

# Short- and long-run aspects of fiscal policy in a deep recession<sup>1</sup>

Consider a small open economy (SOE) facing a real interest rate,  $r_t$ , given from the world market for financial capital. There is no cross-country mobility of labor. The following is assumed to hold under “normal circumstances”:

- aggregate employment,  $N$ , is at the “full employment” level,  $N = (1 - \bar{u})L$ , where  $\bar{u}$  is the NAIRU and  $L$  is the aggregate labor supply, a given constant;
- real GDP,  $Y_t$ , equals its given trend level,  $\bar{Y}_t$ , which grows at a constant exogenous rate  $g > 0$  due to technical progress;
- $r_t = r$ , where  $r$  is a constant and  $r > g$ .

Time is discrete. Further notation is:

$G_t$  = real government spending on goods and services in period  $t$ , possibly just public investment,

$T_t$  = real net tax revenue (= gross tax revenue – transfer payments) in period  $t$ ,

$GBD_t$  = real government budget deficit in period  $t$ ,

$B_t$  = real public debt (all short-term) at the start of period  $t$ .

Suppose that any government budget deficit has to be exclusively financed by issuing debt (and any budget surplus by redeeming debt). We have

$$GBD_t = rB_t + G_t - T_t, \quad (\text{GBD})$$

and

$$B_{t+1} = (1 + r)B_t + G_t - T_t. \quad (\text{DGBC})$$

As an implication,

$$B_{t+1} = B_t + GBD_t. \quad (1)$$

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<sup>1</sup>The problems discussed in this note are inspired by a brief article by Paul Krugman in NYT, July 7, 2010.

Suppose that  $B_0 > 0$  and  $G_t = \gamma \bar{Y}_t$ ,  $t = 0, 1, \dots$ , where  $0 < \gamma < 1$ . Define the “net tax burden” as  $\tau_t \equiv T_t/Y_t$ . Then we can find the minimum constant net tax burden,  $\bar{\tau}$ , which is consistent with fiscal sustainability. The steps are as follows. The debt-income ratio  $b_t \equiv B_t/Y_t$  changes over time according to

$$b_{t+1} \equiv \frac{B_{t+1}}{Y_{t+1}} = \frac{(1+r)B_t + G_t - T_t}{(1+g)Y_t} = \frac{1+r}{1+g}b_t + \frac{\gamma - \tau_t}{1+g}. \quad (2)$$

Since we are looking for a constant net tax burden, we replace  $\tau_t$  by  $\tau$  in this equation. The solution to the resulting linear difference equation with constant coefficients is

$$b_t = (b_0 - b^*) \left( \frac{1+r}{1+g} \right)^t + b^*, \quad \text{where } b^* = \frac{\tau - \gamma}{r - g}, \quad (3)$$

where  $b_0 > 0$  is historically given.

Only if the current fiscal policy,  $(\gamma, \tau)$ , implies a non-exploding debt-income ratio, is the policy sustainable. As an implication of  $r > g$ , this requires that  $\tau$  and  $\gamma$  are such that  $b^* \geq b_0$ . That is, the requirement is that

$$\tau - \gamma \geq (r - g)b_0.$$

The minimum sustained net tax burden consistent with fiscal sustainability is therefore

$$\bar{\tau} = \gamma + (r - g)b_0. \quad (4)$$

In passing we note how, according to (4),  $\bar{\tau}$  depends on  $r$ ,  $g$ ,  $\gamma$ , and  $b_0 \equiv B_0/Y_0$ , respectively:

$$\begin{aligned} \frac{\partial \bar{\tau}}{\partial r} &= b_0 > 0, \\ \frac{\partial \bar{\tau}}{\partial g} &= -b_0 < 0, \\ \frac{\partial \bar{\tau}}{\partial \gamma} &= 1, \\ \frac{\partial \bar{\tau}}{\partial b_0} &= r - g > 0. \end{aligned}$$

Now consider an alternative scenario. In period  $t = -1$  a financial crisis in the world economy causes a world-wide economic downturn. The SOE is hit by a huge negative demand shock and gets into a substantial recession with  $Y_{-1}$  far below  $\bar{Y}_{-1}$ . In response the government decides an “expansionary fiscal policy” instead of “laissez faire”, where:

- “laissez faire” means maintaining  $G_t = \gamma \bar{Y}_t$ ,  $t = 0, 1, \dots$ ;
- “expansionary fiscal policy” entails a discretionary increase in  $G$  of size  $\Delta G$ , beginning in period 0 and maintained during the slump to stimulate economic activity, that is,  $G_t = \gamma \bar{Y}_t + \Delta G$ , where  $\Delta G$  is a positive constant.

Let the tax and transfer rules in the economy imply that net tax revenue in period 0 is given by the function  $T = T(Y)$ ; thus,  $T_0 = T(Y_0)$ . Assume that under the current slump conditions marginal net tax revenue is  $T'(Y) = 0.50$  whereas the spending multiplier is  $\partial Y/\partial G = 1.5$  (a cautious estimate given the slump conditions).

For a given  $\Delta G > 0$ , we will now find expressions for the approximate effect of the expansionary fiscal policy on  $GBD_0$  and  $B_1$ , respectively, in comparison with *laissez faire*. We have  $\partial B_0/\partial G_0 = 0$ , since  $B_0$  is predetermined at the beginning of period 0. Hence, with  $t = 0$ , from (GBD) follows

$$\frac{\partial GBD_0}{\partial G_0} = 1 - T'(Y_0) \frac{\partial Y_0}{\partial G_0} = 1 - 0.5 \cdot 1.5 = 1 - 0.75 = 0.25,$$

and from (1)

$$\frac{\partial B_1}{\partial G_0} = \frac{\partial GBD_0}{\partial G_0} = 0.25.$$

The effects of the rise in  $G_0$  are then given by

$$\Delta GBD_0 \approx \frac{\partial GBD_0}{\partial G_0} \Delta G = 0.25 \Delta G, \quad (5)$$

and

$$\Delta B_1 \approx \frac{\partial B_1}{\partial G_0} \Delta G = \frac{\partial GBD_0}{\partial G_0} \Delta G = 0.25 \Delta G.$$

Suppose that both  $\partial Y/\partial G$  and  $T'(Y)$  are approximately the same in period 1 as in period 0. For a given  $r_1$ , we now find an expression for the approximate effect of the expansionary fiscal policy on  $B_2$  in comparison with *laissez faire*. At  $t = t_1$ , we have, from (DGBC),

$$B_2 = (1 + r_1)B_1 + G_1 - T(Y_1).$$

The expansionary fiscal policy affects  $B_2$  both directly via a higher  $G_1$  (partly offset by the associated extra tax revenue) and indirectly via the higher  $G_0$  which generated a higher  $B_1$ . The total effect is

$$\begin{aligned} \Delta B_2 &\approx (1 + r_1)\Delta B_1 + \Delta G_1 - T'(Y_1)\Delta Y_1 \approx (1 + r_1)0.25\Delta G + (1 - T'(Y_1))\frac{\partial Y_1}{\partial G_1}\Delta G \\ &= (1 + r_1)0.25\Delta G + 0.25\Delta G = (2 + r_1)0.25\Delta G. \end{aligned} \quad (6)$$

Suppose the slump in the world economy as well as the SOE is over in period 2 and onwards. Suppose further that compared with the expansionary fiscal policy, *laissez faire* during the slump would have implied not only higher unemployment, but also more people experiencing *long-term* unemployment. As a result some workers would have become de-qualified, loose self-confidence, and in effect be driven out of the effective labor force. In addition, *laissez faire* during the slump is also likely to cause private capital investment to be smaller than otherwise during the slump.

Let the loss in “full employment” output from period 2 and onwards implied by *laissez faire* be  $\Delta Y$  per period, where  $\Delta Y$  is a positive constant.<sup>2</sup> Finally, let the ensuing loss in net tax revenue be  $\tau \cdot \Delta Y$  per period, where  $\tau$  is a positive constant (possibly close to  $\bar{\tau}$  in (4)).

With  $r_t = r_2$ ,  $t = 2, 3, \dots$ , and given  $\Delta G$  and  $\tau$ , let us find an expression for the value of  $\Delta Y$  required for the expansionary fiscal policy to “pay for itself” in period 2 and onwards in the sense that the averted loss in net tax revenue exactly offsets the extra interest payments. On the one hand the extra interest payments on the debt incurred by the expansionary fiscal policy is  $r_2 \Delta B_2$  per period. On the other hand, by the expansionary fiscal policy a loss in “full employment” output from period 2 and onwards equal to  $\Delta Y$  per period is avoided. Hence, for the expansionary fiscal policy to “pay for itself” in period 2 and onwards it is required that  $r_2 \Delta B_2 \leq \tau \Delta Y$ , that is,

$$\Delta Y \geq \frac{r_2 \Delta B_2}{\tau}.$$

As a numerical example, let  $\tau = 0.30$ ,  $r_1 = 0.02$ , and  $r_2 = 0.03$ . Then the required  $\Delta Y$  is

$$\begin{aligned} \Delta Y &\geq \frac{r_2 \Delta B_2}{\tau} = \frac{r_2(2 + r_1)0.25\Delta G}{\tau} = \frac{0.03(2 + 0.02)0.25\Delta G}{0.3} \\ &= \frac{2.02}{4 \cdot 10} \Delta G = 0.0505 \Delta G. \end{aligned}$$

So if the loss in “full employment” output from period 2 and onwards, incurred under *laissez faire*, is not smaller than about 5% of the fiscal stimulus,  $\Delta G$ , the expansionary fiscal policy “pays for itself” in the sense of not implying a worsening of the long-run fiscal stance.

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<sup>2</sup>It is possible that  $\Delta Y$  is more or less constant for a long time because two offsetting effects are operative. Because of technical progress the loss of output per lost worker is growing over time. On the other hand the pool of long-term unemployed generated by the slump will over time be a decreasing share of the labor force due to exits by the old and entrance by young people in the labor force.

Whether this is likely to hold in practice or not will depend on the concrete circumstances.

Obvious simple extensions of the analysis are:

### Extension 1

As a generalization of the above, suppose the slump in the world economy as well as in the SOE lasts  $n$  periods (a “period” need not be a year, but may for instance be a quarter of a year). Suppose further that:

- $r_t = \tilde{r}$ , for  $t = 1, 2, 3, \dots, n - 1$ ;
- both  $\partial Y/\partial G$  and  $T'(Y)$  in the SOE are approximately unchanged throughout the slump in the world economy.

Then, in analogy with (6) we can find the approximate effect of the expansionary fiscal policy on  $B_n$  in comparison with *laissez faire* in the following way. First, the effect on  $B_3$  is:

$$\begin{aligned}
\Delta B_3 &\approx (1 + \tilde{r})\Delta B_2 + (1 - T'(Y_2))\frac{\partial Y_2}{\partial G_2}\Delta G \\
&= (1 + \tilde{r}) \left[ (1 + \tilde{r})(1 - T'(Y_0))\frac{\partial Y_0}{\partial G_0}\Delta G + (1 - T'(Y_1))\frac{\partial Y_1}{\partial G_1}\Delta G \right] + (1 - T'(Y_2))\frac{\partial Y_2}{\partial G_2}\Delta G \\
&= (1 + \tilde{r}) \left[ (1 + \tilde{r})(1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G + (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G \right] + (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G \\
&= (1 + \tilde{r}) [(1 + \tilde{r}) + 1] (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G + (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G \\
&= [(1 + \tilde{r})^2 + 1 + \tilde{r} + 1] (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G.
\end{aligned}$$

By forward substitution, for any  $t = 1, 2, \dots, n$ , that is, as long as the recession lasts, we have

$$\begin{aligned}
\Delta B_t &\approx \sum_{i=0}^{t-1} (1 + \tilde{r})^i (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G = (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G \frac{1 - (1 + \tilde{r})^t}{1 - (1 + \tilde{r})} \\
&= (1 - T'(Y))\frac{\partial Y}{\partial G}\Delta G \frac{(1 + \tilde{r})^t - 1}{\tilde{r}},
\end{aligned}$$

from the sum formula for a finite geometric series. Consequently, the approximate extra future interest cost per period, caused by the higher debt at time  $n$  when the crisis is over

and the real interest rate is possibly at a higher level,  $\bar{r}$ , is

$$\bar{r}\Delta B_n \approx \bar{r}(1 - T'(Y)\frac{\partial Y}{\partial G})\Delta G \frac{(1 + \tilde{r})^n - 1}{\tilde{r}}.$$

This is the financial cost of the expansionary fiscal policy throughout the recession in comparison with *laissez faire*.

There is also a financial benefit, however, coming from fewer people ending up in long-term unemployment during the recession. This implies less de-qualification in the labor force than otherwise. Let us assume that: (i) the loss in “full employment” output from period  $n$  and onwards caused by the extra long-term unemployment that would result from *laissez-faire* policy equals  $\Delta Y$  per period, where  $\Delta Y$  is a positive constant; (ii) the ensuing loss in net tax revenue is  $\tau \cdot \Delta Y$  per period, where  $\tau$  is a positive constant (possibly close to  $\bar{r}$  from b)). Then the break-even value of  $\Delta Y$  is

$$\Delta Y = \frac{\bar{r}\Delta B_n}{\tau} = \frac{\bar{r}(1 - T'(Y)\frac{\partial Y}{\partial G})[(1 + \tilde{r})^n - 1]}{\tau\tilde{r}}\Delta G.$$

By inserting appropriate values for  $\bar{r}$ ,  $\tilde{r}$ ,  $\tau$ ,  $T'(Y)$ ,  $\frac{\partial Y}{\partial G}$ , and  $n$ ,  $\Delta Y/\Delta G$  can be calculated. It should be underlined that  $\Delta Y/\Delta G$  in this exercise does not stand for a conventional spending multiplier, but for the effect on supply, *after* the depression, of spending *during* the depression.

The longer duration of the economic slump and the (extra) budget deficit caused by the expansionary fiscal policy increases the debt-buildup (both directly and indirectly, through compound interest). Therefore the size of the break-even value of  $\Delta Y$  is an increasing function of  $n$ . On the other hand, the preventive effect of the expansionary fiscal policy vis-a-vis long-term unemployment also lasts longer.

## Extension 2

Suppose the slump in the world economy still lasts  $n$  periods and the slump in the SOE does the same under *laissez faire*, but may be shortened by expansionary fiscal policy through the following mechanism. Let aggregate demand depend on the “general level of confidence”, modeled as a shift parameter. Owing to the adverse shock in period  $-1$ , this parameter turned low. Precautionary saving resulted and firms postponed investment plans because of the slack in aggregate demand. But the shorter the time interval of mass unemployment, the sooner is confidence reestablished. This suggests the hypothesis that by shortening the length of the time interval with mass unemployment, the expansionary

fiscal policy (involving an appropriate size of  $\Delta G$ ) makes the depression last only  $\tilde{n} < n$  periods. From this perspective, the calculation in Extension 1 gives a conservative estimate of the benefits of the expansionary fiscal policy.

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