Definition 1 *Given an financial economy with production* $E = \left\langle \mathbb{I}, \left(C^i, u^i, e^i, \alpha^i\right)_{i\in\mathbb{I}}, V, Y\right\rangle$ *and a production plan* $y \in Y$, *a* ***Radner equilibrium with production plan*** y *is a tuppel*

$$\left(\left(\bar{c}^i, \bar{\theta}^i, \bar{\gamma}^i\right)_{i\in\mathbb{I}}, \left(y, \bar{\theta}^0\right), \bar{p}, \bar{q}, \bar{q}_0\right)$$

such that

1. (a) $\left(\bar{c}^i, \bar{\theta}^i, \bar{\gamma}^i\right)$ solves the problem

$$\max_{(c,\theta,\gamma)} u^i(c) \text{ st. } (c,\theta,\gamma) \in \mathbb{B}_i\left(\bar{p}, \bar{q}, \bar{q}^0; y, \bar{\theta}^0, V\right)$$

   (b) $\sum_{i\in\mathbb{I}} \bar{c}^i = \sum_{i\in\mathbb{I}} e^i + y$

   (c) $\sum_{i\in\mathbb{I}} \bar{\theta}^i + \bar{\theta}^0 = 0$

   (d) $\sum_{i\in\mathbb{I}} \bar{\gamma}^i = 0$

where we have that

$$\begin{aligned}
&\mathbb{B}_i\left(p, q, q^0; y, \theta^0, V\right) \\
= &\left\{(c,\theta,\gamma) \mid \begin{array}{c} p(0)\left(c(0) - e(0) - \alpha^i y(0)\right) \leq \alpha^i \cdot \left(-q\theta^0\right) + \gamma^i \cdot (-q_0) - q\theta \\ p(t)\left(c(t) - e(t) - \alpha^i y(t)\right) \leq \alpha^i V_t\theta^0 + \gamma^i\left(p(t)\,y(t) + V_t\theta^0\right) + V_t\theta \end{array}\right\} \\
= &\left\{(c,\theta,\gamma) \mid p\left(c - e - \alpha^i y\right) \leq W_y \begin{bmatrix} \gamma^i \\ \theta + \alpha^i\theta^0 \end{bmatrix}\right\}
\end{aligned}$$

where $\begin{bmatrix} \gamma \\ \theta + \alpha^i\theta^0 \end{bmatrix} \in \mathbb{R}^{J+1}$ is the extended portfolio vector and

$$W_y = \begin{bmatrix} -q_0 & -q_1 & \cdots & -q_J \\ p(1)\,y(1) + V_1\theta^0 & V_{11} & \cdots & V_{1J} \\ \vdots & \vdots & \ddots & \vdots \\ p(T)\,y(T) + V_T\theta^0 & V_{T1} & \cdots & V_{TJ} \end{bmatrix}$$

is the production extended marketed space: the marketed space extended with the dividends of the asset generated by the production plan y of the firm.

Definition 2 *Given an economy* $E = \left(\left(X^i, u^i, e^i, \alpha^i\right), Y\right)$ *a* ***Walras equilibrium*** *is a tuppel* $(\bar{x}, \bar{y}, \bar{p}) \in \mathbb{R}^{In} \times \mathbb{R}^{Ln} \times \mathbb{R}^n$ *such that*

1. (a) $\bar{p} \cdot \bar{x}^i = \bar{p} \cdot e^i + \alpha^i \bar{p} \cdot \bar{y}^l$, and $u^i\left(\bar{x}^i\right) \geq u^i\left(x^i\right)$ for every x^i such that $\bar{p} \cdot x^i = \bar{p} \cdot e^i + \alpha^i \bar{p} \cdot \bar{y}^l$

   (b) $\bar{p} \cdot \bar{y}^l \geq p \cdot y^l$ for every $y^l \in Y$

   (c) $\sum_i \bar{x}^i - e^i = \sum_l \bar{y}^l$

Given demand correspondance

$$\hat{f}^i\left(p, p \cdot e^i\right)$$
$$\hat{g}^j\left(p\right)$$

we have that an equilibrium when

$$0 \in \sum_i \hat{f}\left(p, p \cdot e^i\right) - e^i - \sum_j \hat{g}^j\left(p\right)$$

Produktions mulighedsområde

$$Y = \left\{ y \in \mathbb{R}^3 | y_0 + y_1 + y_2 \leq 0 \wedge y_0 \leq 0 \wedge y_1, y_2 \geq 0 \right\}$$

Finansielle aktiver og initial beholdninger

	Asset		Init.	end.
	1	2	e^a	e^b
0			2	2
1	1	0	2	0
2	0	1	0	2

Nyttefunktioner

$$u^a(c) = \frac{1}{3} \ln c(0) + \frac{1}{3} \ln c(1) + \frac{1}{3} \ln c(2) = u^b(c)$$

Initiale aktieandele

$$\begin{pmatrix} \alpha^a \\ \alpha^b \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$$

Walras ligevægt

$$\left(\left(\bar{c}^a, \bar{c}^b \right), \bar{y}, \bar{p} \right) = \left(\left(\left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3} \right), \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3} \right) \right), \left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3} \right), (1,1,1) \right)$$

indkomst overførsler

$$\begin{aligned} \bar{r}^a &= \left(-\frac{2}{9}, -\frac{8}{9}, \frac{10}{9} \right) \\ \bar{r}^b &= \left(\frac{2}{9}, \frac{8}{9}, -\frac{10}{9} \right) \end{aligned}$$

Radner ligevægt med $y = \left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$ når $\bar{\gamma}^a = \bar{\gamma}^b = \bar{\theta}^0 = 0$

$$\begin{aligned} & \left(\left(\bar{c}^i, \bar{\theta}^i, \bar{\gamma}^i \right)_{i \in \mathbb{I}}, \left(y, \bar{\theta}^0 \right), \bar{p}, \bar{q}, \bar{q}_0 \right) \\ = & \left(\begin{array}{c} \left(\left(\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}, \begin{pmatrix} -\frac{8}{9} \\ \frac{10}{9} \end{pmatrix}, 0 \right), \left(\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}, \begin{pmatrix} \frac{8}{9} \\ -\frac{10}{9} \end{pmatrix}, 0 \right) \right) \\ \left(\begin{pmatrix} -\frac{4}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right), \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{4}{3} \end{array} \right) \end{aligned}$$

Radner ligevægt med $y = \left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$ når $\bar{\gamma}^a = -\bar{\gamma}^b = -\frac{1}{3}$ og $\bar{\theta}^0 = \left(-\frac{2}{3}, -\frac{2}{3}\right)$

$$
\begin{aligned}
&\left(\left(\bar{c}^i, \bar{\theta}^i, \bar{\gamma}^i\right)_{i\in\mathbb{I}}, \left(y, \bar{\theta}^0\right), \bar{p}, \bar{q}, \bar{q}_0\right) \\
&= \left(\begin{array}{c}
\left(\left(\left(\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}, \begin{pmatrix} -\frac{6}{9} \\ \frac{12}{9} \end{pmatrix}, -\frac{1}{3}\right), \left(\begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}, \begin{pmatrix} \frac{12}{9} \\ -\frac{6}{9} \end{pmatrix}, \frac{1}{3}\right)\right) \\
\left(\begin{pmatrix} -\frac{4}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}, \begin{pmatrix} -\frac{2}{3} \\ -\frac{2}{3} \end{pmatrix}\right), \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, 0
\end{array}\right)
\end{aligned}
$$