

**The Effects of Tax Competition  
When Politicians Create Rents  
to Buy Political Support**

**SUPPLEMENTARY APPENDIX**

by

Wolfgang Eggert and Peter Birch Sørensen

15th July 2007

Address for correspondence:

Peter Birch Sørensen

Department of Economics, University of Copenhagen

Studivstraede 6, 1455 Copenhagen K, Denmark

E-mail: [peter.birch.sorensen@econ.ku.dk](mailto:peter.birch.sorensen@econ.ku.dk)

**SUPPLEMENTARY APPENDIX TO PAPER ON  
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**1. Derivation of conditions for political equilibrium**

This section provides a detailed derivation of the political equilibrium conditions (3.1) and (3.2) stated in section 3.1 of the main text of the paper.

Conditions (3.1) and (3.2) describe the case where the recruitment constraint  $W \geq w$  is not strictly binding so that  $\eta = 0$ . In this case the first-order conditions (A.11) through (A.13) stated in Appendix 2 of the paper simplify to

$$[\alpha_i p_i + (\alpha - \alpha_i) p_o] u'_g - \alpha \lambda = 0, \quad (1)$$

$$\begin{aligned} & [\alpha_i p_i + (1 - \alpha_i) p_o] g' + p_o (u_g - u_p) - \lambda (\tau k + W) \\ & + \frac{k}{n(1 - \alpha) k'} [\bar{k} \alpha_i u'_g (p_i - p_o) + \bar{k} \alpha p_o (u'_g - u'_p) + \lambda \tau (1 - \alpha) k'] = 0, \end{aligned} \quad (2)$$

$$\begin{aligned} & \lambda (1 - \alpha) (k + \tau k') - (1 - \alpha) k p_o u'_p \\ & - \frac{1}{n} [\bar{k} \alpha_i u'_g (p_i - p_o) + \bar{k} \alpha p_o (u'_g - u'_p) + \lambda \tau (1 - \alpha) k'] = 0. \end{aligned} \quad (3)$$

Solving (1) for  $\lambda$  we get

$$\lambda = \left( \frac{\alpha_i p_i + (\alpha - \alpha_i) p_o}{\alpha} \right) u'_g. \quad (4)$$

Inserting (4) into (3) and using the fact that  $\bar{k} \equiv k(1 - \alpha)$ , we find

$$\begin{aligned} & \bar{k} \left( 1 + \frac{\tau k'}{k} \right) \left( \frac{\alpha_i p_i + (\alpha - \alpha_i) p_o}{\alpha} \right) u'_g - \bar{k} p_o u'_p \\ & - \frac{\bar{k}}{n} [\alpha_i u'_g (p_i - p_o) + \alpha p_o (u'_g - u'_p)] - \frac{\bar{k}}{n} \frac{\tau k'}{k} \left( \frac{\alpha_i p_i + (\alpha - \alpha_i) p_o}{\alpha} \right) u'_g = 0. \end{aligned} \quad (5)$$

Dividing through by  $p_o \bar{k}$  and using the definitions  $\delta \equiv \alpha_i (p_i - p_o) / p_o$  and  $\varepsilon \equiv -\left(\frac{n-1}{n}\right) \frac{\tau k'}{k}$ , we can rewrite (5) as

$$\begin{aligned}
(1 - \varepsilon) \left( \frac{\alpha + \delta}{\alpha} \right) u'_g - u'_p - \frac{1}{n} [\delta u'_g + \alpha (u'_g - u'_p)] &= 0 \iff \\
u'_g \left( \frac{\alpha + \delta}{\alpha} \right) \left( 1 - \varepsilon - \frac{\alpha}{n} \right) - \left( \frac{n - \alpha}{n} \right) u'_p &= 0 \iff \\
u'_g &= \left( \frac{\alpha}{\alpha + \delta} \right) \left( \frac{1 - \frac{\alpha}{n}}{1 - \frac{\alpha}{n} - \varepsilon} \right) u'_p, \tag{6}
\end{aligned}$$

which is seen to be identical to equation (3.1) in the text.

To derive equation (3.2) in the text, we start by rewriting (2) in the following way, again using  $\bar{k} \equiv k(1 - \alpha)$ :

$$\begin{aligned}
[\alpha_i p_i + (1 - \alpha_i) p_o] g' + p_o (u_g - u_p) - \lambda (\tau k + W) \\
+ \frac{\tau k}{n \frac{\tau k'}{k}} \left[ \alpha_i u'_g (p_i - p_o) + \alpha p_o (u'_g - u'_p) + \lambda \frac{\tau k'}{k} \right] &= 0. \tag{7}
\end{aligned}$$

From the government budget constraint (2.18) in the text it follows that

$$\tau k = \left( \frac{\alpha}{1 - \alpha} \right) W \implies \tau k + W = \frac{W}{1 - \alpha}. \tag{8}$$

Substituting (4) and (8) into (7), dividing through by  $p_o$ , and using the definitions  $\delta \equiv \alpha_i (p_i - p_o) / p_o$  and  $\varepsilon \equiv -\left(\frac{n-1}{n}\right) \frac{\tau k'}{k}$ , we get

$$\begin{aligned}
(1 + \delta) g' + (u_g - u_p) - \left( \frac{W}{1 - \alpha} \right) \left( \frac{\alpha + \delta}{\alpha} \right) u'_g \\
- \left( \frac{n - 1}{n} \right) \left( \frac{\alpha W}{n \varepsilon (1 - \alpha)} \right) \left[ \delta u'_g + \alpha (u'_g - u'_p) - \left( \frac{n \varepsilon}{n - 1} \right) \left( \frac{\alpha + \delta}{\alpha} \right) u'_g \right] &= 0 \iff \\
(1 + \delta) g' + (u_g - u_p) + \left( \frac{W}{1 - \alpha} \right) \left( \frac{\alpha + \delta}{\alpha} \right) \left( \frac{\alpha}{n} - 1 \right) u'_g \\
- \left( \frac{n - 1}{n} \right) \left( \frac{\alpha W}{n \varepsilon (1 - \alpha)} \right) [(\alpha + \delta) u'_g - \alpha u'_p] &= 0. \tag{9}
\end{aligned}$$

Using (6) to eliminate  $u'_p$  from (9), we obtain

$$\begin{aligned}
& (1 + \delta)g' + (u_g - u_p) + \left(\frac{W}{1 - \alpha}\right) \left(\frac{\alpha + \delta}{\alpha}\right) \left(\frac{\alpha}{n} - 1\right) u'_g \\
& - \left(\frac{n - 1}{n}\right) \left(\frac{\alpha W}{n\varepsilon(1 - \alpha)}\right) \left[ (\alpha + \delta)u'_g - (\alpha + \delta) \left(\frac{1 - \frac{\alpha}{n} - \varepsilon}{1 - \frac{\alpha}{n}}\right) u'_g \right] = 0 \iff \\
& (1 + \delta)g' + (u_g - u_p) = W \left(\frac{X}{1 - \alpha}\right) \left(\frac{\alpha + \delta}{\alpha}\right) u'_g, \tag{10} \\
& X \equiv 1 - \frac{\alpha}{n} + \left(1 - \frac{1}{n}\right) \left(\frac{\alpha^2}{n\varepsilon}\right) \left[1 - \left(\frac{1 - \frac{\alpha}{n} - \varepsilon}{1 - \frac{\alpha}{n}}\right)\right].
\end{aligned}$$

The expression for  $X$  may be rewritten as

$$\begin{aligned}
X &= 1 - \frac{\alpha}{n} + \left(1 - \frac{1}{n}\right) \left(\frac{\alpha^2}{n}\right) \left(\frac{1}{1 - \frac{\alpha}{n}}\right) \\
&= 1 - \frac{\alpha}{n} + \left(\alpha - \frac{\alpha}{n}\right) \left(\frac{\alpha}{n - \alpha}\right) \\
&= 1 + \frac{\alpha^2}{n - \alpha} - \frac{\alpha}{n} \left(1 + \frac{\alpha}{n - \alpha}\right) \\
&= 1 - (1 - \alpha) \left(\frac{\alpha}{n - \alpha}\right). \tag{11}
\end{aligned}$$

From (11) it follows that

$$\begin{aligned}
\frac{X}{1 - \alpha} &= \frac{1}{1 - \alpha} - \left(\frac{\alpha}{n - \alpha}\right) \\
&= \frac{n - \alpha - \alpha(1 - \alpha)}{(1 - \alpha)(n - \alpha)} \\
&= \frac{(1 - \alpha)(n - \alpha) + \alpha(n - \alpha) - \alpha(1 - \alpha)}{(1 - \alpha)(n - \alpha)} \\
&= 1 + \frac{\alpha(n - 1)}{(1 - \alpha)(n - \alpha)}. \tag{12}
\end{aligned}$$

Inserting (12) into (10), dividing through by  $(1 + \delta)u'_g$ , and using the fact that maximisation of profits implies  $w = F_L$ , we finally end up with

$$\frac{g'}{u'_g} + \frac{u_g - u_p}{u'_g(1 + \delta)} = \left(\frac{\alpha + \delta}{\alpha + \alpha\delta}\right) \left[1 + \frac{\alpha(n - 1)}{(1 - \alpha)(n - \alpha)}\right] \left(\frac{W}{w}\right) F_L, \tag{13}$$

which is seen to be identical to equation (3.2) in the text.

## 2. The simulation model

This section documents the model used to generate the simulation results reported in section 3.3.

Using the specifications in (3.10) and (3.11) to derive expressions for  $k(r + \tau)$ ,  $u_g - u_p$ ,  $u'_g$ ,  $u'_p$ , and  $g'(\alpha)$ , we obtain the following model describing the situation where the public sector recruitment constraint  $W \geq w$  is not strictly binding, i.e., the situation where public sector workers generally earn rents:

$$W + r\bar{k} - (w + r\bar{k}) \left\{ \frac{(\alpha + \delta) \left[ 1 - \frac{\alpha}{n} - \varepsilon \right]}{\alpha \left( 1 - \frac{\alpha}{n} \right)} \right\}^{1/\sigma_c} = 0 \quad (14)$$

$$\begin{aligned} \frac{\theta (W + r\bar{k})^{\sigma_c}}{\alpha^{\sigma_g}} + \left( \frac{1}{1 - \sigma_c} \right) \left[ W + r\bar{k} - (w + r\bar{k}) \left( \frac{W + r\bar{k}}{w + r\bar{k}} \right)^{\sigma_c} \right] \\ - \left( \frac{\alpha + \delta}{\alpha + \alpha\delta} \right) \left[ 1 + \frac{\alpha(n-1)}{(1-\alpha)(n-\alpha)} \right] W = 0 \end{aligned} \quad (15)$$

$$\varepsilon = \left( \frac{n-1}{n} \right) \left( \frac{t}{1-\beta} \right) \quad (16)$$

$$t = \frac{\tau}{r + \tau} \quad (17)$$

$$k = \left( \frac{\beta A}{r + \tau} \right)^{1/(1-\beta)} \quad (18)$$

$$w = (1 - \beta) A k^\beta \quad (19)$$

$$\tau(1 - \alpha)k - \alpha W = 0 \quad (20)$$

$$(1 - \alpha)k - \bar{k} = 0 \quad (21)$$

$$SW = \frac{\alpha (W + r\bar{k})^{1-\sigma_c}}{1 - \sigma_c} + \frac{(1 - \alpha) (w + r\bar{k})^{1-\sigma_c}}{1 - \sigma_c} + \frac{\theta \alpha^{1-\sigma_g}}{1 - \sigma_g} \quad (22)$$

Equations (14) and (15) correspond to the political equilibrium conditions (3.1) and (3.2), while (18) and (19) are the capital demand function and the private sector wage rate implied by profit maximisation, respectively. (20) is the government budget constraint, and (21) is the international capital market equilibrium condition in a setting with symmetric countries. Equation (16) gives the tax base

elasticity implied by the Cobb-Douglas production function, and the auxiliary variable  $t$  in (17) is the effective capital income tax rate. The final equation (22) calculates the level of social welfare. The nine equations (14) through (22) determine the nine endogenous variables  $W$ ,  $w$ ,  $r$ ,  $\alpha$ ,  $\tau$ ,  $k$ ,  $\varepsilon$ ,  $t$  and  $SW$ , given the values of the parameters  $\bar{k}$ ,  $A$ ,  $\delta$ ,  $\sigma_c$ ,  $\sigma_g$ ,  $\theta$ ,  $\beta$  and  $n$ .

As indicated, the model above is valid only as long as the public sector recruitment constraint is not binding. When this constraint becomes binding, equation (14) must be replaced by the condition  $W = w$ , and (15) simplifies to

$$\frac{\theta (W + r\bar{k})^{\sigma_c}}{\alpha^{\sigma_g}} - \left( \frac{\alpha + \delta}{\alpha + \alpha\delta} \right) \left[ 1 + \frac{\alpha(n-1)}{(1-\alpha)(n-\alpha)} \right] w = 0 \quad (23)$$

In the case of tax competition among small jurisdictions ( $n \rightarrow \infty$ ) equation (23) may be written as

$$\frac{\theta (W + r\bar{k})^{\sigma_c}}{\alpha^{\sigma_g}} - \left( \frac{1}{1-\alpha} \right) \left( \frac{1}{1-\varepsilon(1-\alpha)} \right) w = 0 \quad (24)$$

which is just a version of (3.4) in the text.

To compute the full solution to the above non-linear system, we used ConOpt 3.0 in GAMS and checked robustness using MINOS5.