Nyttefunktioner

$$u^a(c) = \sum_{t=0}^{2} \alpha_1(t) \ln c_1(t) + \alpha_2(t) \ln c_2(t)$$

$$u^b(c) = \sum_{t=0}^{2} \gamma_1(t) \ln c_1(t) + \gamma_2(t) \ln c_2(t)$$

Parametre

	$\alpha_1(t)$	$\alpha_2(t)$	$\gamma_1(t)$	$\gamma_2(t)$
0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{8}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{1}{12}$
2	$\frac{3}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

Walras efterspørgselsfunktioner

$$\hat{f}^a\left(p, p \cdot e^a\right) = \left(\alpha_1(t) \frac{p \cdot e^a}{p_1(t)}, \alpha_2(t) \frac{p \cdot e^a}{p_2(t)}\right)_{t=0}^2$$
$$\hat{f}^b\left(p, p \cdot e^b\right) = \left(\gamma_1(t) \frac{p \cdot e^b}{p_1(t)}, \gamma_2(t) \frac{p \cdot e^b}{p_2(t)}\right)_{t=0}^2$$

Aggregerede netto efterspørgselsfunktion

$$z(p) = \left(\alpha_1(t) \frac{p \cdot e^a}{p_1(t)} + \gamma_1(t) \frac{p \cdot e^b}{p_1(t)} - e_1^a(t) - e_1^b(t), \alpha_2(t) \frac{p \cdot e^a}{p_2(t)} + \gamma_2(t) \frac{p \cdot e^b}{p_2(t)} - e_2^a(t) - e_2^b(t)\right)_{t=0}^2$$

Initialbeholdning

$$\bar{e}^a = \bar{e}^b = ((15, 9), (8, 4), (8, 4))$$

Allokering

$$\bar{c}^a = ((12, 12), (4, 4), (12, 4))$$
$$\bar{c}^b = ((18, 6), (12, 4), (4, 4))$$

Netto handler

$$\bar{z}^a = ((-3, 3), (-4, 0), (4, 0))$$
$$\bar{z}^b = ((3, -3), (4, 0), (-4, 0))$$

Initialbeholdninger

$$
\begin{aligned}
e^{a} &= ((12,12),(4,4),(12,4)) \\
e^{b} &= ((18,6),(12,4),(4,4))
\end{aligned}
$$

Førsteordens betingelser

$$
\begin{aligned}
\begin{pmatrix} \left(\frac{\alpha_1(0)}{c_1(0)},\frac{\alpha_2(0)}{c_2(0)}\right) \\ \left(\frac{\alpha_1(1)}{c_1(1)},\frac{\alpha_2(1)}{c_2(1)}\right) \\ \left(\frac{\alpha_1(2)}{c_1(2)},\frac{\alpha_2(2)}{c_2(2)}\right) \end{pmatrix} &= \lambda(0)\begin{pmatrix} p(0) \\ 0 \\ 0 \end{pmatrix} + \lambda(1)\begin{pmatrix} 0 \\ p(1) \\ 0 \end{pmatrix} + \lambda(2)\begin{pmatrix} 0 \\ 0 \\ p(2) \end{pmatrix} \\
p(0)(c(0)-e(0)) &= r(0) \\
p(1)(c(1)-e(1)) &= r(1) \\
p(2)(c(2)-e(2)) &= r(2)
\end{aligned}
$$

Spotmarked efterspørgselsfunktioner

$$
\begin{aligned}
g^{a}(p,r^{a}) &= \left(\frac{\alpha_1(t)}{\alpha_1(t)+\alpha_2(t)}\frac{p(t)\,e^{a}(t)+r^{a}(t)}{p_1(t)},\frac{\alpha_2(t)}{\alpha_1(t)+\alpha_2(t)}\frac{p(t)\,e^{a}(t)+r^{a}(t)}{p_2(t)}\right)_{t\in\mathbb{T}} \\
g^{b}(p,r^{b}) &= \left(\frac{\gamma_1(t)}{\gamma_1(t)+\gamma_2(t)}\frac{p(t)\,e^{b}(t)+r^{b}(t)}{p_1(t)},\frac{\gamma_2(t)}{\gamma_1(t)+\gamma_2(t)}\frac{p(t)\,e^{b}(t)+r^{b}(t)}{p_2(t)}\right)_{t\in\mathbb{T}}
\end{aligned}
$$

Indirekte nyttefunktioner

$$\begin{aligned}
v^a(p, r^a) &= \sum_{t=0}^{2} \alpha_1(t) \ln\left(\frac{\alpha_1(t)}{\alpha_1(t)+\alpha_2(t)} \frac{p(t)\,e^a(t)+r^a(t)}{p_1(t)}\right) + \alpha_2(t) \ln\left(\frac{\alpha_2(t)}{\alpha_1(t)+\alpha_2(t)} \frac{p(t)\,e^a(t)+r^a(t)}{p_2(t)}\right) \\
&= \sum_{t=0}^{2} (\alpha_1(t)+\alpha_2(t)) \ln(p(t)\,e^a(t)+r^a(t)) + K^a(p) \\
v^b(p, r^b) &= \sum_{t=0}^{2} \gamma_1(t) \ln\left(\frac{\gamma_1(t)}{\gamma_1(t)+\gamma_2(t)} \frac{p(t)\,e^b(t)+r^b(t)}{p_1(t)}\right) + \gamma_2(t) \ln\left(\frac{\gamma_2(t)}{\gamma_1(t)+\gamma_2(t)} \frac{p(t)\,e^b(t)+r^b(t)}{p_2(t)}\right) \\
&= \sum_{t=0}^{2} (\gamma_1(t)+\gamma_2(t)) \ln\left(p(t)\,e^b(t)+r^b(t)\right) + K^b(p)
\end{aligned}$$

Første ordens betingelser

$$\begin{aligned}
\frac{\alpha_1(t)}{p(t)\,e^a(t)+r^a(t)} + \frac{\alpha_2(t)}{p(t)\,e^a(t)+r^a(t)} &= \lambda\beta(t) \\
\beta(0)\,r^a(0)+\beta(1)\,r^a(1)+\beta(2)\,r^a(2) &= 0
\end{aligned}$$

Optimale indkomstoverførsler

$$\begin{aligned}
r^a(t) &= \left(\frac{(\alpha_1(t)+\alpha_2(t))}{\beta(t)} \sum_{s\in\mathbb{T}} \beta(s)\,p(s)\,e^a(s) - p(t)\,e^a(t)\right)_{t=0}^{2} = \left(\frac{(\alpha_1(t)+\alpha_2(t))\,\bar{w}^a}{\beta(t)} - w^a(t)\right)_{t=0}^{2} \\
r^b(t) &= \left(\frac{(\gamma_1(t)+\gamma_2(t))}{\beta(t)} \sum_{s\in\mathbb{T}} \beta(s)\,p(s)\,e^b(s) - p(t)\,e^b(t)\right)_{t=0}^{2} = \left(\frac{(\gamma_1(t)+\gamma_2(t))\,\bar{w}^b}{\beta(t)} - w^b(t)\right)_{t=0}^{2}
\end{aligned}$$

Ligevægtsbetingelser for indkomstoverførsler

$$\left(\alpha_1(t)+\alpha_2(t)\right)\sum_{s\in\mathbb{T}}\beta(s)\,p(s)\,e^a(s)-\beta(t)\,p(t)\,e^a(t)=-\left(\gamma_1(t)+\gamma_2(t)\right)\sum_{s\in\mathbb{T}}\beta(s)\,p(s)\,e^b(s)-\beta(t)\,p(t)\,e^b(t)$$

Indkomst fra salg af initialbeholdninger

	w^a	w^b
0	24	24
1	16	32
2	64	32
$\bar{w}^i$	$24+\beta(1)\,16+\beta(2)\,64$	$24+\beta(1)\,32+\beta(2)\,32$

Ligningssystem til bestemmelse af ligevægtspriser på indkomstoverførsler

$$\begin{aligned}(24+16\beta(1)+64\beta(2))+2\,(24+32\beta(1)+32\beta(2))-288\beta(1) &= 0\\ 2\,(24+16\beta(1)+64\beta(2))+(24+32\beta(1)+32\beta(2))-576\beta(2) &= 0\end{aligned}$$

Løsning

$$\bar{\beta}=\left(\bar{\beta}(0),\bar{\beta}(1),\bar{\beta}(2)\right)=\left(1,\frac{1}{2},\frac{1}{4}\right)$$

Gradienter i punkterne $\bar{r}^a=\bar{r}^b=(0,0,0)$

$$\begin{aligned}D_r v^a\left(\bar{p},\bar{r}^a\right) &= \left(\frac{\alpha_1(t)+\alpha_2(t)}{\bar{p}(t)\,e^a(t)+\bar{r}^a(t)}\right)=\left(\frac{1}{48},\frac{1}{96},\frac{1}{192}\right)\parallel\left(1,\frac{1}{2},\frac{1}{4}\right)\\ D_r v^b\left(\bar{p},\bar{r}^b\right) &= \left(\frac{\gamma_1(t)+\gamma_2(t)}{\bar{p}(t)\,e^b(t)+\bar{r}^b(t)}\right)=\left(\frac{1}{48},\frac{1}{96},\frac{1}{192}\right)\parallel\left(1,\frac{1}{2},\frac{1}{4}\right)\end{aligned}$$

Spotmarkeds efterspørgsel

$$\begin{aligned}g^a\left(\bar{p},\bar{r}^a\right) &= \left((12,12),(4,4),(12,4)\right)\\ g^b\left(\bar{p},\bar{r}^b\right) &= \left((18,6),(12,4),(4,4)\right)\end{aligned}$$