 0$, no BGP with $g_A > 0$ exists in this case.

Only in the singular case $\varphi = 1 - \psi$ can preferences and policy affect long-run growth. Indeed, in this case, where $n = 0$ is needed to avoid a forever increasing growth rate, along a BGP we find $g_c = \psi \mu L_A$ and $u = (\varepsilon - (1 - \alpha)\psi)\mu L_A$, where $\varepsilon > (1 - \alpha)\psi$ is presupposed.

To conclude, with zero substitution between the production factors in the R&D sector, one "degree of freedom" is lost. As an implication, in the generic case there is no scope for preferences and policy affecting growth. Only in a knife-edge case can preferences and policy affect growth. Thus, the robust case is in this regard in conformity with semi-endogenous growth models without non-renewable resources à la Jones (1995), and the non-robust case is in conformity with fully endogenous growth models without non-renewable resources à la Romer (1990).

6 Conclusion

To the extent that non-renewable resources are necessary inputs in production, sustained growth requires the presence of resource-augmenting technical progress. New growth theory has deepened our understanding of mechanisms that influence the amount and direction of technical change. Applying new growth theory to the field of resource economics and the problems of sustainability yields many insights. The findings emphasized in this article are the following. The standard approach to modelling endogenous technical change in a set-up with non-renewable resources ignores that also R&D may need the resource (directly or indirectly). This implies a bias in favour of sustainability and growth. Indeed, sustained per capita growth requires stronger parameter restrictions when the resource is "growth essential", than when it is not. When the resource is "growth essential", then a policy aiming at stimulating long-run growth generally has to reduce the long-run depletion rate. In this sense promoting long-run growth and "supporting the environment" go hand in hand.

New growth theory has usually, as a simplifying device, considered population growth as exogenous. Given this premise, a key distinction - sometimes even controversy - arises between what is called fully endogenous growth and what is called semi-endogenous growth. In mainstream new growth theory, where non-renewable resources are completely left out of the analysis, this distinction tends to coincide with three other distinctions: (a) that between models that suffer from non-robustness due to a problematic knife-edge condition and models that do not; (b) that between models that imply a scale effect on growth and models that do not; and (c) models that imply policy-dependent long-run growth and models that do not. When non-renewable resources are taken into account and enter the growth engine (directly or indirectly), these dissimilarities are modified: (i) the non-robustness problem vanishes because of the disappearance of the critical knife-edge condition; yet, fully endogenous growth does not become more plausible than before, rather the contrary; (ii) the problem of a scale-effect on growth disappears; (iii) due to the presence of two very different assets, producible capital and non-producible resource deposits, even in the semi-endogenous growth case there is generally scope for policy having long-run growth effects.

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