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Optimal tax progressivity in imperfect labour markets

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Abstract

All modern labour market theories capable of explaining involuntary unemployment as an equilibrium phenomenon imply that increased income tax progressivity reduces unemployment, but they also imply that higher progressivity tends to reduce work effort and labour productivity. This suggests that there may be an optimal degree of tax progressivity where the marginal welfare gain from reduced involuntary unemployment is just offset by the marginal welfare loss from lower productivity. This paper sets up four different models of an imperfect labour market in order to identify the degree of tax progressivity which would maximise the welfare of the representative wage earner. Simulations with these models suggest that the optimal degree of tax progressivity could be quite large, although the results are sensitive to the generosity of unemployment benefits and to the after-tax wage elasticity of work effort. © 1999 Elsevier Science B.V. All rights reserved.

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1. The costs and benefits of labour tax progressivity

Recent tax reforms in the OECD area have tended to reduce the progressivity of the labour income tax. This policy has typically been defended as a means of

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stimulating employment. In the standard textbook model of a competitive labour market, a cut in the marginal tax rate will indeed encourage labour supply by inducing substitution away from leisure, assuming that the average tax rate is kept constant so as to eliminate any income effect on the number of hours worked. Yet it is somewhat ironic that the move towards less tax progressivity took place at the same time as advances in labour economics made clear that reduced progressivity may actually be harmful to employment in imperfect labour markets. [1]

For example, in union bargaining models where unions trade off higher real net wages against higher employment of their members, a rise in the marginal tax rate will ceteris paribus stimulate employment by making it less costly for the union to 'buy' more jobs through wage moderation, since a given fall in the pre-tax wage rate will now lead to a smaller fall in the after-tax wage.

Stronger tax progressivity also generates higher employment in efficiency wage models where employers pay wages above the market-clearing level as a means of inducing higher productivity of their workers. The reason is that a rise in the marginal tax rate reduces the effectiveness of a high (pre-tax) wage rate as an instrument for encouraging high labour productivity, assuming that workers care about after-tax rather than pre-tax wages. As we shall see below, the standard job search model of the labour market likewise implies that higher tax progressivity will promote employment by lowering equilibrium wage rates.

The prediction that tax progressivity encourages wage moderation in imperfect labour markets has recently been tested by a number of authors including Malcomson and Sartor (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995), Lockwood et al. (1995), Tyrväinen (1995), Graafland and Huizinga (1996), Hansen et al. (1996), Nymoen and Rødseth (1996), Wulfsberg (1996) and Aronsson et al. (1997). Covering different countries and different time periods, all of these econometric studies find that an increase in the marginal tax rate tends to reduce the pre-tax real wage, for a given average tax rate.

While modern labour market theories suggest that tax progressivity may be good for employment, they do *not* imply that progressivity is a free lunch. All of the models referred to above imply that there is some efficiency cost of raising the marginal tax rate on labour income. In union models with endogenous work hours, a higher marginal tax rate induces the union to set shorter working hours, thus distorting the labour-leisure choice in the same way as taxes distort individual labour supply decisions in the competitive model. In efficiency wage models a higher marginal tax rate reduces labour productivity by lowering the employer's optimal efficiency wage relative to the rate of unemployment benefit, and in search models a rise in the marginal tax rate may lower the efficiency of the job matching process by reducing workers' expected marginal return to job search or

[1] The effects of taxation in modern labour market models are surveyed in Bovenberg and van der Ploeg (1994), Sørensen (1997), and Pissarides (1998).

by generating so much labour market tightness that the filling of vacant jobs becomes excessively expensive for employers. Yet there is one crucial difference between the standard competitive model and modern labour market models of equilibrium unemployment: in the competitive set-up, a move from proportional to progressive taxation will *always* reduce efficiency by generating more substitution towards leisure activities, assuming an unchanged average tax rate. By contrast, the modern theories of imperfect labour markets suggest that some amount of tax progressivity may actually *promote* economic efficiency by helping to reduce the amount of involuntary unemployment. Accordingly, there may be an 'optimal' degree of tax progressivity where the various distortionary effects of a rise in the marginal tax rate are just offset by the gain from reduced unemployment.

The present paper illustrates this point by developing and simulating four simple models of imperfect labour markets. It invites the reader to conduct the following thought experiment: assuming that a group of identical citizens facing the same risk of unemployment were to agree on a labour income tax schedule behind a Rawlsian 'veil of ignorance', before they know whether they will end up being employed or going unemployed, how much progressivity would they build into this tax schedule? Furthermore, how much would they gain in expected utility by introducing the optimal degree of tax progressivity rather than adopting a purely proportional income tax? The simulations reported below suggest that the most popular theories of involuntary unemployment may justify a considerable amount of tax progressivity, and that progressivity may yield non-negligible welfare gains once it is recognized that the pre-tax labour market equilibrium is distorted for non-tax reasons.

I deliberately abstract from heterogeneous earnings capacities in order to highlight the pure efficiency case for tax progressivity in imperfect labour markets. Of course, a complete analysis of the optimal degree of tax progressivity would also allow for earnings inequalities and for society's concern about equity as well as efficiency. Hence, my analysis should certainly not be taken too literally for policy purposes. However, following the pioneering contribution by Ramsey (1927), the public finance literature has a long tradition of studying the pure efficiency aspects of indirect taxation by considering optimal commodity taxation in an economy inhabited by a single representative consumer [see Sandmo (1976) for a survey]. The present paper may be seen as a parallel to this literature, focusing on the pure efficiency aspects of direct rather than indirect taxation.

The paper is inspired by Holmlund and Kolm (1995) and by the comments on that paper by Calmfors (1995). In their extensive study of the effects of progressive taxation on wage setting in Sweden, Holmlund and Kolm (op. cit.) briefly analyse optimal tax progressivity in the context of a union bargaining model where employers have the 'right to manage'. The present paper extends their analysis by also considering tax progressivity in an efficient bargaining model, in an efficiency wage model and in a search model, by allowing for the general equilibrium effect of unemployment on the average tax rate via the government budget constraint, and by offering a new method for quantifying the welfare gains from tax progressivity.

In the sections below, I set up simple versions of the union bargaining model, the efficiency wage model, and the job search model with the purpose of analysing the employment and welfare effects of labour income taxation at alternative levels of unemployment compensation. These models are subsequently calibrated and simulated in order to estimate the optimal degree of labour tax progressivity and the welfare gain from progressivity.

2. General assumptions

The models abstract from saving and investment and assume that the total consumption of an employed worker equals his after-tax wage income $Wh^{o} - T(Wh^{o})$, where W is the pre-tax real wage rate, h^{o} is official working hours, and $T()$ is the total tax liability which depends on recorded labour income Wh^{o}. The utility of an employed worker U^{e} varies positively with total consumption and negatively with the actual effort h exerted on the job:

$$U^{e} = Wh^{o} - T(Wh^{o}) - \frac{h^{1+\delta}}{1+\delta}, \quad \delta > 0 \tag{1}$$

In the bargaining model actual working hours h coincide with the official working hours h^{o} determined in the bargain between the union and the employer. In the efficiency wage model h^{o} is institutionally fixed at unity, but the actual number of hours worked may fall below this level because perfect monitoring of actual work effort is infeasible. The search model assumes that actual as well as official working hours are fixed at unity, so the last term on the RHS of (1) is simply left out in that model.

The flow utility of an unemployed worker U^{u} is equal to the after-tax rate of unemployment benefit which is given by $c[Wh^{o} - T(Wh^{o})]$, where the exogenous policy parameter $c < 1$ is the net replacement ratio embodied in the system of unemployment compensation.

Worker utility is thus taken to be linear in income, implying risk neutrality. This assumption is made not only for analytical convenience, but also because introduction of risk aversion might bias the analysis in favour of tax progressivity, given that the model does not incorporate a capital market allowing unemployed workers to engage in consumption smoothing. [2]

The total consumption of the representative worker is a CES aggregate of n different consumption goods, with a constant substitution elasticity $\sigma > 1$ between

[2] Since tax progressivity reduces equilibrium unemployment, risk averse workers will tend to favour more progressivity when they cannot smooth consumption during periods of unemployment.

any two goods. When workers allocate their total consumption across the different goods so as to maximise utility, the monopolistically competitive producer of each individual good will be faced with a downward-sloping demand curve with a numerical price elasticity equal to σ, and a profit-maximising firm will charge a price which is a mark-up m over the marginal cost of production, where the mark-up factor equals $m = \sigma/(\sigma - 1)$. [3] In a symmetric general equilibrium, all firms charge the same price which I normalize at unity. The output Q of the representative firm is given by the following Cobb–Douglas production function where the capital stock is implicitly taken to be fixed, where L measures effective labour input in hours, and where N is the number of persons employed:

$$Q = L^{\alpha}, \quad L = hN, \quad 0 < \alpha < 1 \tag{2}$$

By choice of units, the total labour force is set equal to one so that $N = 1 - u$, with u denoting the rate of unemployment.

Following Holmlund and Kolm (1995, p. 439), I assume an income tax schedule satisfying $Wh^{o} - T(Wh^{o}) = k(Wh^{o})^{\upsilon}$, k and υ being positive constants. It is easy to show that $\upsilon = (1 - t^{m})/(1 - t^{a})$, where $t^{m} = T'$ is the marginal tax rate, and $t^{a} = T/Wh^{o}$ is the average tax rate. The parameter υ is the so-called Coefficient of Residual Income Progression (CRIP). Originally introduced by Musgrave and Thin (1948), the CRIP parameter υ measures the elasticity of after-tax income with respect to pre-tax income. Under a purely proportional tax system where $t^{m} = t^{a}$, we have $\upsilon = 1$, whereas a progressive tax system $(t^{m} > t^{a})$ involves a value of υ below unity. As demonstrated by Jakobsson (1976), the CRIP measure of tax progressivity has some attractive theoretical properties. First of all, a tax schedule with a lower value of υ is unambiguously more redistributive in the sense of implying a Lorenz curve for the distribution of after-tax income which is everywhere inside the Lorenz curve generated by a tax schedule with a higher value of υ. Second, a tax schedule with a constant value of υ implies that an equal proportionate rise in all pre-tax incomes—which will not affect (the Lorenz curve for) the distribution of incomes before tax—will also leave the (Lorenz curve for) the distribution of after-tax income unaffected.

The purpose of the analysis below is to identify the value of υ which will maximise the expected utility of the representative worker in the various models of imperfect labour markets. [4]

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[3] These results follow from the standard set-up with monopolistic competition originally introduced by Dixit and Stiglitz (1977).

[4] As one referee pointed out to me, the differentiable tax schedule I consider is not necessarily the optimal schedule, since a tax schedule with a 'kink' at some appropriate income level (implying that υ is not held constant at all income levels) might produce a superior allocation. I chose to work with the analytically convenient tax schedule specified above partly because of the theoretical justifications for this schedule given by Jakobsson (1976), and partly to facilitate comparison of my estimates of optimal tax progressivity to those offered by Holmlund and Kolm (1995).

3. The union bargaining model

Drawing on the assumptions above, this section sets up a simple union bargaining model inspired by Lockwood and Manning (1993), Bovenberg and van der Ploeg (1994), Holmlund and Kolm (1995), and Koskela and Vilmunen (1996). The economy is divided into a large number of production sectors, each having its own labour union engaging in bargaining with the representative firm in the sector. If a worker fails to find a job in the sector organized by his current union, he will look for employment outside the sector, expecting to enjoy the utility level

$$U^{\mathrm{a}} = (1-u)\overline{U}^{\mathrm{e}} + uU^{\mathrm{u}} \tag{3}$$

with $\overline{U}^{\mathrm{e}}$ denoting the utility level of workers employed in other sectors, and the employment rate $1-u$ measuring the probability of finding a job outside the current sector. The utility Ω of the representative utilitarian union equals the aggregate rents from employment obtained by its members, i.e., $\Omega = (U^{\mathrm{e}} - U^{\mathrm{a}})N$, where U^{e} is given by (1) with $h = h^{\mathrm{o}}$.

The objective of the representative monopolistically competitive firm is its level of real profits, Π. With threat points of zero, the union and the firm engage in Nash bargaining striving to

$$\text{Maximise } \lambda \log\left[(U^{\mathrm{e}} - U^{\mathrm{a}})N\right] + (1-\lambda)\log \Pi \tag{4}$$

The parameter λ measures the bargaining strength of the union, and U^{a} is taken as exogenously given in the decentralised bargaining process.

If the firm has the 'right to manage', it unilaterally sets the level of employment N so as to maximise profits, given the wage rate W and the working time h^{o} negotiated with the union. However, it is well known that the right-to-manage institution leaves the two bargaining parties with unexploited gains from trade (see, e.g., McDonald and Solow, 1981). I therefore also consider the case of efficient bargaining where the union and the firm bargain over the total amount of labour input L as well as over W and h^{o}. [5]

Solving the maximisation problem in (4) using (1) through (3), assuming profit-maximising mark-up pricing by firms, and imposing the symmetry condition that all firms pay the same wages and charge the same prices in equilibrium, one may derive the following equations for the equilibrium rates of unemployment,

[5] In fact, since workers and hours are perfect substitutes in the firm's production function, the firm cares only about the total labour input L and delegates the decision on individual working hours to the union in both bargaining scenarios.

work hours and wages, where the parameter k indicates the relative bargaining power of the union:

$$u = \frac{\upsilon k(1+\delta)(m-\alpha)}{(\alpha+mk)(1+\delta)(1-c) - \alpha\upsilon(1+k)}, \quad k \equiv \lambda/(1-\lambda) \tag{5}$$

$$h^{\delta} = \left(\frac{\alpha\upsilon(1+k)}{\alpha+mk}\right) W(1-t^{\mathrm{a}}) \tag{6}$$

Right-to-manage:

$$W = \frac{\alpha}{m}[h(1-u)]^{\alpha-1} \tag{7}$$

Efficient bargaining:

$$W = \left(\frac{\alpha+mk}{m(1+k)}\right)[h(1-u)]^{\alpha-1} \tag{8}$$

It is easy to show from (5) that a higher net replacement rate c increases equilibrium unemployment, whereas a more progressive tax system (a lower value of υ) reduces it.

The number of working hours h for an employed union member is given by (6). The left-hand side is the money metric marginal disutility of work. In a competitive laissez-faire economy this magnitude would be equal to labour's marginal product W, but with monopolistic practices in labour and product markets ($k > 0$ and $m > 1$) as well as taxation, we see that working hours are distorted. Thus, a union concerned about the employment of its members will restrict working hours below the level which the individual employed member would have preferred at the going (net) wage rate, and which is given by $h^{\delta} = \upsilon W(1-t^{\mathrm{a}}) = W(1-t^{\mathrm{m}})$. According to (6) the marginal tax rate drives an additional wedge between the marginal disutility of work and the marginal productivity of labour. These observations make clear that more tax progressivity (a rise in the marginal tax rate) does have a first-order efficiency cost by causing a further distortion of labour supply.

Eq. (8) reproduces the well-known result that efficient bargaining will drive the real wage above labour's marginal revenue product which is given by $(\alpha/m)[h(1-u)]^{\alpha-1}$.

In simple union models with exogenous individual work hours an increase in tax progressivity will always reduce the pre-tax real wage rate. However, it can be shown from (5) through (8) that this result will not necessarily hold in the present model with endogenous working hours. On the one hand, a rise in the marginal tax rate tends to reduce union wage claims by making it less costly for the union to 'buy' more jobs through wage moderation. On the other hand, the higher marginal tax rate also reduces the optimal number of individual work hours, and this works in favour of higher wage claims by making the union less interested in 'selling' work hours to employers. When the marginal tax rate becomes sufficiently high, the latter effect will dominate, as previously pointed out by Calmfors (1995, pp. 464–465), but higher progressivity will nevertheless reduce unemployment by inducing unions to set shorter individual working hours.

4. The efficiency wage model

In union models involuntary unemployment arises from the 'excessive' wage claims of unions exploiting their monopolistic control of labour supply. Efficiency wage models abstract from the existence of unions, focusing instead on the incentives for employers to set wages above the market-clearing level in order to induce higher labour productivity. This section develops a 'shirking' model of efficiency wage setting, combining ideas from Shapiro and Stiglitz (1984) and Pisauro (1991). Following Shapiro and Stiglitz, I assume that employers cannot perfectly monitor the work effort of their employees. Workers therefore have an incentive to 'shirk' in the workplace, but employers may offset this incentive by paying high wages, thereby imposing a utility loss on a worker who is fired in case he is caught shirking.

Unlike Shapiro and Stiglitz, but following Pisauro (op. cit.), I assume that the amount of effort exerted in the workplace is a continuous variable. Statutory work hours are fixed at unity, but the worker may vary the actual amount of effort (h) exerted on the job. The individual worker faces an increasing risk of being fired on grounds of dissatisfactory performance the smaller his work effort relative to some 'normal' effort level h^*. Specifically, the probability s that a worker will be fired during any time period is given by

$$s = \bar{s} + \eta\left(\frac{h^* - h}{h^*}\right), \quad \bar{s} > 0, \quad 0 < \eta < 1 \tag{9}$$

where $\bar{s}$ is an exogenous job destruction rate, η is the exogenous probability that the worker will be monitored, and $(h^* - h)/h^*$ is the degree to which his performance falls short of the average work norm.

Let J^{e} denote the discounted utility value of being employed, and let J^{u} indicate the discounted value of being unemployed. The present value of being employed equals the discounted value of the 'return' to employment which consists of the instantaneous flow utility of an employed person (U^{e}) minus the expected 'capital loss' $s(J^{\mathrm{e}} - J^{\mathrm{u}})$ arising from the instantaneous probability s that the worker is fired. Assuming an exogenous discount rate ρ and a steady state in which all variables are constant over time, we thus have

$$J^{\mathrm{e}} = \frac{U^{\mathrm{e}} - s(J^{\mathrm{e}} - J^{\mathrm{u}})}{\rho} \tag{10}$$

By analogy, the value of being unemployed equals the discounted value of the 'return' to unemployment which is made up of the instantaneous flow utility of an unemployed individual $U^{u} = cW(1-t^{a})$ plus the expected capital gain $a(J^{e}-J^{u})$ stemming from the instantaneous probability a that an unemployed person will find a job:

$$J^{u} = \frac{U^{u} + a(J^{e}-J^{u})}{\rho} \tag{11}$$

The value of unemployment J^{u} is exogenous to the individual employed worker, being determined by general labour market conditions, and the wage rate included in U^{e} is set by the employer. Given the wage rate and the utility function (1) with $h^{o}=1$, an employed worker chooses his level of effort so as to maximise the value of employment specified in (10), subject to (9). The solution to this problem implies that the representative worker's effort is an increasing function of the marginal after-tax wage rate $W(1-t^{m})$. Taking this relationship into account, the representative firm chooses its wage rate and level of employment so as to maximise profits. In a symmetric steady state equilibrium we have $h^{*}=h$ and $u=s/(s+a)$. Using these relationships, the solution to the problems of the representative worker and the representative firm can be shown to imply the following expressions for the equilibrium levels of unemployment, effort, and wages:

$$u = \frac{sv(1+\delta)}{\delta(1+\delta)(\rho+s)(1-c)-\rho v(1+\delta)-\eta v} \tag{12}$$

$$h = \left(\frac{\eta v W(1-t^{a})}{\delta(\rho+s)}\right)^{\frac{1}{1+\delta}} \tag{13}$$

$$W = \frac{\alpha}{m} h^{\alpha}(1-u)^{\alpha-1} \tag{14}$$

Notice that once one imposes the symmetry condition $h=h^{*}$, it follows from (9) that $s=\bar{s}-\eta$. The variable s in (12) and (13) can therefore be treated as exogenous.

Once again, it follows from (12) that higher tax progressivity reduces unemployment whereas a higher net replacement ratio increases it. Eq. (13) implies that increased tax progressivity will induce lower work effort. A higher marginal tax rate makes it profitable for employers to lower their wage rates, because the ensuing fall in the after-tax wage and the concomitant fall in work effort will now be lower than before so that the cost saving from lower wage rates exceeds the cost increase arising from lower productivity. As a result, the level of involuntary unemployment falls, but the price to be paid is a lower level of productivity.

5. The search model

Models of search unemployment come in many varieties such as those presented in Diamond (1982), Pissarides (1985, 1990), Mortensen (1986), Howitt (1988), Blanchard and Diamond (1989), and Ljungqvist and Sargent (1995). The search model set up below is essentially the popular model developed by Pissarides (1990, chap. 1; 1996), although in Section 6 I extend the Pissarides model by incorporating the government budget constraint to allow for general equilibrium effects of unemployment on the average tax rate.

The key assumption in search models is that imperfect information forces unemployed job seekers and firms with vacant jobs to go through a costly and time-consuming search process before they can be matched. In the basic Pissarides model, the per period number of matches M between unemployed workers U and firms with vacancies V is described by the matching technology

$$M = \mu U^{\epsilon} V^{1-\epsilon} \Leftrightarrow q = \mu \theta^{-\epsilon}, \quad q \equiv M/V, \quad \theta \equiv V/U, \quad \mu > 0, \quad 0 < \epsilon < 1 \tag{15}$$

where q is the probability that a vacant job will be filled during each period, and θ is a measure of labour market tightness. With working hours fixed at unity, the output of the representative firm is $Q = N^{\alpha}$ so a worker's marginal product is $\alpha N^{\alpha-1}$. Given an exogenous job separation rate s and assuming a steady state, the representative firm's present value of an additional occupied job (J^{o}) is

$$J^{\mathrm{o}} = \frac{\alpha N^{\alpha-1} - W - s(J^{\mathrm{o}} - J^{\mathrm{v}})}{\rho} \tag{16}$$

where J^{v} is the present value of a vacant job. According to (16), the present value of an extra occupied job equals the discounted value of the additional profit $\alpha N^{\alpha-1} - W$ produced by an extra worker minus the expected capital loss $s(J^{\mathrm{o}} - J^{\mathrm{v}})$ stemming from the instantaneous probability s that the job will be destroyed. Assuming a fixed cost γ of keeping a job open, the present value of a vacant job is determined by

$$J^{\mathrm{v}} = \frac{-\gamma + q(J^{\mathrm{o}} - J^{\mathrm{v}})}{\rho} \tag{17}$$

where $q(J^{\mathrm{o}} - J^{\mathrm{v}})$ is the expected capital gain per period arising from the probability q that a vacant job will be filled.

The rent to a firm from a job match is $J^{\mathrm{o}} - J^{\mathrm{v}}$, and a job seeker's rent from a match is $J^{\mathrm{e}} - J^{\mathrm{u}}$ which in turn is given by (10) and (11). When a firm and an unemployed job seeker have been matched, they negotiate the wage rate in order to maximise the Nash bargaining product of these rents, i.e., they solve the problem

$$\text{Maximise } \lambda \log(J^{\mathrm{e}} - J^{\mathrm{u}}) + (1-\lambda)\log(J^{\mathrm{o}} - J^{\mathrm{v}}) \quad \text{w.r.t. W} \tag{18}$$

taking the ‘outside options’ J^{u} and J^{v} as exogenously given by general labour market conditions.

In long run equilibrium, free entry of firms implies $J^{v}=0$ and a steady state requires $u=s/(s+a)$, where $a=M/U=(M/V)/(V/U)=\mu\theta^{1-\varepsilon}$ according to (15). Solving the problem in (18), using the equilibrium relationships just mentioned, and imposing symmetry between wages inside and outside the representative firm, one finds the following relationships where (20) is a long run labour demand curve and (21) is a wage curve:

$$u=\frac{s}{s+\mu\theta^{1-\epsilon}} \tag{19}$$

$$W=(\alpha/m)(1-u)^{\alpha-1}-(\gamma/\mu)(\rho+s)\theta^{\epsilon} \tag{20}$$

$$W=\frac{\gamma \upsilon k[(\rho+s)\theta^{\epsilon}+\mu\theta]}{\mu(1-c)} \tag{21}$$

By comparative static analysis of this system one may confirm our previous finding that higher tax progressivity reduces unemployment. The reason is that a higher marginal tax rate on labour income raises the cost to the employer of providing the worker with some given increase in the after-tax wage rate, and at the same time it reduces the cost to the worker of conceding more profit to the employer by accepting a lower pre-tax wage rate. Given that wage negotiators strive to maximise some weighted average of the rents accruing to the two parties (cf. 18), it then becomes optimal to shift more of the total rents towards the employer via a fall in the wage rate.

One important implication of the search model is that equilibrium unemployment is not necessarily too high from a social welfare perspective. As we have seen, the level of employment and unemployment adjusts via entry and exit of firms offering vacant jobs. When a new firm enters the market offering a vacancy, the resulting increase in labour market tightness generates a positive external effect on workers, making it (slightly) easier for unemployed workers to find a job and increasing their bargaining power when they do. At the same time, the new entrant also imposes a negative externality on existing firms by making it (slightly) more difficult for them to fill their vacancies, and by reducing their bargaining power when they have found a match. It is not obvious whether the positive or the negative externality from additional entry of firms (leading to lower unemployment) will dominate. Indeed, when the matching function is homogeneous of degree one, as we assume in (15), and when the discount rate approaches zero, it can be shown that the level of unemployment generated by the market will be socially optimal (i.e., the positive and negative externalities from the entry of another firm will just cancel out) when the parameter ε in (15) equals $1/2$ (Pissarides, 1990, chap. 7). However, this result was derived for an economy without public unemployment compensation. In the presence of (generous) unemployment benefits, there is a presumption that equilibrium unemployment will tend to be inefficiently high, and indeed the simulations reported below do suggest that this is the case when plausible parameter values are plugged into the search model.

6. Government

The models developed above determine a labour market equilibrium for given values of the policy variables t^a, υ, and c. However, since the level of wages, employment and unemployment affect the tax base as well as government spending on unemployment benefits, the average tax rate t^a becomes endogenous in general equilibrium, due to the government budget constraint. In all of the models the government budget constraint takes the form

Wage tax revenue + 100% profits tax
= net expenditure on unemployment benefits
+ exogenous government expenditure

Thus the government is assumed to tax away all pure profits, allowing a fall in the average tax rate on labour income when profits go up. This is an analytically convenient way of accounting for the income effect of changes in profits, and it is essentially equivalent to assuming that workers receive dividends from firms in proportion to their labour and benefit income. With homogeneous agents, this seems to be the most natural assumption to make.

The government budget constraint given above endogenously determines the average tax rate. In the simulations, the exogenous component of government expenditure has been calibrated so as to imply an average effective tax rate of 55% in the benchmark equilibrium with proportional taxation and an equilibrium unemployment rate of 10%. An average tax rate of 55% may seem very high, but in a West European context it is in fact quite realistic, once it is recognized that indirect consumption taxes serve as indirect taxes on labour, since consumption taxes erode the real purchasing power of nominal wages (see Sørensen, 1997, Table 1).

7. Measuring the welfare effects of tax progressivity

Starting out from a purely proportional tax system ($\upsilon = 1$), the welfare gain from tax progressivity may be measured by the ex ante compensating variation, defined as the maximum amount of income which the representative worker would be willing to pay ex ante to move from a proportional tax system to a tax system characterized by the degree of progressivity υ.[6] For example, in the efficiency

[6] This welfare measure is in the spirit of Helms (1985). I am grateful to Johan Stennek for drawing my attention to Helm's work on welfare measurement in the presence of uncertainty.

wage model or in the search model where the worker's discounted utility is determined by (10) and (11), the ex ante compensating variation CV(v) associated with the move from proportional taxation to a tax system involving a degree of tax progressivity v is found from the equation

$$[1-u(v)]J^{e}(v)+u(v)J^{u}(v)-(1/\rho)CV(v)=[1-u(1)]J^{e}(1)+u(1)J^{u}(1) \qquad (22)$$

The term on the RHS of (22) is the expected lifetime utility of the representative worker under proportional taxation where $v=1$. Eq. (22) thus determines the amount of income CV(v) which could be taken away from the worker under the tax system with progressivity v without reducing his expected utility below the level of expected utility obtainable under a proportional tax system. The optimal degree of tax progressivity is defined as the value of v which maximises CV(v).

8. Simulation results

Table 1 characterizes the optimal tax policy in the different labour market models, given the parameter values reported in the note to the table. The assumed values of α and m imply that the numerical wage elasticity of labour demand is $m/(m-\alpha)=1.5$. This elasticity captures substitution between labour and the (fixed) capital stock as well as the fall in labour demand arising from the negative output response to higher labour costs. [7] According to (6), the magnitude $1/\delta$ measures the net wage elasticity of individual work hours in the union model. Most empirical studies find this elasticity to be rather low, with many estimates centering around 0.1. [8] Accordingly, I have set $\delta=10$. In the search model I have set $\mu=3.27$ and $\varepsilon=0.66$, based on empirical estimates of the matching function (15) in the Danish study by Albæk and Hansen (1995). I also adopted the conventional assumption that $k=1$ (equal bargaining power of the worker and the firm) in the search model. The value of s was set equal to the average firing rate in the Danish labour market over the period 1980–1991 (Albæk and Sørensen, 1995). The subjective utility discount rate ρ had to be fixed at the fairly high level of 0.25 to satisfy the theoretical constraint $0<\eta<1$ on the monitoring probability η in the efficiency wage model. In all models and scenarios, the remaining unobservable parameters (k in the union bargaining model, η in the efficiency wage model, and γ in the search model) were calibrated so as to generate 10% unemployment in the benchmark equilibrium with proportional taxation. Eqs. (7) and (8) make clear that the negotiated wage rates in the right-to-manage model

[7] Symons and Layard (1983) estimated this elasticity for six large OECD countries. Their estimates ranged from 0.4 to 2.6, with four of the six countries having elasticities greater than one.

[8] The recent Danish study by Smith (1995) is quite typical in this respect.

Table 1
Optimal tax progressivity

Net replacement rate (c)	Bargaining model			Efficiency wage model			Search model		
	v^*	u	CV	v^*	u	CV	v^*	u	CV
0.5	0.74	0.070	0.047	0.80	0.078	0.026	0.52	0.078	0.030
0.6	0.72	0.067	0.057	0.78	0.074	0.035	0.41	0.072	0.048
0.7	0.70	0.062	0.074	0.75	0.068	0.054	0.31	0.065	0.074

v^* = optimal degree of tax progressivity, as measured by CRIP. CV = welfare gain from the optimal degree of tax progressivity, measured as a fraction of private consumption.
Calibration: $\alpha = 0.7$, $m = 2.1$, $\delta = 10$, $s = 0.12$, $\rho = 0.25$, $\varepsilon = 0.66$. The unobservable parameters have been calibrated so as to generate $u = 0.1$ and $t^a = 0.55$ for $v = 1$ and $c = 0.6$.

and in the efficient-bargaining model will differ significantly only if k is substantially above zero. The calibration procedure just described generated a rather low value of $k = 0.016$ which implied only negligible differences in the quantitative predictions of the two versions of the union bargaining model. Hence, the rounded figures in the first three columns of Table 1 are valid for both variants of this model.

Assuming three different values of the net replacement rate, Table 1 indicates the optimal degree of tax progressivity, measured by the CRIP v. The table also shows the concomitant second-best optimal level of unemployment as well as the estimated maximum welfare gain from tax progressivity, expressed as a fraction of total private consumption. Since the optimal values of v are well below unity, the optimal tax system seems to involve a substantial degree of progressivity. Recalling that the benchmark equilibrium unemployment rate is 10% under proportional taxation, we see that the move from proportional to optimal progressive taxation reduces unemployment quite considerably in all three models. The ensuing reduction in public spending on unemployment benefits paves the way for a fall in the average tax rate which combines with lower unemployment risks to produce the welfare gains from tax progressivity reported in the table. By normal standards, these welfare gains are large, ranging from about 3% to about 7% of private consumption. Table 1 makes clear that the optimal tax policy is sensitive to the generosity of unemployment benefits. A higher net replacement rate tends by itself to generate higher unemployment, and hence there is a need for more tax progressivity to offset this effect.

The simple labour market models considered here can be criticised for failing to account for some important distortions caused by high marginal tax rates. Indeed, each model focuses only on one particular tax distortion. The union bargaining model incorporates the labour-leisure distortion, while the efficiency wage model illustrates the tendency of tax progressivity to discourage work effort on the job, and the search model highlights how the labour market tightness generated by tax progressivity may impair the efficiency of the job matching process. Yet there are

Table 2
Sensitivity of optimal tax progressivity to the compensated net wage elasticity of effort

Elasticity of marginal disutility of effort with respect to effort (δ)	Bargaining model[a]			Efficiency wage model		
	Compensated net wage elasticity of effort	Optimal value of υ	Welfare gain from tax progressivity[b] (CV)	Compensated net wage elasticity of effort	Optimal value of υ	Welfare gain from tax progressivity[b] (CV)
10	0.1	0.72	0.057	0.009	0.78	0.035
8	0.13	0.81	0.034	0.11	0.86	0.020
6	0.17	0.91	0.013	0.14	0.93	0.011
4	0.25	0.99	0.001	0.20	0.94	0.057

[a] Right-to-manage version.
[b] Measured by the ex ante compensating variation, expressed as a fraction of private consumption under a purely proportional tax system.
Note: For each alternative value of δ, the models have been recalibrated to ensure that $u = 0.10$ and $t^a = 0.55$ for $\upsilon = 1$. The value of c is 0.60 in all scenarios.

clearly other ways in which taxes distort economic behaviour, since high marginal tax rates could hamper human capital investment, labour mobility, and entrepreneurship, and could cause a reallocation of labour towards jobs with high non-pecuniary benefits and towards the informal economy.

The simulations underlying Table 1 may therefore seriously underestimate the efficiency costs of tax progressivity. As noted earlier, the union bargaining model and the efficiency wage model incorporate an endogenous variable h which may be interpreted as working hours or 'effort', or as a broader productivity index indicating the quality as well as the quantity of work. By varying the elasticity δ of the marginal disutility of work effort with respect to the effort level, it is possible to vary the (compensated) elasticity of effort with respect to the marginal after-tax wage rate. In the simulations above, this elasticity is about 0.1, but if the negative efficiency effects of the marginal tax rate on the quantity and quality of work are more serious, the compensated elasticity of 'effort' w.r.t. the marginal after-tax wage rate should be given a higher value.

In Table 2, I have calculated the optimal degrees of tax progressivity for such higher values of the 'effort' elasticity, assuming a net replacement ratio of 0.6, and once again calibrating the free (unobservable) parameters of the models so as to generate an unemployment rate of 10% in the benchmark case of proportional taxation. [9] Table 2 reveals that even fairly modest increases in the compensated marginal net wage elasticity of effort imply significant reductions in the optimal degree of tax progressivity and tend to reduce the welfare gains from progressivity

[9] When changing δ in the efficiency wage model, I recalibrate ρ so that $u = 0.1$ for $\upsilon = 1$, keeping η constant to ensure fulfilment of the theoretical constraint $0 < \eta < 1$.

considerably. Not surprisingly, a high effort elasticity thus weakens the pure efficiency case for tax progressivity.

9. Concluding remarks

The analysis in this paper should not be taken to imply that governments of unemployment-ridden countries should necessarily increase the progressivity of the labour income tax. Rather, it should be seen as a plea for a more balanced approach to the analysis of the costs and benefits of tax progressivity. If labour markets are highly distorted for non-tax reasons, tax progressivity may be a second-best means of counteracting the tendency of the market to generate excessive unemployment.

One shortcoming of the simplified labour market models applied above is that these models can only illuminate the effects of a general change in the degree of tax progressivity *at all income levels* achieved through, say, a combination of changes in the tax base and in marginal tax rates. As Andersen and Rasmussen (1999) have pointed out, a cut in the tax burden on low income earners financed by higher marginal tax rates on all earned income would not only increase the degree of tax progressivity at all income levels; it might also affect the level and structure of employment by lowering the average tax rate on low income earners while raising the average tax rate on high income earners. Certainly, the welfare effects for the different income groups could differ substantially. In Sørensen (1997), I have tried to throw additional light on the effects of tax progressivity in a more elaborate simulation model allowing for worker heterogeneity.

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