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Economic Policy Research Unit
Department of Economics
University of Copenhagen
Øster Farimagsgade 5, Building 26
DK-1353 Copenhagen K
DENMARK
Tel: (+45) 3532 3010
Fax: (+45) 3532 3000
Web: http://www.econ.ku.dk/epru/

The Basic Environmental Economics of The Circular Economy

Peter Birch Sørensen

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Peter Birch Sørensen

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Abstract: This paper sets up a Ramsey model with exhaustible natural resources to study the optimal recycling of polluting raw materials and household waste products. During the process of economic development it is optimal for the economy to go through an initial "linear" phase with no recycling followed by a "circular" phase where some materials and waste products are recycled to alleviate growing natural resource scarcity and environmental degradation. Ensuring the optimal degree of recycling in a market economy requires a Pigouvian tax on non-recycled raw materials combined with a subsidy to recycling of household waste and a tax on man-made wealth to internalize the environmental cost of capital accumulation.

Key words: Circular economy, linear economy, optimal recycling, Hotelling rule, Pigouvian taxation, Environmental Kuznets Curve

JEL classification: Q53, Q58, H21

Correspondence to:

Peter Birch Sørensen
Department of Economics, University of Copenhagen
Øster Farimagsgade 5, Building 26, 1353 Copenhagen K, Denmark
E-mail: pbs@econ.ku.dk

THE BASIC ENVIRONMENTAL ECONOMICS OF THE CIRCULAR ECONOMY

By Peter Birch Sørensen

1. A New Catch Phrase: The "Circular Economy"

Environmental economists have strived to clarify the meaning of "sustainable development" ever since the concept was introduced by the World Commission on Environment and Development (1987). Recent years have seen the emergence of a new catch phrase that might also benefit from clarification: the "circular economy". Proponents of the circular economy argue for a transition from the current "linear economy" characterized by excessive exploitation of natural resources and harmful accumulation of polluting waste to an economy where existing products are reused and raw materials and waste products are recycled as much as possible.

This idea of the circular economy has been pushed for some time by think tanks such as the Ellen Macarthur Foundation (2012) and is becoming increasingly popular among environmentalists and many governments and business leaders. The idea has featured in the last two Five Year Plans of the Chinese government (Zhijun and Nailing, 2007), and the European Commission (2015) has recently proposed an EU action plan for the circular economy.

Skeptics would say that while a certain degree of recycling may be warranted, you can have too much of a good thing. As Baumol (1977) pointed out long ago, recycling requires the use of resources which may at some point generate more harm than good to the environment. Others might comment that there is nothing new under the sun since the ideas underlying the circular economy paradigm were anticipated long ago by writers such as Boulding (1966) who criticized the existing "cowboy economy" and called for a new "economics of spaceship Earth" where "Man must find his place in a cyclical ecological system which is capable of continuous reproduction of material form.." (Boulding, 1966, p. 7).

This paper argues that the circular economy paradigm does in fact contain a rational core, but it does not call for a new and different approach to environmental policy. On the one hand I show that at some stage during the process of economic development it becomes socially optimal to move from a "linear" to a "circular" economy. On the other hand I demonstrate that if it is possible to implement an appropriate mix of Pigouvian taxes and subsidies, there is no need for other policy instruments to promote the circular economy.

To illustrate these points I set up a Ramsey-type model where production of final goods uses (human and physical) capital and raw materials extracted from a stock of exhaustible natural resources. The use of materials pollutes the environment which is also harmed by waste products from household consumption. Firms may choose to invest capital in a technology that enables them to recycle a part of their raw materials, and consumers may choose to collect some of the household waste and sell it to firms which may transform it into usable raw materials. The recycling of materials and waste benefits the environment, but in the early stage of economic development where natural resources are abundant and capital is scarce, firms and households have no incentive to engage in recycling. However, at some point the growing scarcity of natural resources relative to man-made capital will induce a move from the linear economy with no recycling to an "immature circular economy" where some recycling of raw materials or some recycling of household waste is initiated. Later on the rising price of newly extracted materials and continuing capital accumulation will support a further move to a "mature circular economy" where both households and firms engage in recycling. In a laissez-faire economy the volume and timing of recycling will be inoptimal from a social viewpoint since agents do not internalize the environmental effects of their activities. These market failures may be corrected through a tax on non-recycled raw materials combined with a subsidy to household waste collection and a Pigouvian wealth tax which internalizes the effects of capital accumulation on the environment.

The paper adds to a relatively small environmental economics literature on recycling. An early contribution was made by Smith (1972) who focused on the reuse of household waste. Schultze (1974) illustrated how the recycling of raw materials could ameliorate the exhaustion of non-renewable resources, and Lusky (1975, 1976) studied the allocation of household time between work in the labour market and recycling activity, showing (in his 1975 paper) how the optimal amount of recycling might be secured through a tax on consumption. Hoel (1978) analyzed the optimal path of economic development and the role of recycling when natural resource extraction harms the environment. The simplicity of his model generated the stark conclusion that resource extraction and recycling will never take place simultaneously, whereas the present analysis finds that the two activities can go on at the same time. The more recent papers by Di Vita (2001, 2007) investigate how endogenous technical change driven by R&D may affect the recycling of waste and thereby consumer welfare, and Pittel et al. (2010) set up a Ramsey-type model of exogenous growth with recycling of waste to study how the optimal level of recycling may be implemented through government subsidies. Like the present paper, the article by Andersen (2007) makes the point that the policy problems discussed within the circular economy paradigm can be tackled via the classical Pigouvian policy instruments emphasized in conventional environmental economics. However, none of the contributions mentioned above included an explicit modelling of recycling activity by households as well as firms and they generally did not focus on explaining the transition from a "linear" to a "circular" economy. The present paper extends my previous work on the circular economy (Sørensen, 2017) by including household waste collection and a market for household waste in the analysis and by offering a rationale for an environmentally motivated wealth tax.

Section 2 sets up my model which is used in section 3 to derive the first-best allocation of resources. Section 4 characterizes the initial linear stage of the economy and sections 5 and 6 describe the first-best transition from a linear to an immature circular economy and further on to a mature circular economy. Section 7 analyzes the resource allocation and recycling activity generated by a market economy, and section 8 derives a set of Pigouvian taxes and subsidies that will enable the market to attain the optimal volume and timing of recycling. The concluding section 9 discusses the insights from and the limitations of the analysis.

2. A Ramsey Model with Recycling

We consider an economy inhabited by a representative family dynasty with an infinite horizon. In each period the family derives utility $u(C)$ from consumption of final goods C and utility $v(E)$

from the quality of the environment *E*. The family may also engage in the collection and sorting of household waste products that may be recycled to firms which can transform them into raw materials in production. Since the provision of recyclable waste *Z* requires time and effort, it generates disutility $h(Z)$. At time zero the present value of the family's lifetime utility *U* is therefore given by

$$U = \int_0^\infty \left[u(C) + v(E) - h(Z) \right] e^{-\rho t} dt, \quad u' > 0, \; u'' < 0, \quad v' > 0, \; v'' < 0, \quad h' > 0, \; h'' > 0, \tag{1}$$

where $\rho > 0$ is the constant rate of time preference, and the variables *C*, *E*, and *Z* are understood to be functions of time *t*.

Since transforming waste into new raw materials is costly, only a fraction $1-c$ of the waste products collected by households can be turned into usable materials, so the volume R^H of raw materials that is based on recycled household waste is

$$R^H = (1-c)Z, \qquad 0 < c < 1. \tag{2}$$

The total quantity *M* of raw material used in the production of final goods is

$$M = N + R^H + R, \tag{3}$$

where *N* is new raw material extracted through mining of an exhaustible natural resource, and *R* is the volume of recycled raw material.

Recycling raw material in production requires investing a capital stock K^R in the recycling process. The flow of recycled material is given by the following recycling technology:

$$R = g\left(K^R / M\right) M, \qquad g(0) = 0, \quad g' > 0, \quad g'' < 0, \quad \lim_{K^R/M \to \infty} g\left(K^R / M\right) = 1. \tag{4}$$

According to the last assumption in (4) a complete recycling of all materials ($g = 1$) would require an infinitely high capital intensity of the recycling process and is therefore infeasible due to the Second Law of Thermodynamics discussed by Georgescu-Roegen (1971). The assumption $g(0) = 0$ reflects that no recycling is possible if no capital is invested in recycling equipment.

Abstracting from new discoveries, it follows from (3) that the reserve stock of the exhaustible natural resource stock (*S*) evolves as

$$\dot{S} = -N = -\left(M - R^H - R\right). \tag{5}$$

For simplicity, I will assume that raw materials may be extracted at zero cost.

The output of final goods *Y* is given by the linearly homogeneous production function

$$Y = F\left(K^Y, M\right), \quad F_K > 0, \quad F_{KK} < 0, \quad F_M > 0, \quad F_{MM} < 0, \tag{6}$$

where the subscripts indicate first and second partial derivatives, and where K^Y is the stock of capital used in final goods production. The total stock of man-made capital (*K*) in the economy is

$$K = K^Y + K^R. \tag{7}$$

We may think of *K* as a composite of physical and human capital where optimizing behaviour ensures that investment in the two forms of capital yields the same marginal return. Ignoring depreciation and denoting the amount of new investment by *I*, the change in the capital stock over time is

$$\dot{K} = I. \tag{8}$$

The total output of final goods may be used for consumption or for investment:

$$Y = C + I. \tag{9}$$

By normalization, one unit of household consumption generates one unit of polluting waste, so the net pressure of household activity on the environment is the amount of non-recycled household waste $C - Z$ which reduces the quality of the environment by $\omega \cdot \left(C - Z\right)$ units. The quality of the environment also deteriorates by an amount γ for each unit of raw materials in final goods production that is not recycled. The ability of the environment to assimilate waste and regenerate itself is proportional to the existing stock of environmental goods (proxied by *E*), with a proportionality factor δ. Hence the change in environmental quality over time is

$$\dot{E} = \delta E - \omega(C - Z) - \gamma(M - R), \qquad \omega > 0, \quad \gamma > 0, \quad \rho > \delta > 0. \tag{10}$$

As we shall see below, the assumption $\rho > \delta$ ensures that the shadow value of environmental quality is finite.

3. The First-Best Allocation

A utilitarian social planner will maximize the lifetime utility function (1) subject to the constraints implied by (2) through (10), given the predetermined initial values of *K*, *S*, and *E*. The current-value Hamiltonian for this optimal control problem can be written as

$$\begin{aligned} H = u(C) + v(E) - h(Z) + \mu \overbrace{\left[F\left(K - K^R, M\right) - C \right]}^{\dot{K}} \\ + \lambda \overbrace{\left\{ (1-c)Z - \left[1 - g\left(K^R / M\right)\right] M \right\}}^{\dot{S}} + \eta \overbrace{\left\{ \delta E - \omega(C - Z) - \gamma\left[1 - g\left(K^R / M\right)\right] M \right\}}^{\dot{E}} \end{aligned} \tag{11}$$

where μ, λ, and η are the current shadow values of the state variables *K*, *S*, and *E*, respectively. The control variables are *C*, K^R, and *M*, and the first-order conditions for the solution to the social planning problem are found to be

$$u'(C) = \mu + \omega\eta, \tag{12}$$

$$Z = 0 \quad \text{if} \quad h'(0) \geq \lambda(1-c) + \omega\eta, \tag{13a}$$

$$h'(Z) = \lambda(1-c) + \omega\eta \quad \text{if} \quad h'(0) < \lambda(1-c) + \omega\eta, \tag{13b}$$

$$mF_M = \frac{\lambda + \gamma\eta}{\mu}, \qquad m \equiv \frac{1}{1 - (1-\varepsilon)g}, \qquad \varepsilon \equiv g'\left(K^R / M\right) \frac{\left(K^R / M\right)}{g\left(K^R / M\right)}, \tag{14}$$

$$K^R = 0 \quad \text{if} \quad g'(0)\left(\frac{\lambda + \gamma\eta}{\mu}\right) \leq F_K, \tag{15a}$$

$$g'\left(K^R/M\right)\left(\frac{\lambda+\gamma\eta}{\mu}\right)=F_K \quad \text{if} \quad g'(0)\left(\frac{\lambda+\gamma\eta}{\mu}\right)>F_K, \tag{15b}$$

$$\dot{\mu}=\left(\rho-F_K\right)\mu, \tag{16}$$

$$\dot{\lambda}=\rho\lambda, \tag{17}$$

$$\dot{\eta}=(\rho-\delta)\eta-v'(E). \tag{18}$$

Eq. (12) states that the marginal benefit from consumption must equal its marginal social opportunity cost which consists of the value μ of the additional man-made capital that could have been accumulated by foregoing consumption plus the marginal welfare gain $\omega\eta$ from the improvement of environmental quality achieved by postponing consumption. Eq. (13a) says that recycling of household waste is not worthwhile if the marginal social gain from doing so is not at least as large as the marginal disutility from collecting waste even when no waste is initially collected. The marginal social gain from waste collection is the sum of the gain $\lambda(1-c)$ from the resulting alleviation of raw material scarcity and the marginal welfare gain $\omega\eta$ when the waste is not dumped in the environment. When recycling of waste is indeed worthwhile, the marginal disutility from waste collection should equal this marginal social gain, as stated in (13b).

Eq. (14) is a condition for optimal use of materials, requiring their marginal productivity to equal the marginal social cost of their use, accounting for the degree of recycling. The variable *m* is a "recycling multiplier" reflecting that a unit of materials can be used more than once when there is recycling. Each time an extra unit of materials enters the production process, a fraction $(1-\varepsilon)g$ of it can be used again, so an initial unit increase of materials input results in a total increase of $m\equiv 1/\left[1-(1-\varepsilon)g\right]$ units.[1] The presence of the dampening elasticity ε in the expression for *m* reflects that adding an extra unit of materials to the recycling process while keeping the recycling

[1] To verify this, note that $m=1+(1-\varepsilon)g+\left[(1-\varepsilon)g\right]^2+\left[(1-\varepsilon)g\right]^3+....=1/\left[1-(1-\varepsilon)g\right]$.

equipment K^R constant reduces the effectiveness of the process, thereby reducing the share of materials that can be recycled. Diminishing returns in the recycling process imply that the elasticity $\varepsilon \equiv (g'/g)(K^R/M)$ is smaller than 1.[2] The fraction $(\lambda+\gamma\eta)/\mu$ appearing in (14) and (15) is the marginal social cost of using an additional unit of non-recycled raw material in production, measured in units of the capital good (since we are dividing by μ). It consists of the marginal cost of depleting the natural resource stock, captured by the shadow price λ/μ, plus the marginal welfare cost $\gamma\eta/\mu$ of the damage to the environment when an extra unit of non-recycled materials is put through the production process.

The optimal degree of recycling is determined by (15a) and (15b) where the term $g'(0)(\lambda+\gamma\eta)/\mu$ is the marginal social gain from investing a unit of capital in recycling, starting from a level of zero investment. This gain reflects the alleviation of natural resource scarcity and the improvement of environmental quality resulting from initiating recycling. The right-hand side of (15a) and (15b) is the marginal social opportunity cost of reallocating capital from final goods production to recycling, given by the marginal productivity of capital in final goods production. Thus eq. (15a) says that if the marginal social gain from recycling is smaller than its marginal opportunity cost, society should not invest in recycling. But if $g'(0)(\lambda+\gamma\eta)/\mu > F_K$ so that some amount of recycling is worthwhile, eq. (15b) says that investment in recycling should be carried to the point where its marginal social benefit equals its marginal social opportunity cost.

Using the dynamic optimality conditions (16) through (18), we can derive a *wealth accumulation rule* determining how much wealth society should transfer from the present to the future, and a *portfolio composition rule* indicating how society should allocate its wealth between man-made capital and natural capital. The wealth accumulation rule is found by differentiating both sides of (12) with respect to time and inserting (16) and (18) in the resulting expression. This yields the

[2] The recycling process specified in (4) can be thought of as resulting from a linearly homogeneous "recycling function" $R = R(K^R, M) = g(K^R/M)M$ where $g(K^R/M) \equiv R(K^R/M, 1)$. With diminishing returns to each of the inputs in the recycling function $R(K^R, M)$, the function $g(K^R/M)$ will also display diminishing returns to the capital intensity K^R/M.

following modified Keynes-Ramsey rule for an optimal intertemporal allocation of consumption, where σ is the elasticity of the marginal utility of consumption:

$$\frac{\dot{C}}{C}=\frac{1}{\sigma}\left(\Omega-\rho\right),\qquad \sigma\equiv-\frac{u''C}{u'}>0,\qquad \Omega\equiv\left(\frac{\mu}{\mu+\omega\eta}\right)F_K+\left(\frac{\omega\eta}{\mu+\omega\eta}\right)\left(\frac{v'(E)}{\eta}+\delta\right). \tag{19}$$

The variable Ω in (19) is the marginal social return to saving which is a weighted average of the marginal return to investment in man-made capital (F_K) and the marginal return to investment in improved environmental quality $\frac{v'(E)}{\eta}+\delta$ ("environmental capital"). The return to investment in man-made capital enters with a coefficient less than one because the welfare gain from the higher future consumption made possible by a larger future capital stock is partly offset by the welfare loss from the additional pollution caused by the rise in future consumption. On the other hand, since a rise in saving requires a fall in current consumption that reduces the current dumping of household waste, it generates an immediate welfare gain from a cleaner environment reflected in the term $v'\left(E\right)/\eta$ and a further gain from a greater future assimilative capacity of the environment which is captured by the parameter δ. We see that the weight $\omega\eta/\left(\mu+\omega\eta\right)$ assigned to the return to investment in "environmental capital" in the definition of Ω increases with the value of such capital (η) relative to the total value $\mu+\omega\eta$ of the additional man-made and environmental capital accumulated via an extra unit of saving. Note that when one abstracts from the importance of the environment for human welfare, i.e., when $\eta=0$, eq. (19) collapses to the standard Keynes-Ramsey rule $\dot{C}/C=\left(1/\sigma\right)\left(F_K-\rho\right)$.

The portfolio composition rule can be found by differentiating the first equation in (14) with respect to time and inserting (14), (16), (17) and (18) into the resulting expression to obtain

$$F_K=\frac{\dot{F}_M}{F_M}+\left(\frac{\gamma\eta}{\lambda+\gamma\eta}\right)\left(\frac{v'\left(E\right)}{\eta}+\delta\right)+\frac{\dot{m}}{m}. \tag{20}$$

The left-hand side of (20) is the marginal social rate of return on investment in man-made capital, given by its marginal productivity. In optimum this must equal the marginal social rate of return on investment in natural capital appearing on the right-hand side of (17). The investment in natural capital involves postponing the extraction of an extra unit of materials from "today" until "tomorrow". A part of the gain from doing so consists in the rise of the marginal productivity of materials as they become scarcer over time. This is captured by the first term on the right-hand side of (20). The second term reflects that postponement of extraction implies a lower current use of materials which generates an environmental gain, partly because the postponement of emissions directly benefits consumers and partly because the lower current emission of waste products from production increases the future assimilative capacity of the environment. We see that the environmental gain carries a heavier weight the greater the importance of improving environmental quality relative to the importance of alleviating natural resource scarcity, i.e., the larger the fraction $\gamma\eta / \left(\lambda + \gamma\eta\right)$. Finally, there is a gain from postponement of extraction to the extent that the "materials multiplier" *m* increases over time so that materials can be used more effectively in the future. This is captured by the third term on the right-hand side of (20). From the definition of *m* stated in (14) it follows that if the elasticity ε is roughly constant, we have $\frac{\dot{m}}{m} \approx \frac{\left(1-\varepsilon\right)\dot{g}}{1-\left(1-\varepsilon\right)g}$ so that (20) may be written as

$$F_K = \frac{\dot{F}_M}{F_M} + \left(\frac{\gamma\eta}{\lambda+\gamma\eta}\right)\left(\delta + \frac{v'\left(E\right)}{\eta}\right) + \frac{\left(1-\varepsilon\right)\dot{g}}{1-\left(1-\varepsilon\right)g}. \tag{21}$$

Recalling that $g < 1$ and $\varepsilon < 1$ because of diminishing returns to recycling, we see from (21) that an increase over time in the recycling rate *g* increases the marginal gain from postponing the extraction and use of materials, which is intuitive.

4. The Early Stage of Economic Development: The Linear Economy

At an early stage of economic development where the stock of man-made capital is low, natural resources are still abundant, and environmental quality still has not suffered seriously from large flows of pollution and waste, the shadow value μ of man-made capital will be high due to its scarcity, whereas the shadow values λ and η of natural and environmental capital will be relatively low. Presumably the marginal social gain $g'(0)(\lambda+\gamma\eta)/\mu$ from investing in recycling of raw materials will then be smaller than the marginal social return F_K to investing in final goods production, since the marginal productivity of capital in final goods production is high when the capital stock is low. Moreover, with low values of λ and η the marginal social gain $\lambda(1-c)+\omega\eta$ from recycling of household waste is likely to be smaller than the marginal disutility $h'(0)$ from initiating waste collection. In these circumstances (13a), (15a) plus (2) and (4) imply that

$$Z=0 \quad \text{and} \quad K^R=0 \quad \Rightarrow \quad R^H=0 \quad \text{and} \quad R=0. \tag{22}$$

Given the likely initial conditions at the early stage of development, it will thus be socially optimal for the economy to start out in a "linear" phase with no recycling. In this phase it follows from (10) and (22) that the quality of the environment will evolve as

$$\dot{E}=\delta E-\omega C-\gamma M. \tag{23}$$

5. The Transition to an Immature Circular Economy

As the economy moves forward through the linear phase, it is likely to reach a point where some amount of recycling becomes optimal. To see this, note that (16) through (18) imply

$$\mu(t)=\mu(0)e^{-\int_0^t\left[F_K(z)-\rho\right]dz}, \tag{24}$$

$$\lambda(t)=\lambda(0)e^{\rho t}, \tag{25}$$

$$\eta(t)=\int_t^\infty v'(E(z))e^{-(\rho-\delta)(z-t)}dz, \tag{26}$$

In the linear phase where man-made capital is scarce, its marginal productivity F_K may be assumed to exceed the rate of time preference ρ, so according to (24) its marginal shadow value will fall over time as more capital is accumulated. At the same time (25) shows that the marginal shadow value of the natural resource rises steadily over time at the rate ρ as the resource gets scarcer. Eq. (26) states that the marginal shadow value of environmental quality equals the present value of the future marginal utilities of environmental quality.[3] When the marginal social cost of materials use is low, the optimality condition (14) will encourage a large input of materials in final goods production. In the absence of recycling one would therefore expect that the pollution from materials use and household waste ($\gamma M + \omega C$) will exceed the absorption capacity of the environment (δE), causing the environment to deteriorate. Since the marginal utility of environmental quality increases as the quality goes down, it follows from (26) that the fall in environmental quality will drive up its shadow value η over time.

Thus the linear economy is likely to be characterized by falling values of μ and F_K and rising values of η and λ as capital and pollution accumulates and the natural resource stock diminishes. With the passing of time the economy will therefore either reach a point where $g'(0)(\lambda + \gamma\eta)/\mu = F_K$ beyond which some recycling of raw materials in production becomes optimal, or a point where $h'(0) = \lambda(1-c) + \omega\eta$ beyond which it becomes beneficial to initiate some recycling of household waste. In general these two points in time will not coincide, so as one of them is passed, the economy will enter an intermediate phase which may be characterized as an "immature circular economy" where only one of the two forms of recycling takes place.

6. The Mature Circular Economy

In the immature circular economy the shadow value of the natural resource (λ) will continue to rise in accordance with (25), and as long as savings remain positive the resulting accumulation of capital will continue to drive down the values of μ and F_K. The initiation of one form of recycling will tend to alleviate the deterioration of environmental quality as the value of either *R* or *Z* in eq. (10) turns positive, so we cannot rule out that the quality of the environment will start to improve in

[3] Note that since $\rho > \delta$ by assumption, the integral in (26) is finite. The presence of the parameter δ in the effective discount rate $\rho - \delta$ reflects that an improvement in current environmental quality increases the future ability of the environment to absorb waste, thereby increasing the future quality of the environment.

the immature circular economy, thereby driving down its shadow value η. However, unless there is a strong increase in environmental quality, the continuing increase in λ and the further fall in μ and F_K will sooner or later take the economy to a phase which may be termed the "mature circular economy" where some recycling of household waste as well as some recycling of raw materials becomes socially optimal due to the growing scarcity of natural resources relative to man-made capital.

If the deterioration of environmental quality has not been halted already in the immature circular economy, the activation of both forms of recycling in the mature circular economy suggests that the environment may start to improve during this phase.

7. **Resource Allocation in the Market Economy**

Let us now compare the resource allocation generated by competitive markets to the socially optimal allocation described above. Suppose the natural resource stock is owned by a representative competitive mining firm which extracts a flow of new raw materials *N* per period. Since extraction is costless and raw materials can be sold at the real market price *p*, the mining firm can pay out the following net dividend D^M to its owners in each period:

$$D^M = pN. \tag{27}$$

The market value V^M of the mining firm is the present value of its future dividend payouts which is

$$V_t^M = \int_t^{\infty} D_z^M e^{-\int_t^z r_q dq} dz, \tag{28}$$

where r is the real market interest rate. The mining firm draws up a plan for the future levels of extraction that will maximize its market value (28) at time $t = 0$ subject to the stock-flow constraint $\dot{S} = -N$ and the predetermined initial reserve stock S_0. The first-order conditions for the solution to this problem yield the classical Hotelling rule stating the equilibrium resource price increases at the rate of interest:

$$r = \frac{\dot{p}}{p}. \tag{29}$$

The mining firm sells the extracted raw materials to the representative competitive firm in the final goods industry and the price of materials adjusts to ensure that supply equals demand so that

$$N = M - R^H - R. \tag{30}$$

The final goods firm uses the production technology (6) and the recycling technology (4) (when recycling is profitable). The firm may choose to buy a quantity Z^D of waste products collected by households. When the market is active, household waste is traded at the unit price p^H, but only a fraction $1-c$ of the waste can be transformed into usable raw material, as stated in (2). The government may levy a unit tax at the rate τ^M on the use of non-recycled materials. The real net dividend D^Y paid out by the final goods firm after deduction of investment expenditure may therefore be written as

$$\begin{aligned} D^Y &= Y - p\left(M - R^H - R\right) - p^H Z^D - \tau^M\left(M - R\right) - I \\ &= F\left(K - K^R, M\right) - \left(p + \tau^M\right)\left[1 - g\left(K^R / M\right)\right]M + \left[p\left(1-c\right) - p^H\right]Z^D - I, \end{aligned} \tag{31}$$

where we have used the facts that $R^H = \left(1-c\right)Z^D$ and $M - R = \left(1-g\right)M$. By analogy to (28), the market value V^Y of the final goods firm is

$$V_t^Y = \int_t^\infty D_z^Y e^{-\int_t^z r_q dq} dz. \tag{32}$$

Given (31) and its initial total stock of capital, the final goods firm chooses K^R, M, Z^D and I with the aim of maximizing (32) at $t = 0$ subject to the stock-flow constraint $\dot{K} = I$. The first-order conditions for the solution to this problem can be shown to imply that

$$F_K = r, \tag{33}$$

$$mF_M = p + \tau^M, \tag{34}$$

$$K^R = 0 \quad \text{if} \quad \left(p + \tau^M\right) g'(0) \le F_K, \tag{35a}$$

$$\left(p + \tau^M\right) g'\left(K^R / M\right) = F_K \quad \text{if} \quad \left(p + \tau^M\right) g'(0) > F_K. \tag{35b}$$

$$p^H = p(1-c) \quad \text{and} \quad Z^D = Z. \tag{36}$$

Eq. (33) is the standard condition for profit maximization that the marginal productivity of capital must equal the real rate of interest. Eq. (34) says that materials are used until their marginal productivity equals their tax-inclusive price, accounting for the multiplier effect of recycling captured by the variable *m*. According to (35a) no capital is invested in recycling unless the resulting saving on materials expenses exceeds the marginal revenue from investing capital in final goods production. In the early stage of development where natural resources are abundant and man-made capital is scarce, the materials price *p* will be low and the marginal productivity of capital in final goods production will be high, so (35a) suggests that the economy will go through an initial linear phase with no recycling of raw material. However, the Hotelling rule (29) implies that the materials price will rise over time, and as capital accumulates its marginal productivity will fall. At some point recycling of raw material therefore becomes profitable, and the economy will enter a circular phase where the profit-maximizing level of materials recycling is determined by the arbitrage condition (35b) which requires identical marginal returns to investment in recycling and investment in final goods production.

Since each unit of household waste can be transformed into $1-c$ units of raw material, the firm's demand for household waste becomes infinitely elastic at the unit price of waste $p^H = (1-c)p$. At this price firms are therefore willing to purchase any amount of waste that households find it optimal to supply, as stated in (36). Let us therefore consider what takes to induce households to collect waste products for raw material production.

In each period the representative household receives net dividends from firms plus a government lump-sum transfer *B*. If it finds it worthwhile to collect any waste, the household also receives a total revenue $\left(p^H + s\right)Z$ from the sale of waste products to final goods producers, where *s* is a government subsidy to waste collection. At the same time the government may choose to levy a wealth tax at the rate τ^V per unit of wealth. Since the dividends D^M and D^Y are measured net of any new capital that households inject in firms as they invest their savings, the household budget constraint may thus be written as

$$C = D^M + D^Y + B + \left(p^H + s\right)Z - \tau^V V. \tag{37}$$

Total household wealth V is

$$V \equiv V^M + V^Y. \tag{38}$$

From the expressions for V^M and V^Y in (28) and (32) it follows that total wealth evolves as

$$\dot{V} \equiv \dot{V}^M + \dot{V}^Y = r\left(V^M + V^Y\right) - D^M - D^Y = rV - \left(D^M + D^Y\right). \tag{39}$$

Combining (37) and (39), we obtain the dynamic household budget constraint:

$$\dot{V} = \left(r - \tau\right)V + B + \left(p^H + s\right)Z - C. \tag{40}$$

The government balances its budget in each period, implying

$$B = \tau^M\left(M - R\right) + \tau^V V - sZ. \tag{41}$$

The household chooses a time path for C and Z to maximize the present value of its lifetime utility (1) subject to the budget constraint (40) and the initial stock of wealth, taking the lump-sum transfer B as given. The first-order conditions for the solution to this problem imply that

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left(r - \tau^V - \rho\right), \qquad \sigma \equiv -\frac{u''C}{u'} > 0, \tag{42}$$

$$Z = 0 \quad \text{if} \quad h'(0) \geq \left(p^H + s\right)u'(C), \tag{43a}$$

$$h'(Z) = \left(p^H + s\right)u'(C) \quad \text{if} \quad h'(0) < \left(p^H + s\right)u'(C). \tag{43b}$$

Eq. (42) is a version of the familiar Keynes-Ramsey rule for wealth accumulation, accounting for the wealth tax which reduces the private return to saving. The magnitude $\left(p^{H}+s\right)u'(C)$ in (43) is the household's welfare gain from the extra consumption that can be financed by collecting and selling an extra unit of household waste. If this marginal welfare gain is smaller than the marginal disutility from collecting even one unit of waste, the household will not want to collect any waste at all, as stated in (43a). But if some waste collection is worthwhile, the household will collect waste up to the point where the marginal disutility from doing so equals the marginal welfare gain from selling it and consuming the proceeds, as indicated in (43b).

As we have seen, the equilibrium market price of household waste is $p^{H}=(1-c)p$ which will rise systematically over time as the price of newly extracted raw materials increases in line with the Hotelling rule (29). In the early phase of economic development the materials price may well be too low to compensate households for the inconvenience of waste collection, but at some point the price will become high enough to induce households to initiate some waste collection, thereby activating the market for household waste products. This point in time is unlikely to coincide with the time when it becomes optimal for firms to start recycling raw materials, so the market economy will go through an "immature" circular phase where either raw materials or household waste is recycled before the continuing rise in *p* and the continuing fall in *r* takes it to a "mature" phase where both forms of recycling take place. The next section investigates how the government may ensure the optimal level and timing of recycling in the market economy.

8. Securing Incentives for Optimal Recycling

On a first-best path of economic development the economy must obey the wealth accumulation rule (19) and the portfolio composition rule (20). In addition, there must be an optimal allocation of resources between recycling activity and other activity at any point in time. According to (13) through (15) the latter requires that

$$Z=0 \quad \text{if} \quad h'(0)\geq u'(C)\left[\frac{\lambda(1-c)+\omega\eta}{u'(C)}\right], \tag{44a}$$

$$h'(Z) = u'(C)\left[\frac{\lambda(1-c)+\omega\eta}{u'(C)}\right] \quad \text{if} \quad h'(0) < u'(C)\left[\frac{\lambda(1-c)+\omega\eta}{u'(C)}\right], \tag{44b}$$

$$K^R = 0 \quad \text{if} \quad g'(0)F_M(K,M) \le F_K(K,M), \tag{45a}$$

$$g'\left(K^R/M\right)mF_M\left(K-K^R,M\right) = F_K\left(K-K^R,M\right) \quad \text{if} \quad g'(0)F_M(K,M) > F_K(K,M). \tag{45b}$$

Eqs. (44a) and (45a) are just restatements of the conditions that recycling should not be undertaken unless its marginal social benefit exceeds its marginal social cost, and (44b) and (45b) require that marginal social benefits should equal marginal social costs when recycling takes place. Specifically, (45b) states that the marginal productivity of capital invested in recycling of raw materials (measured in terms of the resulting increase in the output of final goods) should equal the marginal productivity of capital invested directly in final goods production. This optimality condition will actually be met in a competitive market economy, since substitution of (34) into (35b) yields (45b). However, in a laissez-faire economy where $s = 0$, the amount of household waste collection will not be socially optimal, since the privately optimal level of *Z* implied by (43b) will generally not coincide with the first-best level determined by (44b). Moreover, in the laissez-faire economy the timing of the transition from the linear to the circular economy will be "wrong", so the volume of both forms of recycling at any given point in time will be distorted relative to the first-best allocation.

To see this, note that the wealth accumulation rule for the market economy can be written in the following way by substituting (33) into (42):

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\left(F_K - \tau^V - \rho\right). \tag{46}$$

When $\tau^V = 0$ this condition for the privately optimal growth in consumption will clearly deviate from the socially optimal consumption growth rate given by (19). Furthermore, differentiating (34) with respect to time and inserting (29), (33) and (34) in the resulting equation, we obtain the following expression characterizing the portfolio composition in the market economy:

$$F_K = \frac{\dot{F}_M}{F_M} + \frac{\dot{m}}{m} + \frac{r\tau^M - \dot{\tau}^M}{P}, \qquad P \equiv p + \tau. \tag{47}$$

Comparing (20) to (47) we see that, in a laissez-faire economy where $\tau^M = \dot{\tau}^M = 0$, the marginal private gain from postponing resource extraction given by the right-hand side of (47) will tend to be lower than the marginal social rate of return on the right-hand side of (20) which includes the environmental gain from slower extraction. In the initial linear phase of the laissez-faire economy natural resource extraction will therefore tend to be too rapid relative to the first-best pace of extraction. Intuitively one might also expect the transition to the circular phase to occur too late in the laissez-faire economy, but this cannot be taken for granted since the more intensive use of raw materials in the linear laissez-faire economy also means that the scarcity of natural resources increases more rapidly over time. What we *can* say is that the transition will not take place at the optimal time and that the levels of materials use and recycling in each period will deviate from their first-best levels.[4]

The tendency towards excessive exploitation of natural resources may be corrected by imposing a Pigouvian tax on non-recycled materials at a rate equal to the present value of the marginal environmental cost of materials use. Specifically, this Pigou tax must be levied at the following rate, where η and λ are the shadow values of the environment and of the natural resource stock prevailing along the economy's first-best time path, and where mec_z^M is the marginal external cost of using a unit of non-recycled materials in some future period *z*, measured as a fraction of its tax-inclusive price P_z:

$$\tau_t^M = \int_t^\infty mec_z^M P_z e^{-\int_t^z r_q dq} dz, \qquad mec_z^M \equiv \left(\frac{\gamma\eta}{\lambda + \gamma\eta}\right)\left(\frac{v'(E)}{\eta} + \delta\right). \tag{48}$$

To see that this tax rate will indeed ensure the optimal allocation of capital between investment in man-made capital and natural capital, note that (48) implies

[4] In Sørensen (2017) I present a more detailed analysis of this issue in a simplified model that focuses on recycling of raw materials but abstracts from household waste.

$$\dot{\tau}^M = r\tau^M - mec^M \cdot P = r\tau^M - \left(\frac{\gamma\eta}{\lambda + \gamma\eta}\right)\left(\frac{v'(E)}{\eta} + \delta\right)P. \tag{49}$$

When (49) is inserted in (47), the resulting portfolio composition rule for the market economy becomes identical to the portfolio composition rule (20) for the planned economy both in the linear phase where $m = 1$ and $\dot{m} = 0$ and in the circular phase with $m > 1$ and $\dot{m} \neq 0$.

Moreover, the government can steer the market economy towards the first-best level of wealth accumulation by levying a wealth tax at the rate

$$\tau^V = \left(\frac{\omega\eta}{\mu + \omega\eta}\right)\left[F_K - \left(\frac{v'(E)}{\eta} + \delta\right)\right], \tag{50}$$

since such a tax will ensure that the privately optimal savings rule (46) coincides with the wealth accumulation rule (19) for the planned economy. The intuition for the optimality of the wealth tax (50) may be explained as follows: The additional future consumption F_K made possible when an extra unit of capital is accumulated will generate an environmental cost $\left(\omega\eta / \left(\mu + \omega\eta\right)\right)F_K$ because the higher future consumption generates additional household waste.[5] On the other hand, the fall in current consumption needed to make room for increased investment implies a postponement of the dumping of household waste. The welfare gain from this postponement of pollution is $\left(\omega\eta / \left(\mu + \omega\eta\right)\right)\left(\delta + v'\left(E\right) / \eta\right)$ where the term $v'\left(E\right) / \eta$ captures the short-run gain in environmental quality and δ reflects the improved future assimilative capacity of the environment when less waste is dumped today. Thus the right-hand side of (50) represents the net environmental damage cost of an extra unit of saving and investment. Our analysis provides a rationale for an environmentally motivated wealth tax that will internalize this marginal external cost of wealth accumulation. Note that if the term in the square bracket in (50) is negative, a wealth subsidy rather than a tax will be needed to ensure wealth accumulation at the optimal rate. This could reflect a

[5] Note from (12) that $\mu + \omega\eta = u'\left(C\right)$, so the environmental cost $\omega\eta / \left(\mu + \omega\eta\right)$ is measured in units of the numeraire consumption good.

situation where the current environment is under serious stress from current pollution so that postponement of further emissions through postponement of consumption has a high social value.

While the wealth tax in (50) will induce households to save at the proper rate, it will not support collection of household waste at the optimal level. The problem is that the equilibrium market price $p^H = p(1-c)$ for waste does not reward households for the environmental gain from recycling it. To correct this market failure the government must offer a subsidy that equates the private marginal gain from waste collection given by the right-hand sides of the expressions in (43) to the marginal social gain stated on the right-hand sides of the corresponding expressions in (44). Recalling that the equilibrium market price for waste is $p^H = p(1-c)$, this requires a subsidy to waste collection at the rate $s = \left[(\lambda / u') - p\right](1-c) + (\omega\eta / u')$, but since $pu' = \lambda$ when the regulated market economy is on the first-best time path,[6] this reduces to the following subsidy rate (where we have used (12) to arrive at the last equality):

$$s = \frac{\omega\eta}{\mu + \omega\eta} = \frac{\omega\eta}{u'(C)}. \tag{51}$$

A subsidy at this rate rewards households for the external environmental benefit $\omega\eta / u'(C)$ generated when an additional unit of household waste is recycled in the production process rather than being dumped in the environment.

In summary, the government may implement the first-best transition from the linear to the circular economy by an appropriate combination of a Pigou tax on non-recycled raw materials in production, a Pigouvian tax/subsidy on wealth, and a Pigouvian subsidy to household waste collection.

[6] The shadow value λ measures the welfare gain to the consumer if an extra unit of the natural resource were to be discovered at zero cost along the economy's first-best time path. In the market economy such a discovery would enable the mining firm to pay out an extra dividend amounting to *p* that would enable the household to increase its consumption by a corresponding amount, generating a utility gain of pu'. If the market economy is on the first-best time path, it follows that $pu' = \lambda$.

9. Concluding Remarks

The analysis in this paper has not established that all economies will necessarily go through a “linear” phase before entering a “circular” phase. Whether an economy will follow this course of development will depend on its initial conditions. However, if the early stage of development is characterized by a scarcity of man-made capital relative to natural resources, we have seen that the economy will most likely start out from the linear stage, but at some point it will be induced to move on to a circular stage with some amount of recycling as man-made capital becomes more abundant relative to natural resources. From a social viewpoint the incentive to enter the circular stage will be strengthened in the plausible case where the linear stage involves a decline in the quality of the environment.

The mechanisms that take the economy from the linear to the circular phase could provide part of the explanation for the Environmental Kuznets Curve according to which pollution increases during the early stage of economic development and declines as the economy matures. In our model the reversal of the tendency for environmental quality to decline is driven by rising natural resource scarcity and an increasing stock of capital which tend to reduce the use of polluting raw materials and to increase recycling of materials and waste as the relative price of raw material goes up and as the rising supply of capital reduces the opportunity cost of investing in recycling.[7] Our model also indicates that policy makers have an incentive to tighten environmental policy when the growth process causes increasing damage to the environment.

According to our analysis one important driver of the transition from the linear to the circular economy is a continuing rise in the relative price of exhaustible natural resources. In reality we have not observed an unbroken rising trend in real raw materials prices. Slade (1982) identified a U-shaped time trend in many important resource prices which could be explained as the net effect of a race between technical progress in extraction technologies and growing scarcity of easily accessible reserve stocks. The more recent research by Lee et al. (2006) suggests that real resource prices have tended to be stationary around deterministic trends with structural breaks. In contrast to

[7] There is a long-standing debate whether an Environmental Kuznets Curve (EKC) actually exists, cf. Barbier (1997) and Stern (2004). Brock and Taylor (2010) argue that a Solow growth model with exogenous technical progress in pollution abatement can account for the different EKC patterns observed across countries when one allows for cross-country differences in initial conditions and economic structures.

the basic Hotelling mechanism emphasized by our model, an important driver of recycling could be technical progress in recycling technologies. Incorporating (endogenous) technical change into the model would be a natural extention of the present analysis.

Our simplified model has abstracted from many other aspects of recycling that may be important in practice. For example, the treatment of household waste is typically regulated by local governments, and differences in regulation across jurisdictions may hamper the creation of an efficient market for collection, sorting and recycling of waste, as emphasized by the Advisory Board for Circular Economy (2017) in Denmark. Moreover, the optimal degree of recycling of waste will depend on landfill space constraints (Highfill and McAsey, 1997) and on the social value of the waste as a fuel in the production of heat and electricity (Gradus et al., 2017). The choice between home separation of different types of waste by households and post-collection separation of waste such as plastic may also be important for the total costs of recycling, as documented by Dijkgraaf and Gradus (2017) in the case of the Netherlands. Designing optimal recycling policies thus involves a host of practical issues that are not included in the present analysis.

Furthermore, in practice governments may not have the information and administrative capacity to implement the fine-tuning Pigouvian taxes and subsidies needed to implement the first-best recycling policy, and at any rate the existence of numerous non-environmental market distortions as well as concerns about the distributional effects of Pigouvian taxes and subsidies will inevitably force policy makers into a second-best context. Nevertheless, the analysis in this paper suggests that a well-designed package of environmental taxes and subsidies is an important prerequisite for an efficient degree of recycling. At the more basic level, the paper has tried to illustrate that the policy issues studied by proponents of the new circular economy paradigm can be fruitfully analysed by using the tools of conventional environmental economics.

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