Problem Set 9

Solve before the classes April 29 and May 2.

Exercise 1

Exercise 17.B.1 in Mas-Colell, Whinston and Green.

Exercise 2

On page 588, the book defines $f(p) = \dfrac{1}{\alpha(p)}(p + z^+(p))$ for all $p \in \Delta$. By definition, $\Delta = \{p \in \mathbb{R}^L_+ \mid p_1 + \ldots + p_L = 1\}$, and $z^+_\ell(p) = \max\{z_\ell(p), 0\}$, and $\alpha(p) = \sum_{\ell=1}^L [p_\ell + z^+_\ell(p)]$.

(i) Graph the function $\max\{y, 0\}$ with y a real variable, and observe that this is a continuous function of y with non-negative values. Now explain, that $z(p)$ continuous in p implies $z^+(p)$ continuous in p (do not forget that L coordinates are involved).

(ii) Prove that $p \in \Delta$ implies $\alpha(p) \geq 1$. Explain, that continuity of $z(p)$ implies continuity of $\alpha(p)$.

(iii) Explain that for any $p \in \Delta$, $z^+(p)$ is an L-dimensional vector while $\alpha(p)$ is a real number. Explain then, that $f(p)$ is an L-dimensional vector.

(iv) Finally verify, that $f(p) \in \Delta$ for any $p \in \Delta$. I.e., $f_\ell(p) \geq 0$ and $\sum_{\ell=1}^L f_\ell(p) = 1$.

Exercise 3

(i) A consumer has CES utility on two goods, $u(x_1, x_2) = (x_1^{-2} + K x_2^{-2})^{-1/2}$, where $K > 0$ is a fixed constant. Solve this consumer's problem at prices $p = (p_1, p_2)$ and income w.

(ii) Let the consumer have initial endowments $\omega = (1, 0)$, and find the demand function $x(p)$.

(iii) A similar consumer has utility $u(x_1, x_2) = (K x_1^{-2} + x_2^{-2})^{-1/2}$ with the same constant K, and initial endowments $\omega = (0, 1)$. Find this consumer's $x(p)$ (use your result from (ii)).

(iv) Now we aim to find all equilibria in an exchange economy consisting of those two consumers, where $K = (12/37)^3$. Let $p_2 = 1$, and use the variable $q = p_1^{1/3}$. Show that there is market clearing exactly when $0 = 12q^3 - 37q^2 + 37q - 12$. Show that $q = 1$ solves this equation, and find the two remaining solutions.