Produktions mulighedsområde

$$Y = \left\{ y \in \mathbb{R}^3 | y_0 + y_1 + y_2 \leq 0 \wedge y_0 \leq 0 \wedge y_1, y_2 \geq 0 \right\}$$

Finansielle aktiver og initial beholdninger

	Asset		Init.	end.
	1	2	e^a	e^b
0			2	2
1	1	0	2	0
2	0	1	0	2

Nyttefunktioner

$$u^a(c) = \frac{1}{3}\ln c(0) + \frac{1}{3}\ln c(1) + \frac{1}{3}\ln c(2) = u^b(c)$$

Initiale aktieandele

$$\begin{pmatrix} \alpha^a \\ \alpha^b \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$$

Radner ligevægt givet produktions plan $y = \left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$$\begin{pmatrix} c^a \\ c^b \\ p \end{pmatrix} = \begin{pmatrix} \left(\frac{14}{9}, \frac{20}{9}, \frac{2}{9}\right) \\ \left(\frac{10}{9}, \frac{4}{9}, \frac{22}{9}\right) \\ (1,1,1) \end{pmatrix}$$

Indirekte nyttefunktioner når $r^i = (\alpha^i p(t)\, y(t))$

$$\begin{aligned} v^a(p, r^a) &= \sum_t \frac{1}{3}\ln\left(e^a(t) + \frac{r^a(t)}{p(t)}\right) \\ v^b(p, r^b) &= \sum_t \frac{1}{3}\ln\left(e^b(t) + \frac{r^b(t)}{p(t)}\right) \end{aligned}$$

Gradienten for transformationsfunktionen i $y = \left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$$D_y F\left(-\frac{4}{3}, \frac{2}{3}, \frac{2}{3}\right) = (1, 1, 1)$$

Netto indkomst overførsler i ligevægt

$$\begin{aligned} r^a &= \left(-\frac{4}{9}, \frac{2}{9}, \frac{2}{9}\right) \\ r^b &= \left(-\frac{8}{9}, \frac{4}{9}, \frac{4}{9}\right) \end{aligned}$$

Gradienterne til de indirekte nyttefunktioner i ligevægt

$$\begin{aligned} D_r v^a\left(p, \bar{r}^a\right) &= \left(\frac{1}{3p\left(t\right)} \frac{1}{\left(e^a\left(t\right) + \frac{r^a(t)}{p(t)}\right)}\right) = \left(\frac{3}{14}, \frac{3}{20}, \frac{3}{2}\right) \\ D_r v^b\left(p, \bar{r}^b\right) &= \left(\frac{1}{3p\left(t\right)} \frac{1}{\left(e^b\left(t\right) + \frac{r^b(t)}{p(t)}\right)}\right) = \left(\frac{3}{10}, \frac{3}{4}, \frac{3}{22}\right) \end{aligned}$$

Givet ændring i produktionsplan $dy = \left(-1, \frac{1}{2}, \frac{1}{2}\right)$ ændring i nytte

$$\begin{aligned} D_r v^a\left(p, \bar{r}^a\right) \cdot dy &= \left(\frac{3}{14}, \frac{3}{20}, \frac{3}{2}\right) \cdot \left(-1, \frac{1}{2}, \frac{1}{2}\right) = -\frac{3}{14} + \frac{1}{2}\frac{3}{20} + \frac{1}{2}\frac{3}{2} > 0 \\ D_r v^a\left(p, \bar{r}^a\right) \cdot dy &= \left(\frac{3}{10}, \frac{3}{4}, \frac{3}{22}\right) \cdot \left(-1, \frac{1}{2}, \frac{1}{2}\right) = -\frac{3}{10} + \frac{1}{2}\frac{3}{4} + \frac{1}{2}\frac{3}{22} > 0 \end{aligned}$$

Givet initial beholdninger $e^a = e^b = (2,2,0)$ er gradienterne for de indirekte nyttefunktioner givet ved

$$\begin{aligned} D_r v^a\left(p, \bar{r}^a\right) &= \left(\frac{3}{14}, \frac{3}{20}, \frac{3}{2}\right) \\ D_r v^b\left(p, \bar{r}^b\right) &= \left(\frac{3}{10}, \frac{3}{22}, \frac{3}{4}\right) \end{aligned}$$

Givet ændring i produktionsplan $dy = \left(-1, \frac{1}{2}, \frac{1}{2}\right)$ ændring i nytte

$$\begin{aligned} D_r v^a\left(p, \bar{r}^a\right) \cdot dy &= \left(\frac{3}{14}, \frac{3}{20}, \frac{3}{2}\right) \cdot (-1, 0, 1) = -\frac{3}{14} + \frac{3}{2} > 0 \\ D_r v^a\left(p, \bar{r}^a\right) \cdot dy &= \left(\frac{3}{10}, \frac{3}{22}, \frac{3}{4}\right) \cdot (-1, 0, 1) = -\frac{3}{10} + \frac{3}{4} > 0 \end{aligned}$$

Maximering af nytte ved valg af produktionsplan:

$$\begin{aligned} u^a(y) &= \ln\left(2 - \frac{1}{3}\left(y(1) + y(2)\right)\right) + \ln\left(2 + \frac{1}{3}y(1)\right) + \ln\left(\frac{1}{3}y(2)\right) \\ u^b(y) &= \ln\left(2 - \frac{2}{3}\left(y(1) + y(2)\right)\right) + \ln\left(2 + \frac{2}{3}y(1)\right) + \ln\left(\frac{2}{3}y(2)\right) \end{aligned}$$

Første ordens betingelser:
a:

$$\begin{aligned} -\frac{1}{3}\frac{1}{2 - \frac{1}{3}\left(y_1 + y_2\right)} + \frac{1}{3}\frac{1}{2 + \frac{1}{3}y_1} &= -\mu_1 \\ -\frac{1}{3}\frac{1}{2 - \frac{1}{3}\left(y_1 + y_2\right)} + \frac{1}{3}\frac{1}{\frac{1}{3}y_2} &= -\mu_2 \\ \mu_1 &\geq 0 \\ \mu_2 &\geq 0 \end{aligned}$$

$\Rightarrow$ der giver løsningen $y = (-3, 0, 3)$

b:

$$
\begin{aligned}
-\frac{2}{3}\frac{1}{2-\frac{2}{3}(y_1+y_2)} + \frac{2}{3}\frac{1}{2+\frac{2}{3}y_1} &= -\mu_1 \\
-\frac{2}{3}\frac{1}{2-\frac{2}{3}(y_1+y_2)} + \frac{1}{3}\frac{1}{\frac{2}{3}y_2} &= -\mu_2 \\
\mu_1 &\geq 0 \\
\mu_2 &\geq 0
\end{aligned}
$$

$\Rightarrow$ der giver løsningen $y = \left(-\frac{3}{2}, 0, \frac{3}{2}\right)$