Simple Utility Functions with Giffen Demand*

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February 16, 2006

Abstract

Simple utility functions with the Giffen property are presented: locally, the demand curve for a good is upward sloping. The utility functions represent continuous, monotone, convex preferences.

Keywords: Preferences, Giffen Good.

JEL Classification: D11 (Consumer Economics: Theory)

*This work grew out of teaching experiences. I am particularly grateful to a class of 2003 that gave feedback through their written exam. I also thank the anonymous referee.

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1 Introduction

Most microeconomics textbooks mention Giffen goods as a theoretical possibility within standard demand theory. They illustrate most other theoretical properties by more convincing fully specified examples, but usually explain the Giffen property in a picture with two indifference curves. The (student) reader must mentally fill in the gap.

Below some simple, standard utility functions with the Giffen property are presented. From a technical point of view, the trick is to use modified Leontief preferences with a widened angle at the indifference curve kink. This permits demand to be downward sloping in income, equivalent to the Giffen effect since the substitution effect is zero at the kink.[1]

In recent literature, [6] advocates for simple analytical examples of Giffen goods. The example in [6] does not satisfy convexity, prompting [7] to reference the textbook example in [9]. [9] defines the utility function on a strict subset of $\mathbb{R}^2_+$, and [7] conveys the impression that it has no extension on $\mathbb{R}^2_+$ satisfying all the standard properties. But [9] provides the extension in a figure, referring to an analytical definition in [8]. This early example satisfies only weak convexity, while [4] offers an example with strictly convex preferences.

The Slutsky equation has led some empirical researchers to search for the Giffen effect among goods consumed in large quantities. However, as observed by [5], price changes in important goods are often associated with income changes, making it difficult to empirically isolate the Giffen effect. However, in the below examples, the Giffen effect arises in situations where the substitution effect is nil. Then a good is Giffen if and only if it is inferior, without regard to the quantity consumed.

2 Examples

Suppose a consumer has utility function $u(x) = \min\{u_1(x), u_2(x)\}$ where u_1 and u_2 are utility functions. Three interpretations are natural. First,[2] the bundle x may allow the

[1] The idea of upward sloping demand is attributed to Giffen by [3]. [3] suggests that a bread price increase may so adversely affect poor labor families, that their only possible response is to increase the bread consumption. The essential mechanism is the negative income effect.

[2] This is similar in spirit to [1]'s consumer theory. [2] follows [1] more serenely, discussing the conditions for the Giffen effect.

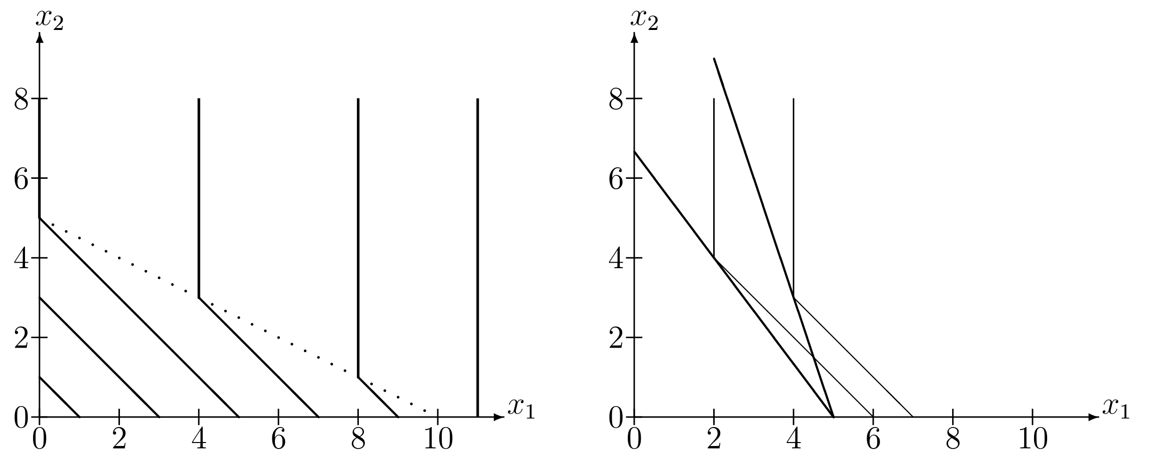

Figure 1: The left panel depicts indifference curves for Example 1, with $A = 2$ and $B = 10$. The straight dotted line indicates the locus of indifference curve kinks. The right panel illustrates the Giffen effect. The consumer's income is $m = 60$, and $p_1 = 12$. When $p_2 = 4$, the consumer demands $x = (4, 3)$. A price increase to $p_2 = 9$ changes the demand to $x = (2, 4)$. The demand for good 2 rises from 3 to 4 as its price rises.

consumer to perform two activities, thinking in amount u_1, and walking in amount u_2. The activities are complements, providing utility $u = \min\{u_1, u_2\}$. Second, suppose the bundle is purchased before the consumer knows whether the true utility function is u_1 or u_2. The infinitely risk averse consumer has ex ante utility function $u = \min\{u_1, u_2\}$. Third, the consumer may have multiple selves, one with utility u_1 and another with u_2. The consumer maximizes a Rawlsian welfare aggregate $u = \min\{u_1, u_2\}$.

Let the consumption set be the usual $\mathbb{R}^2_+$. Given income $m > 0$ and price vector $p = (p_1, p_2) \in \mathbb{R}^2_{++}$, the consumer chooses $x = (x_1, x_2)$ to maximize $u(x)$ subject to the budget constraint $p_1 x_1 + p_2 x_2 \leq m$. The maximand is the Marshallian demand $x(p, m)$.

Example 1. Let $u_1(x_1, x_2) = x_1 + B$ and $u_2(x_1, x_2) = A(x_1 + x_2)$ where $A > 1$ and $B > 0$. The left panel of Figure 1 illustrates some indifference curves which have kinks on the straight line $x_2 = (B - (A-1)x_1)/A$, sloping down from bundle $(0, B/A)$ to bundle $(B/(A-1), 0)$. The demanded $x(p, m)$ lies on this kink line when $p_1 > p_2$ and $Bp_2/A < m < Bp_1/(A-1)$. In this region, the demand function for good 2 is $x_2(p_1, p_2, m) = [Bp_1 - (A-1)m]/[Ap_1 - (A-1)p_2]$. Here, x_2 is an increasing function of p_2, i.e., a Giffen good. The right panel of Figure 1 illustrates the Giffen effect.

Proposition 1 *Assume u_1 and u_2 are utility functions representing continuous, monotone,* *convex preferences on* $\mathbb{R}^2_+$. *Then* $u = \min\{u_1, u_2\}$ *has the same properties. Assume that* $\hat{x} \in \mathbb{R}^2_{++}$ *solves* $u_1(\hat{x}) = u_2(\hat{x})$, *that* u_1 *and* u_2 *are* C^1 *at* $\hat{x}$, *and that the marginal rates of substitution* $c_k = [\partial u_k(\hat{x})/\partial x_1] / [\partial u_k(\hat{x})/\partial x_2]$ *satisfy* $c_1 > c_2 > 0$. *Take as given a price vector* $\hat{p} \in \mathbb{R}^2_{++}$ *with* $c_1 > \hat{p}_1/\hat{p}_2 > c_2$, *and let* $\hat{m} = \hat{p}\cdot\hat{x}$. *If* $\partial u_1(\hat{x})/\partial x_2 > \partial u_2(\hat{x})/\partial x_2$ *then good* 2 *is a Giffen good near* $(\hat{p}, \hat{m})$ *for the consumer with utility function* u. *If, instead,* $\partial u_2(\hat{x})/\partial x_1 > \partial u_1(\hat{x})/\partial x_1$, *good* 1 *is the Giffen good.*

Proof. Note that $u(x) \geq \bar{u}$ if and only if $u_1(x) \geq \bar{u}$ and $u_2(x) \geq \bar{u}$. It follows that u represents continuous, monotone, convex preferences. The indifference curve for u through $\hat{x}$ has a kink. By the implicit function theorem applied to $u_1(x) = u_2(x)$, if $\partial u_1(\hat{x})/\partial x_2 \neq \partial u_2(\hat{x})/\partial x_2$, the locus of kinks extends locally through $\hat{x}$ with slope

$$\frac{dx_2}{dx_1} = -\frac{\partial u_1(\hat{x})/\partial x_1 - \partial u_2(\hat{x})/\partial x_1}{\partial u_1(\hat{x})/\partial x_2 - \partial u_2(\hat{x})/\partial x_2}. \tag{1}$$

By the assumptions, the demand $x(p, m)$ is on the kink curve when (p, m) is near $(\hat{p}, \hat{m})$. If $\partial u_1(\hat{x})/\partial x_2 > \partial u_2(\hat{x})/\partial x_2$, then $c_1 > c_2$ implies $0 > dx_2/dx_1 > -c_2$. The kink curve is flatter than the indifference curves, as in Figure 1, so good 2 is Giffen. If $\partial u_2(\hat{x})/\partial x_1 > \partial u_1(\hat{x})/\partial x_1$, likewise $0 > dx_1/dx_2 > -1/c_1$, and good 1 is Giffen. ■

Example 2. In this example, preferences are strictly convex, indifference curves are closed in $\mathbb{R}^2_{++}$, and $u_1(0) = u_2(0) = 0$. Specifically, $u_1(x_1, x_2) = (x_1^{c_1} x_2)^{1/(1+c_1)}$ and $u_2(x_1, x_2) = (x_1^{c_2} x_2)^{1/(2+2c_2)}$ with $c_1 > c_2 > 0$, so u_1 and u_2 represent the familiar Cobb-Douglas preferences. There is a kink at $\hat{x} = (1, 1)$, where $\partial u_1(\hat{x})/\partial x_2 = 1/(1 + c_1) > \partial u_2(\hat{x})/\partial x_2 = 1/(2 + 2c_2)$ if $1 + 2c_2 > c_1$. Then good 2 is Giffen.

The examples exploit the zero substitution effect at a kink in the indifference curve. This extreme can be relaxed through approximation with kink-free functions, as in [4]. More directly, the function $\min\{u_1, u_2\}$ is approximated by the constant elasticity of substitution (CES) function $(u_1^\rho + u_2^\rho)^{1/\rho}$ as $\rho \to -\infty$. In the two examples, u_1 and u_2 are concave functions of x, so also $u(x) = (u_1^\rho(x) + u_2^\rho(x))^{1/\rho}$ is concave in x. The function u therefore represents continous, monotone, convex preferences. When $-\rho$ is sufficiently large, all indifference curves near $\hat{x}$ are sufficiently close to those of $\min\{u_1(x), u_2(x)\}$, and the good is Giffen also in the CES case.

Generally, $u = \min\{u_1, u_2\}$ inherits standard preference properties from u_1 and u_2. One exception, central to the welfare theorems, is local non-satiation. Suppose that $u_1(x_1, x_2) = x_1 - x_2$ and $u_2(x_1, x_2) = x_2 - x_1$. These two functions satisfy local non-satiation, but u does not, for it reaches a satiation utility level of 0 on the diagonal.

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