The Timing of Parimutuel Bets*

Marco Ottaviani† **Peter Norman Sørensen**‡

May 2006

Abstract

We propose a dynamic model of parimutuel betting that addresses the following three empirical regularities: a sizeable fraction of bets is placed early, late bets are more informative than early bets, and proportionally too many bets are placed on longshots. Exploiting a similarity with Cournot oligopoly, we show that bettors have an incentive to bet early when they are large and act on common information. Bettors who are instead privately informed and small have an incentive to bet at the end without access to the information of the others. Longshots (or favorites) are then less (or more) likely to win than indicated by the market odds.

Keywords: Parimutuel betting, timing, oligopoly, information, favorite-longshot bias.

JEL Classification: D82 (Asymmetric and Private Information), D84 (Expectations; Speculations), G13 (Contingent Pricing; Futures Pricing), L13 (Oligopoly and Other Imperfect Markets).

*Earlier versions of this paper were titled "Late Informed Betting and the Favorite-Longshot Bias" and "The Timing of Bets and the Favorite-Longshot Bias". We thank Marcus Asplund, David Goldreich, Denis Gromb, Johnnie Johnson, Maureen O'Hara, Stefano Sacchetto, Stephen Salant, Lones Smith, Xavier Vives, Tim van Zandt, Bill Zame, Bill Ziemba, Jeff Zwiebel, and seminar participants at the 2003 North American Summer Meetings of the Econometric Society, 2004 European Meetings of the Econometric Society, Economic Theory Workshop (Odense), Aarhus, Bristol, Copenhagen, Essex, European University Institute, Helsinki, London Business School, London School of Economics, Michigan, Southampton, Tilburg, University College London, and Venice for helpful comments.

†Economics Subject Area, London Business School, Sussex Place, Regent's Park, London NW1 4SA, England. Phone: +44-20-7262-5050 ext. 3778. Fax: +44-20-7402-0718. E-mail: `mottaviani@london.edu`. Web: http://faculty.london.edu/mottaviani.

‡Department of Economics, University of Copenhagen, Studiestræde 6, DK–1455 Copenhagen K, Denmark. Phone: +45–3532–3056. Fax: +45–3532–3000. E-mail: `peter.sorensen@econ.ku.dk`. Web: http://www.econ.ku.dk/sorensen.

1 Introduction

According to the efficient market hypothesis, the price of a financial asset is an unbiased estimate of its fundamental value, reflecting all the information available to the market (Fama (1991)). It has proven difficult to perform direct tests of this hypothesis in regular financial markets, due to the lack of exogenous measures of fundamental values of the assets traded.[1] Researchers have turned to betting markets, where fundamental values are both observable and typically exogenous, and have found a number of somewhat puzzling empirical facts. In this paper, we propose a theoretical explanation for these regularities.

Our analysis is based on the institutional features of *parimutuel betting* markets.[2] These are mutual markets, in which the total money bet on all outcomes (net of the *track take*)[3] is shared proportionally among those who bet on the winning outcome. Typically, bets are placed in real time, resulting in provisional odds that are publicly displayed and updated at regular intervals until *post time*, when betting is closed. Since the payments are made exclusively on the basis of the final distribution of bets, individual bettors can only take positions without knowing with certainty the odds they face.[4]

In the context of a horse race, market efficiency predicates that the final distribution of parimutuel bets is directly proportional to the market's assessment of the horses' chances of winning. This is because the gross expected payoff of a bet on a horse is equal to the ratio of its probability of winning to the proportion of bets placed on that horse. Equivalently, the expected payoffs on the different horses are equalized when the fraction of money bet on each of them is equal to the probability that the horse wins.

Starting with Griffith (1949), data on parimutuel bets have been used to test this

[1] Typically, the fundamental values of the assets traded in financial markets are not directly observable. In addition, the performance variables used as proxies of value are not exogenous, being themselves affected by market prices.

[2] Since its introduction in the nineteenth century, parimutuel betting has become the most common wagering system at horse-racking tracks throughout the world. Parimutuel schemes are also adopted for betting on outcomes of many sport events (e.g., soccer and basketball) as well as by lotteries.

[3] The track take includes taxes and other expenses for running the race and the betting scheme.

[4] It is worth contrasting parimutuel betting with the alternative scheme of "fixed odds" betting (cf. Dowie (1976)). In fixed odds betting, bookmakers accept bets at specific, but changing, odds throughout the betting period. With fixed odds, the return to any individual bet is therefore not affected by the bets placed subsequently.

proposition. The proportion of money bet on a horse has been shown to track closely its empirical chance of winning, in support of the market efficiency hypothesis. However, three empirical regularities have emerged:

- A large amount of money is placed just before post time, but sizeable amounts are placed much earlier (see e.g. Camerer (1998) and National Thoroughbred Racing Association (2004)).[5] If more information becomes available later, why are so many bets placed well before post time? This is the puzzle of *early betting*.

- Late bets tend to contain more information about the horses' finishing order than earlier bets (see e.g. Asch, Malkiel and Quandt (1982) and Gandar, Zuber and Johnson (2001)). This is the phenomenon of *late informed betting*.

- Horses with short odds (i.e., favorites) tend to win even more frequently than indicated by the final market odds, while horses with long odds (i.e., longshots) win less frequently (see e.g. Thaler and Ziemba (1988) and Snowberg and Wolfers (2005)). This is known as the *favorite-longshot bias*.

In this paper we formulate a theoretical model that sheds light on these facts. The model posits an exogenous initial distribution of bets placed by *outsiders*, so that the rational *insiders* can earn non-negative returns despite the presence of a positive track take. Each insider has some private information, modeled as an informative signal about the outcome of the race. These informational assumptions are similar to those made in the market microstructure literature, and applied to fixed-odds betting by Shin (1991 and 1992). We focus instead on parimutuel betting markets, in the context of which the empirical evidence reported above has been collected.[6]

In the context of this model, we study the strategic incentives that drive the timing of bets placed by the insiders. Our starting point is a prevailing intuition that has been first formulated by Asch, Malkiel and Quandt (1982): "bettors who have inside information

[5] In Camerer's data set roughly half of the money is placed in the last three minutes before post time, and half (often well) before then.

[6] We refer to Ottaviani and Sørensen (2005) for a comparison of the equilibrium outcomes resulting in parimutuel and fixed-odds betting markets. See also the discussion in Section 6.

would prefer to bet late in the period so as to minimize the time that the signal is available to the general public." We show that this result holds provided that insiders are small, in the sense that their individual effect on the odds is negligible. In this case, each insider has an incentive to wait in order to hide one's *private information* and possibly see that of others. When instead the insiders are large and can affect the odds, they have a countervailing strategic incentive to bet early in order to prevent others from exploiting the *market power* stemming from the common information shared among them. In this observation lies the main contribution of this paper.

To understand our explanation of the early betting puzzle, consider a small number of bettors who share the same information about the horses' winning chances, and so are not concerned about revealing this information. Due to the parimutuel structure, the payoff per dollar bet on a profitable horse is a decreasing function of the total bets placed on that horse. Bettors are effectively competing in a market with a downward sloping demand curve, with the final price determined by the final distribution of bets. We show that bettors have then an incentive to act early, in order to prevent competitors from unfavorably changing the odds against them (Proposition 1). This strategic incentive to place early bets is based on the simple fact that parimutuel betting is a Cournot game, and appears not to have been noticed before in the literature.

Our explanation of the second regularity — late informed betting — is based on the presence of private information. We show that when the insiders are "small," in the sense that they are price takers and so have no market power, bets are simultaneously placed at post time (Proposition 3). Our analysis reveals that the incentive to bet late is due to the fact that parimutuel prices are determined by the overall number of orders made in the market, regardless of when the orders are placed. The information revealed by early orders can then be used by other bettors who affect the price to the detriment of the early bettors. Our result on late informed betting formalizes the conditions needed to verify Asch, Malkiel and Quandt's (1982) intuition reported above. This incentive to bet late is similar to the one that operates in auctions with a fixed deadline (see e.g. Roth and

Ockenfels (2002)) and in pre-opening markets (Medrano and Vives (2001)).[7]

Our explanation of the third regularity — the favorite-longshot bias — is based on the informational content of the equilibrium distribution of simultaneous bets, characterized in Proposition 2. We argue that the presence of private information introduces a systematic wedge between the final distribution of bets and the market's beliefs. To understand this, suppose for the moment that some privately informed insiders bet simultaneously at post time. When many (or few) insiders end up betting on the same outcome, which now becomes more of a favorite (or longshot), it means that many (or few) had private information in favor of this outcome. If individual bettors knew that many bets would have been placed on that outcome, they would have been even more likely to bet on that outcome. But the final distribution of bets is not known when betting, so that this inference can only be made after betting is closed. Intuitively, a disproportionately low (or high) fraction of bets are placed on favorites (or longshots), because these bets were placed without knowing the final odds (Proposition 4). Despite its simplicity, this explanation of the favorite-longshot bias has not been proposed before.[8]

Our results have implications for the design of market mechanism for trading contingent claims. As discussed by Economides and Lange (2005), the parimutuel structure has been recently adopted in new markets for contingent claims on economic statistics, such as US nonfarm payroll employment figures and European harmonized indices of consumer prices.[9] These markets allow traders to hedge risks related to these variables. A major advantage of parimutuel markets is that the intermediary managing the system is not exposed to any risk. On the flip side, market participants are subject to risk on the terms

[7] The incentive to delay trade is in contrast to theoretical predictions obtained in the context of regular financial markets with continuous trading, in which every order to buy asset A immediately increases the price of A, thereby eroding the profitability of later buy orders for A. As shown by Holden and Subrahmanyam (1992) and Back, Cao and Willard (2000), competition among a large enough number of insiders in regular financial markets tends to result in almost immediate trade. Note that fixed-odds markets considered by Schnytzer and Shilony (2002) are equivalent to regular financial markets. We return to this comparison with regular financial markets at the end of the paper.

[8] Ottaviani and Sørensen (2004) further analyze this informational explanation. See Section 5.3 for a discussion of the other explanations proposed in the literature on the favorite-longshot bias.

[9] Since October 2002, Deutsche Bank and Goldman Sachs have been hosting Parimutuel Derivative Call Auctions of options on economic statistics. Baron and Lange (2003) report on the performance of these markets.

of trade and might have incentives to delay their orders. If traders are small and have private information, they might trade late and place orders mostly on the basis of their limited information, without access to information revealed by other market participants.[10] Parimutuel markets are not conducive to strong market efficiency, due to the incentive to postpone one's trade and free ride on the private information of others.[11]

In the literature, Koessler and Ziegelmeyer (2002) formulated the first analysis of parimutuel betting under asymmetric information. They assumed that bettors have binary signals and bet sequentially with exogenous order. We instead allow bettors to have continuous signals, as is commonly done in auction theory. In the context of our tractable model, we are able to provide a simple characterization of equilibrium betting, offer insights about the forces driving the endogenous timing of bets, and derive the favorite-longshot bias.[12] We defer a more comprehensive discussion of the literature to Section 5.3.

The paper proceeds as follows. We formulate the model in Section 2. In general, bets not only affect odds but also reveal information to other bettors. We analyze these two effects in isolation by considering two versions of the model in turn. In Section 3 we consider the case in which individual bettors are large enough to have a non-negligible effect on the odds, but are not concerned about revealing information. We show that early betting results in this case. In Section 4 we turn to the second version of the model in which there is a continuum of privately informed bettors, and show that in equilibrium they all postpone their bets to the end. For this purpose, we characterize the equilibrium of the last-period simultaneous betting game and then we endogenize the timing by allowing the bettors to decide when to act. In Section 5 we show that the favorite-longshot bias results when the large bettors act early on the basis of public information as well as when the privately informed bettors delay betting to the last period. We conclude in Section 6 by discussing the predictions of our theory. The proofs are collected in Appendix A. Appendix B contains additional details not intended for publication.

[10] See Wolfers and Zitzewitz (2004) for a broad introduction to the informational content of market-generated forecasts.

[11] Trade will instead be early if it is based on common information and individual traders have market power. But note that these conditions are rather undesirable for the other market participants.

[12] See also Feeney and King (2001) for the characterization of the equilibria in a sequential parimutuel game with complete information and exogenous order.

2 Model

We present a stylized model of parimutuel betting on the *outcome* of a race between two horses, A and B.[13] The winning horse is identified with the *state*, $x \in \{A, B\}$.

Time is discrete and betting is open in a commonly known finite window of time, with periods denoted by $t = 0, 1, \ldots, T$. As explained in detail below, in period $t = 0$ bettors receive information about the state. Betting opens at $t = 1$ and closes at $t = T$ (post time). A publicly observable tote board displays in any period the cumulative amounts bet on each horse until post time.[14]

At the racetrack, some bets are placed for recreational purposes based on idiosyncratic preferences for particular horses, while others are motivated by profit maximization. Even though in reality individual bettors could be motivated by a combination of recreation (private value) and profits (common value), for convenience our model separates recreational from profit-maximizing bettors. In this way, we depart from the literature on preferences for risk taking, but follow an approach commonly adopted in market microstructure models, in which noise traders are usually separated from informed arbitrageurs.[15]

The amount of exogenously given bets placed on outcome x at time $t = 1$ by unmodeled *outsiders* is denoted by $n(x)$.[16] For simplicity, we assume that the amount of outsiders' bets is not random and publicly known. Liquidity or noise traders play a similar role in more traditional models of financial markets.

There are I rational bettors, or *insiders*. The insiders are risk neutral and maximize their expected return.[17] Thanks to the presence of the outsiders, the insiders are able to derive a non-negative expected return from betting. The insiders are assumed to share a common *prior belief* $q = \Pr(A)$ that horse A is the winner, possibly formed after the

[13] We formulate the model for the simplest case with two horses, but clearly our insights apply to more realistic settings with a larger number of horses.

[14] For example, the UK's Tote updates the display every 30 seconds. The assumption of discrete time is realistic and technically convenient, but is not essential for our results.

[15] Similar results holds when bettors are ex-ante identical and are motivated by the sum of a recreational utility and the expected monetary payoff of the gamble (see Ottaviani and Sørensen (2004)).

[16] While in betting markets these outsiders often have recreational motives, in hedging markets their role could be played by market participants with private values from hedging certain risks.

[17] Payoffs are not discounted, since betting takes place within a short time frame.

observation of a common signal. We consider two cases:

- In Section 3 we assume that there is finite number of large insiders who share the same information. In this case each insider is allowed to bet a variable amount on either horse.

- In Section 4 we consider the case with a continuum $[0, I]$ of atomistic insiders, each endowed with some private information (see Section 4.1). In this case we assume that each insider faces an identical wealth constraint, being able to bet an amount normalized to 1. Each insider decides if, when, and whether to bet on $x = A$ or $x = B$.

The total amount bet by the insiders on outcome x is denoted by $m(x)$.

We make the realistic assumption that bets cannot be cancelled once they are made.[18] The total amount bet by insiders and outsiders is placed in a pool, from which a proportional *track take* τ is taken. The remaining money is then evenly distributed to the winning bets. If x is the winner, each unit bet on outcome x yields

$$(1-\tau)\frac{\sum_{y=A}^{B}[n(y)+m(y)]}{n(x)+m(x)}. \tag{1}$$

The insiders know the exact amounts $n(A)$ and $n(B)$ and the other parameters of the model.

3 Early Betting with Market Power

In this section, we show that parimutuel payoffs generate an incentive to bet early. In order to isolate this incentive, we allow the bettors to be "large", i.e., to place bets of arbitrary (non-negative) size. A bettor who can make a sizable bet faces an adverse movement in the odds, and should consider this effect when deciding how much to bet. This market

[18] Typically, patrons are not allowed to cancel their bet after leaving the seller's window, unless they can prove to have made a mistake. The limitations on the right to cancel bets are set in order to prevent manipulation and preserve integrity of the wagering process. See e.g., Canada Department of Justice (1991), paragraph 57, and Delaware Harness Racing Commission (1992), rule 9.4.1.3.

power channel introduces an incentive to bet early, before other bettors place their bet to one's detriment.

Following Hurley and McDonough (1995), we assume that there is a *finite* number I of rational bettors who share the same information about the state. Clearly, in this setting the amounts bet by others cannot then reveal any information.

Assume that the common prior belief, the track take and pre-existing bets are such that $q(1-\tau) > n(A) / [n(A) + n(B)]$, i.e. the pre-existing market probability is sufficiently lower than the prior belief that it is profitable to bet on A. If bettors $i = 1, \ldots, I$ place the amounts $m_1, \ldots, m_I$ on outcome A, bettor i's payoff is

$$U_i(m_i) = q(1-\tau) \frac{n(A) + n(B) + \sum_{i=1}^{I} m_i}{n(A) + \sum_{i=1}^{I} m_i} m_i - m_i.$$

This game is a special version of Cournot's oligopoly model of quantity competition. To see this, interpret the amount m_i as the quantity produced at constant marginal unit cost by firm i. Let the market's inverse demand curve be equal to

$$p(m) = q(1-\tau) \frac{n(A) + n(B) + m}{n(A) + m}, \tag{2}$$

where m is the aggregate quantity.

The I bettors play a dynamic Cournot game. As is well known, in the two-player Stackelberg game, the leader would bet a larger amount and earn greater profits than the follower. By increasing the bet size beyond the static Cournot outcome, the leader pushes the follower to reply with a smaller amount. When the timing is endogenous, one would intuitively expect the insiders to place their bets as soon as possible, in order to profit from the early mover advantage.

Following Hamilton and Slutsky (1990), we consider an "extended game with action commitment", according to which each bettor first commits to the one period in which to bet, and in that period decides how much to bet as a function of the betting history until then. At period t, it can be observed how much was bet by each bettor in all previous periods $1, \ldots, t-1$, excluding the current period t. In our setting, this timing assumption incorporates the fact that the tote board can be observed, in addition to three less appealing properties: (i) each bettor can bet only once, (ii) each bettor cannot change strategy in reaction to others' bets as the game proceeds, and (iii) each bettor can observe precisely the set of bettors who have already bet and are therefore no longer active. We believe that this technical timing assumption can be dropped without consequence for the result, but the assumption conveniently allows us to apply a known result. Matsumura's Proposition 3 allows us to conclude::

Proposition 1 *With I informed bettors, there are two possible subgame perfect equilibrium outcomes. In the first equilibrium, all bettors bet at $t = 1$, when they play the static Cournot outcome. In the second equilibrium, all but one bettor place bets at $t = 1$.*

Proof. See Appendix A. □

In every equilibrium, all (but at most one) bettors place simultaneous bets in the first period, immediately after receiving their public information. Intuitively, market power gives an incentive to move early, in order to capture a good share of the money on the table. This can explain why a sizeable fraction of bets are placed well before betting is closed — our first puzzle.[19]

4 Late Betting with Private Information

In this section, we turn to late informed betting. We assume that there is a continuum $[0, I]$ of small, privately informed insiders, each of whom can make a unit bet. We show that the presence of private information introduces an incentive that operates in the opposite direction to the one identified in the last section. Since bettors are small, their individual payoffs (1) are not affected by any individual's bet. This price-taking assumption is the essential feature of our continuum population assumption.[20]

In Section 4.1 we extend the model to allow the insiders to have private information. In Section 4.2, we analyze the simultaneous betting game that takes place in the last period, $t = T$. We show that there exists a unique equilibrium in period T, and characterize the

[19] An alternative explanation for early betting is based on the exogenous preference of bettors for placing bets early. Our explanation for early betting is instead based on the strategic response of insiders to the presence of outsiders.

[20] With a continuum of bettors, each of them takes the price as given. See our companion paper Ottaviani and Sørensen (2004) for an analysis of simultaneous betting with a finite number of insiders.

equilibrium strategies. In Section 4.3, we solve the dynamic game by using a backwards induction argument. We prove that the insiders postpone their bets until the last period, and characterize the amounts bet on the two horses. In Section 5, we then show that the equilibrium exhibits the favorite-longshot bias.

4.1 Private Information

Private information is believed to be pervasive in horse betting (see e.g. Crafts (1985)). It is modeled as follows. At $t = 0$, before betting begins, each insider i privately observes a *signal* s_i. Conditionally on state x, these signals are assumed to be identically and independently distributed with probability density function $f(s|x)$, where the index i has been dropped.[21]

Upon observation of signal s, the prior belief q is updated according to Bayes' rule into the *posterior belief* $p = \Pr(A|s) = qf(s|A) / [qf(s|A) + (1-q) f(s|B)]$.[22] This transformation of s into p determines the conditional distributions of p given x from the corresponding distributions of s given x. The posterior belief p is then distributed according to the cumulative probability distribution function G on $[0,1]$. By the law of iterated expectations, G must satisfy $q = E[p] = \int_0^1 p\, dG(p)$.[23]

We assume that G is *continuous*[24] with density g and has *full support* equal to the beliefs set $[0,1]$. The full support assumption requires that that there are arbitrarily informative signal realizations and guarantees the existence of an interior equilibrium.

The conditional distributions of the posterior belief p given x play a central role in our analysis. By Bayes' rule, we have $p = qg(p|A)/g(p)$ and $1-p = (1-q)\, g(p|B)/g(p)$. The densities of the posterior beliefs are then $g(p|A) = pg(p)/q$ and $g(p|B) = (1-p)\, g(p)/(1-$

[21] See Watanabe (1997) for an alternative model of parimutuel betting with a continuum of bettors. In his model, the bettors' heterogeneous beliefs are assumed to be common knowledge. In our model instead, beliefs reflect heterogeneous information about the state.

[22] Note that the case of common information analyzed in Section 2 is a degenerate outcome of this model resulting when the signals are completely uninformative, so that $p = q = \Pr(x = A)$ always.

[23] While the conditional distributions of the signal is our primitive, there is no loss of generality in making assumptions directly on G. Any posterior belief distribution with $q = E[p]$ can be derived from the assumption that the distribution of the signal s is the same as the distribution of the posterior p.

[24] In the presence of discontinuities in the posterior belief distribution, equilibria might involve mixed strategies. Our results can be extended to allow for these discontinuities.

q). The monotonicity in p of the likelihood ratio $g(p|A)/g(p|B) = [p/(1-p)][(1-q)/q]$ reflects the property that higher beliefs in outcome A are relatively more likely when outcome A is true. This monotonicity implies that $G(p|A)$ first-order stochastically dominates $G(p|B)$, so that $G(p|A) < G(p|B)$ for all $p \in (0,1)$.

4.2 Equilibrium Betting in the Last Period

Cumulative bets committed by insiders at end of period $T-1$ are considered part of the outside bets $n(A), n(B)$. The population size I refers to the continuum of insiders who did not bet before the last period, and G is the cumulative distribution of their posterior beliefs. We maintain the assumption that G has full support.[25]

In a Bayes-Nash equilibrium, every rational bettor best replies to a correctly predicted fraction of the insiders who bet on each outcome in each state. Denote by $m(y|x)$ the amount bet by the insiders on outcome $y \in \{A, B\}$ when state $x \in \{A, B\}$ is true. If state x is true, the gross payoff to a bet on outcome x is

$$W(x|x) = (1-\tau)\frac{n(A)+n(B)+m(A|x)+m(B|x)}{n(x)+m(x|x)} > 0. \tag{3}$$

Consider the decision problem of a bettor with belief p. The expected return from a unit bet on outcome A is $pW(A|A) - 1$, where p is the probability that horse A wins, $W(A|A)$ is the gross payoff from a unit bet on A and -1 is the cost of this unit bet. Similarly, the expected payoff for a bet on B is $(1-p)W(B|B) - 1$, and for not betting is 0. It follows immediately that there exist thresholds beliefs $0 \leq \hat{p}_B \leq \hat{p}_A \leq 1$ such that the bettor optimally bets on B when $p < \hat{p}_B$, abstains when $\hat{p}_B < p < \hat{p}_A$, and bets on A when $p > \hat{p}_A$.

If the winning probability of horse A implied by the pre-existing bet is not too extreme compared to the track take, in the unique equilibrium insiders bet on both outcomes.

Proposition 2 *Assume that*

$$0 \leq \tau < \min\left\{\frac{n(A)}{n(A)+n(B)}, \frac{n(B)}{n(A)+n(B)}\right\}. \tag{4}$$

[25] We argue below that the case in which G is no longer unbounded ($0 < G(p) < 1$ for all $p \in (0,1)$) results in a rather trivial last-period game.

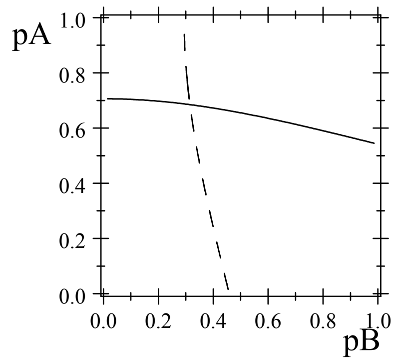

Figure 1: This illustrates the determination of equilibrium in the linear signal example with $q = 1/2$, $n(A) = n(B) = I$, and $\tau = .15$. The solid line represents the set of $(\hat{p}_B, \hat{p}_A)$ solving (5), while the dashed line represents the solutions to (6). Both curves are downward sloping, and the solid line crosses from below the dashed line at the equilibrium values of $(\hat{p}_B, \hat{p}_A)$.

There exists a unique Bayes-Nash equilibrium of the last-period game. Every insider bets on B when $p < \hat{p}_B$*, abstains when* $\hat{p}_B < p < \hat{p}_A$*, and bets on A when* $p > \hat{p}_A$*, where the thresholds* $0 < \hat{p}_B < \hat{p}_A < 1$ *constitute the unique solution to the two indifference conditions*

$$\hat{p}_A = \frac{1}{1-\tau} \frac{n(A) + I\left[1 - G(\hat{p}_A|A)\right]}{n(A) + n(B) + I\left[1 - G(\hat{p}_A|A)\right] + IG(\hat{p}_B|A)} \tag{5}$$

and

$$\hat{p}_B = 1 - \frac{1}{1-\tau} \frac{n(B) + IG(\hat{p}_B|B)}{n(A) + n(B) + I\left[1 - G(\hat{p}_A|B)\right] + IG(\hat{p}_B|B)}. \tag{6}$$

Proof. See Appendix A. □

Note that when the pre-existing bets heavily favor outcome A or the track take is very large, the gross expected payoff of a bet on A is $W(A|A) < 1$ regardless of how many insiders bet on B. If so, no bets are then placed on outcome A in equilibrium. Condition (4) rules out such situations.

Equation (5) is derived from the indifference condition $\hat{p}_A W(A|A) = 1$ by using $m(A|A) = I\left[1 - G(\hat{p}_A|A)\right]$ and $m(B|A) = IG(\hat{p}_B|A)$. As seen in Figure 1, this results in an inverse relationship between $\hat{p}_A$ and $\hat{p}_B$. To see why this is the case, suppose by contradiction that instead $\hat{p}_A$ and $\hat{p}_B$ were to both rise, so that fewer insiders bet on A, and more insiders bet on B. Ceteris paribus, such a change makes it more attractive to bet on A, so that $W(A|A)$ rises. But $\hat{p}_A$ is determined by the indifference between betting on A and not betting, so a rise in $W(A|A)$ implies a fall in $\hat{p}_A$, in contradiction with the initial supposition. Similarly, the indifference condition $(1-\hat{p}_B)W(B|B)=1$ results in the downward sloping relationship (6). The proof of the proposition establishes that these curves cross precisely once, as illustrated in Figure 1.

Example. Our results are conveniently illustrated by the *linear signal example* with conditional densities $f(s|A)=2s$ and $f(s|B)=2(1-s)$ for $s\in[0,1]$, and corresponding cumulative distributions $F(s|A)=s^2$ and $F(s|B)=1-(1-s)^2$. This signal structure can be derived from a binary signal with uniformly distributed precision. With fair prior $q=1/2$, we have $p=s$ so that $G(p|A)=p^2$ and $G(p|B)=2p-p^2$.

In this example with fair prior ($q=1/2$), balanced pre-existing bets ($n(A)=n(B)\equiv n$), and track take $\tau\leq 1/2$, the unique symmetric-policy Nash equilibrium has an explicit expression, with cutoff belief

$$\hat{p}_A = 1-\hat{p}_B = \frac{(1-\tau)\left(1+\frac{n}{I}\right)-\sqrt{\left(1+\frac{n}{I}\right)\left[\tau^2+\frac{n}{I}(1-\tau)^2\right]}}{(1-2\tau)} \in [1/2,1).$$

4.3 Equilibrium in the Dynamic Game

Having characterized the equilibrium in the last period, we now turn to the analysis of the full dynamic game, in which insiders can bet in any period between $t=1$ and $t=T$. We return to the original assumption that $n(A)$ and $n(B)$ denote the known non-random amounts of outsider bets. A behavioral strategy for an insider with a given privately observed signal specifies how much remaining wealth to bet on either horse if any, after each publicly observed history. A perfect Bayesian equilibrium specifies a behavioral strategy for each bettor, such that every bettor's strategy is optimal given the other bettors' strategies. Perfection requires that the continuation strategies should again constitute a perfect Bayesian equilibrium of the remaining game after any publicly observed tote board history, given rationally updated beliefs.

Proposition 3 *Assume that (4) holds. Then:*

(i) In all perfect Bayesian equilibria, the total amounts bet by the insiders are equal to those implied by Proposition 2.

(ii) There exists a perfect Bayesian equilibrium in which all betting is postponed to the last period.

(iii) If the thresholds of Proposition 2 satisfy $\hat{p}_B \leq q \leq \hat{p}_A$, this equilibrium is unique.

Proof. See Appendix A. □

Proposition 3 concords with Asch, Malkiel and Quandt's (1982) empirical finding that late changes in odds predict the finishing order very well. Our theory suggests that both the favorite-longshot bias and late informed betting can be ascribed to the presence of private information.[26]

The proof of (i) also establishes that no information can be revealed by early bets on the equilibrium path. Notice that condition $\hat{p}_B \leq q \leq \hat{p}_A$ required for result (iii) is always satisfied when the model is symmetric. If this condition fails, say $\hat{p}_A < q$, then early betting could take place by insiders with private beliefs $p \in [p_1, p_2]$ where $\hat{p}_A < p_1 < q < p_2$ and $G(p_2|A) - G(p_1|A) = G(p_2|B) - G(p_1|B)$. Such bets do not reveal any information, and are of no consequence for the final distribution of bets, according to (i).

Our analysis of the timing game employs some simplifying assumptions. First, the proof of the proposition is somewhat facilitated by the continuum population assumption, in that a single player's deviation cannot be observed at all. However, the logic of this late timing result is much stronger. If an early bettor on horse A signals favorable information for this horse, then later bettors will find horse A more attractive and horse B less attractive. But from (3), the early bettor's return is decreasing in $m(A|A)$ and increasing in $m(B|A)$, so he does not desire later bettors to follow his lead in this fashion.[27] The incentive of informed traders to postpone their bet to the last minute is thus driven by the fact that

[26] Alternatively, late informed betting could be due by the fact that more information becomes publicly available during the betting period for exogenous reasons. However, an explanation based on public information would not be able to account for the presence of the favorite-longshot bias.

[27] Moreover, postponing the bet gives the option of learning from the other insiders' bets, although this cannot happen in the equilibrium of our model.

in parimutuel betting all trades are executed at the same final price.[28]

Second, we have assumed that the tote board is updated after each period, so that at time t the total amounts wagered at times $1, \ldots, t-1$ is publicly known. In reality, there might be a slight delay in displaying bets on the tote board. With a delay of $D > 1$ periods, our prediction is that informed betting takes place within the last D betting rounds, when betting is essentially simultaneous.

Third, we have assumed that the post time T is certain and commonly known. In reality, sometimes there is uncertainty about T and this affects the incentives to time bets. According to Proposition 3, in equilibrium no bets are placed by the insiders until there is a positive chance of market closure. Some insiders would then bet somewhat before the actual closing time, and so reveal part of their information. As a result, the outcome is closer to the rational expectations equilibrium.

As we have seen in this section, individual bettors have an incentive to bet late if they have private information. This is consistent with the empirical observation that a sizeable fraction of bets are placed slightly before betting is closed. A similar incentive to act late has been observed in the context of other call markets, such as auctions with fixed deadlines (see e.g. Roth and Ockenfels (2002)) and pre-opening markets (Biais, Hillion and Spatt (1999) and Medrano and Vives (2001)). Parimutuel betting markets and open auctions share the feature that bets (or bids) placed before the deadline are publicly observed to competing bettors (or bidders). However, there is an important difference — while in an open auction early bids are valid only until they are topped by a competitor, in parimutuel markets early bets remain binding regardless of subsequent bets and affect the uniform clearing price in conjunction with subsequent bets. We have isolated the effects of this feature in Section 3.

Our theoretical findings are compatible with experimental results recently obtained by Plott, Wit and Yang (2003) in laboratory parimutuel markets. Their experimental subjects were endowed with limited budgets and given private signals informative about

[28] The National Thoroughbred Racing Association (2004) is concerned that last-minute betting is facilitated by new methods of off-track betting, and that last-minute odds changes drive away a significant number of bettors.

the likelihood of the different outcomes. Subjects could place bets up to their budget before the random termination of the markets. Compared to our model, the presence of a random termination time gives bettors an additional incentive to move early in order to reduce the termination risk. Although Bayes' rule was explained to the experimental subjects, not all profitable bets were made and some favorite-longshot bias was observed. Some of the bettors postponed placing their limited budget, gambling that the termination would happen later. As a result, the market odds were not equalized to the posterior odds. This outcome is consistent with our explanation.

5 Favorite-Longshot Bias

The market odds ratio for horse x is defined as the net return to a bet on x if x wins, i.e. $W(x|x) - 1$. The implied market probability for horse x is $(1-\tau)/W(x|x)$, equal to the fraction of bets placed on it. According to a simple formulation of the market efficiency hypothesis, this market probability should aggregate market beliefs into an unbiased estimator of the horse's true probability of winning.

In order to test this proposition, empirical investigations have typically proceeded by pooling data from many horse races. The outcomes of the races to which horses participate are used to estimate the horses' empirical probability of winning depending on their market odds. The oft-observed favorite-longshot bias reveals that the greater the implied market probability of horse x, the greater the empirical average return to a dollar bet on x. Market probabilities thus understate the winning chances of favorites, and overstate the winning chances of longshots.

In accordance with this empirical approach, we compute the market implied probability for horse x in our models and relate it to the probability that horse x wins conditional on the information contained in the distribution of bets.

5.1 Large Commonly Informed Bettors

Consider first the case with large commonly informed bettors, analyzed in Section 3. When the insiders are commonly informed but have market power, they all play a best response to the quantities bet by the opponents. The true chance of horse A is $q = \Pr(x = A)$, and we consider the situation where $q(1-\tau) > n(A)/[n(A)+n(B)]$, i.e. the pre-existing market probability is short of q. Bettors suffer inframarginal losses from increasing their own bets and so cease betting before the price for the last unit bet is equal to the marginal cost.

The market implied probability for outcome A, $[n(A)+m]/[n(A)+n(B)+m]$, is lower than $q(1-\tau)$, which would result in zero profits. In turn it is lower than q, so although we assume that the rational bettors are placing their money on this horse (in this sense a favorite), the true probability of its winning is greater than the market implied probability. This finding is in line with the favorite-longshot bias. This explanation of the favorite-longshot bias is an extension of Isaacs' (1953) explanation derived in the context of a monopolistic bettor to our setting with multiple (oligopolistic) bettors.

5.2 Small Privately Informed Bettors

Consider next the case with small privately informed bettors, analyzed in Section 4. According to Proposition 3, when there is a continuum of small privately informed bettors, the outcome of betting is characterized by the simultaneous equilibrium of Proposition 2. Since higher private beliefs are more frequent when the true outcome is A (i.e., $G(p|B) > G(p|A)$ for all $0 < p < 1$), in equilibrium each individual bets more frequently on outcome x in state x. Therefore, in aggregate we have $m(A|A) > m(A|B)$ and $m(B|B) > m(B|A)$. This implies that horse x has a higher market probability (i.e., is more favored) when state x is true, and that the amounts $m(A|x), m(B|x)$ bet by the insiders fully reveal x. Upon observation of $m(A|x), m(B|x)$, outcome x is revealed to be true with probability one. The implied market probabilities, on the other hand, are never so extreme, since we always have $1-\tau < W(x|x) < \infty$ and therefore $(1-\tau)/W(x|x) \in (0,1)$. We conclude that the equilibrium outcome of our model exhibits the favorite-longshot bias.

Proposition 4 *Assume (4). In the unique Bayes-Nash equilibrium of the simultaneous last-period game, there is a favorite-longshot bias.*

The result hinges on the fact that in a Bayes-Nash equilibrium, each insider does not know the total amount wagered by the other insiders. In a rational expectations equilibrium (REE), it is assumed instead that bettors can adjust their actions until they are satisfied with their bet, given their knowledge of the aggregate distribution of bets.[29] Notice that an REE must be perfectly revealing in this setting,[30] so that in the REE all insiders bet on the winner and we have $m(A|A) = m(B|B) = I$.[31] In our Bayes-Nash equilibrium, some insiders bet instead on the longshot, and not enough insiders bet on the favorite. As a result, the market implied probabilities are driven less towards the truth than in the REE. We conclude that our Bayes-Nash equilibrium results in a stronger favorite-longshot bias than the corresponding REE.

Potters and Wit (1996) have formulated the closest antecedent to our informational explanation for the favorite-longshot bias in parimutuel markets. In their as well as in our model, the favorite-longshot bias arises as a deviation from the rational expectations equilibrium. In Potters and Wit's model, the privately informed bettors are given the chance to adjust their bets at the final market odds, but they ignore the information contained in the bets.[32] In our model instead, bettors fully understand the informational link, but are not allowed to adjust their bets after observing the final market odds. If the market closes immediately after the informed bets are placed, the market's tâtonnement process cannot incorporate this private information and reach a rational expectations equilibrium.

The favorite-longshot bias takes an extreme form in Proposition 4 since the continuum of bettors reveals the true outcome. This assumption allows us to derive the favorite longshot bias in a very stark way, but it is not essential for the result. An immediate way to obtain a less extreme result is to let the race outcome be random, even conditionally

[29] In reality the aggregate amounts bet are observable only after all bets have been placed. REE models assume that traders have more information than is actually available to them.

[30] To see this, note that in an REE bettors also employ threshold strategies. If the state were not revealed, a positive fraction of them would bet on either horse (as above), and the amounts would then reveal the true outcome.

[31] Note that the insiders cannot completely remove the favorite-longshot bias even in the REE, due to their limited wealth. See Manski (2004) for an REE model of the Iowa Electronic Market, in which a deviation from the efficient market hypothesis arises due to limited wealth of traders.

[32] Ali's (1977) Theorem 2 also features bettors with diverse beliefs who ignore others' information.

on the x which aggregates the private information.[33]

In the model presented in this paper the favorite-longshot bias always arises because there is no noise in the final distribution of bets due to the presence of a continuum of insiders. With a finite number of bettors, Ottaviani and Sørensen (2004) show that the sign and extent of the favorite-longshot bias depends on the interaction of noise and information. In the presence of a few bettors with little private information, posterior odds are close to prior odds, even with extreme market odds, so that deviations of market odds from prior odds are mostly due to the noise contained in the signal. In that case, the market odds tend to be more extreme than the posterior odds, resulting in a reverse favorite-longshot bias.[34] As the number of bettors increases, the realized market odds contain less noise and more information. For any given level of market odds, the corresponding posterior odds are then more extreme and so the favorite-longshot bias is more pronounced. The noise vanishes when there is a large enough number of bettors. In the limit considered here, the favorite-longshot bias always results.

5.3 Discussion

We now compare our explanation of the favorite-longshot bias with the main alternatives proposed in the literature.[35] The literature can be broadly classified into two groups, depending on whether the explanation is based on the preferences of bettors or on the market microstructure. In the first group, Griffith (1949) suggested that individuals subjectively ascribe too large probabilities to rare events, while Weitzman (1965) and Ali (1977) hypothesized that individual bettors are risk loving, and so are willing to give up a larger expected payoff when assuming a greater risk (longer odds). By not modeling market structure and information, theories based on preferences cannot explain the timing of bets or the dependence of the extent of the favorite-longshot bias on market rules.

[33] This point can be shown formally by extending the model as follows. Suppose that the outcome is z, while the state x is a noisy binary signal of the outcome z. The private signal is informative about x, but, conditional on x, its distribution is independent of z. In this setting, the symmetric equilibrium features the favorite-longshot bias (see Proposition 5 of Ottaviani and Sørensen (2004)).

[34] This explains why the reverse favorite-longshot bias is observed in parimutuel lotteries, in which bettors do not have private information.

[35] For a more extensive review of these explanations, see the surveys by Hausch and Ziemba (1995), Sauer (1998), and Jullien and Salanié (2002).

The explanations based on market rules apply to either parimutuel or fixed odds betting. For the specific case of parimutuel markets, Isaacs (1953) noted that an informed monopolist bettor would not bet until the marginal bet has zero value. Hurley and McDonough (1995) argued that a sizeable track take and the inability to place negative bets limits the amount of arbitrage by bettors with superior information, and so tends to result in relatively too few bets placed on the favorites. The explanation we have proposed in Section 3 and 5.1 extends Isaacs' argument from monopoly to our oligopoly setting with multiple bettors. Note that in the competitive limit (when the number of bettors I tends to infinity), the favorite-longshot bias only results in the the presence of a positive track take. In our model with a finite number of bettors, the favorite-longshot bias instead results also when $\tau = 0$.

For fixed odds betting markets, Shin (1991 and 1992) explained the favorite-longshot bias as the response of an uninformed bookmaker to private information possessed by insider bettors. In Shin's model, a bookmaker with market power sets odds that display the favorite-longshot bias in order to limit the subsequent losses to the better informed insiders. As we have shown in Section 5.2, a similar bias results from simultaneous betting in parimutuel markets.[36] Models with private information can therefore explain the favorite-longshot bias in both parimutuel and fixed-odds markets as well as why late bets contain more information about the horses' finishing order than early bets.

6 Conclusion

In this paper, we have formulated theoretical explanations for both the timing of informative bets and the favorite-longshot bias observed in parimutuel markets. Our explanations are based on the incentives of rational insiders, who derive positive expected profits in the presence of outsiders who enjoy betting for exogenous reasons. The simple model proposed here therefore combines behavioral elements with traditional equilibrium analysis.

Our key assumption is that insiders are privately informed about the outcome of the

[36] However, the logic that drives the result is different in the two markets. As shown by Ottaviani and Sørensen (2004), in the two market structures the favorite-longshot bias has different comparative statics properties with respect to changes in the track take and other parameters.

race. From the theoretical point of view, it is natural to posit that the "differences in opinion that makes horse races" arise from private information. Indeed, the presence of private information in horse-race betting is corroborated by independent evidence. Once private information is introduced, betting markets can realize their full potential as testbeds for theories of price formation.

We have found that private information is broadly consistent with a number of regularities observed in parimutuel markets. The timing incentives of the insiders depend crucially on the nature of their information. On the one hand, the insiders bet *early* based on the part of the information that is *common* among them. The insiders have an incentive to preempt each other in capturing shares of the profitable bets that are available thanks to the presence of outsiders. Since there is no need to hide information that is already available to the other insiders, common information results in early betting. On the other hand, the insiders bet *late* based on the part of their information that is *private*. The insiders have an incentive to delay betting in order to conceal their private information.[37]

We have identified a scenario with many small insiders in which *all* informed bets are placed at the end of the betting period. Clearly, our insight applies provided that *some* informed bets are placed simultaneously at the end. As a result, the final market odds do not reflect the beliefs of the market but rather tend to be less extreme than the posterior belief based on the information revealed by the final distribution of bets. Essentially, market efficiency fails because the final market odds are not known when the bets are placed.

The parimutuel payoff structure has a built-in insurance against adverse selection, that is not present in fixed-odds markets. In the presence of unfavorable information about a horse, this horse attracts few bets and so automatically becomes more attractive once the parimutuel odds are adjusted to balance the budget. This effect tends to result in a reduced favorite-longshot bias in parimutuel compare than in fixed-odds markets, as observed empirically by Bruce and Johnson (2001). For a comparison of the equilibrium

[37] The analysis of the interplay of these two opposing incentives is an interesting topic for future research. This problem cannot be recast as a special case of other models of endogenous timing with countervailing incentives, such as Brunnermeier and Morgan (2005).

outcomes in parimutuel and fixed-odds markets we refer to Ottaviani and Sørensen (2005).

Our analysis of the parimutuel structure sheds light on the effect of market rules on the incentives for timing trades. In regular financial markets with continuous trading, transactions are completed when orders are placed. Extending Kyle's (1985) model to allow for multiple informed traders who share the same information, Holden and Subrahmanyam (1992) have shown that in equilibrium trading is immediate, and so information is revealed quickly.[38] Competing insiders with noisy information do take some time to reveal their information (Foster and Viswanathan (1996)), but immediate trade tends to result when the number of informed insiders becomes large (Back, Cao and Willard (2000)). In contrast, we have shown that delay results in the competitive limit of a parimutuel market when traders have imperfect information. This difference is driven by the fact that in parimutuel call markets traders commit their orders at a transaction price that depends on the orders placed subsequently. Trading rules play a key role for the aggregation of private information into prices.

[38] In Kyle's (1985) continuous auction model, a single informed trader has an incentive to spread trading over time, in order to optimally hide the information behind the steady flow of noise trading.

References

Ali, Mukhtar M., "Probability and Utility Estimates for Racetrack Bettors," *Journal of Political Economy*, 1977, 85(4), 803–815.

Asch, Peter, Burton G. Malkiel and Richard E. Quandt, "Racetrack Betting and Informed Behavior," *Journal of Financial Economics*, 1982, 10(2), 187–194.

Back, Kerry, C. Henry Cao and Gregory A. Willard, "Imperfect Competition among Informed Traders," *Journal of Finance*, 2000, 55(5), 2117–2155.

Baron, Ken and Jeffrey Lange, "From Horses to Hedging," *Risk (www.risk.com)*, February 2003, 73–77.

Biais, Bruno, Pierre Hillion and Chester Spatt, "Price Discovery and Learning during the Preopening Period in the Paris Bourse," *Journal of Political Economy*, 1999, 107(6), 1218–1248.

Bruce, Alistair C. and Johnnie Johnson, "Efficiency Characteristics of a Market for State Contingent Claims," *Applied Economics*, 2001, 33, 1751–1754.

Brunnermeier, Markus K. and John Morgan, "Clock Games: Theory and Experiments," 2005, Princeton University and University of California at Berkeley, mimeo.

Camerer, Colin F., "Can Asset Markets Be Manipulated? A Field Experiment with Racetrack Betting," *Journal of Political Economy*, 1998, 106(3), 457–482.

Canada Department of Justice, *Pari-Mutuel Betting Supervision Regulations* (SOR/91-365), 1991, http://laws.justice.gc.ca/en/C-46/SOR-91-365/index.html.

Crafts, Nicholas, "Some Evidence of Insider Knowledge in Horse Race Betting in Britain," *Economica*, 1985, 52(207), 295–304.

Delaware Harness Racing Commission, *Rules and Regulations*, 1992, http://www.state.de.us/deptagri/harness/forms/Harness_RuleRegs.pdf.

Dowie, Jack, "On the Efficiency and Equity of Betting Markets," *Economica* 1976, 43(170): 139–150.

Economides, Nicholas and Jeffrey Lange, "A Parimutuel Market Microstructure for Contingent Claims Trading," *European Financial Management*, 2005, 11(1), 25–49.

Fama, Eugene F., "Efficient Capital Markets: II," *Journal of Finance*, 1991, 46(5), 1575–1617.

Feeney, Rob and Stephen P. King, "Sequential Parimutuel Games," *Economics Letters*, 2001, 72, 165–173.

Foster, F. Douglas and S. Vishwanathan, “Strategic Trading When Agents Forecast the Forecasts of Others,” *Journal of Finance*, 1996, 51(4), 1437–1478.

Gandar, John M., Richard A. Zuber and R. Stafford Johnson, “Searching for the Favourite-Longshot Bias Down Under: An Examination of the New Zealand Pari-mutuel Betting Market,” *Applied Economics*, 2001, 33, 1621–1629.

Griffith, R. M., “Odds Adjustment by American Horse-Race Bettors,” *American Journal of Psychology*, 1949, 62(2), 290–294.

Hamilton, Jonathan H. and Steven M. Slutsky, “Endogenous Timing in Duopoly Games: Stackelberg or Cournot Equilibria,” *Games and Economic Behavior*, 1990, 2, 29–46.

Hausch, Donald B. and William T. Ziemba, “Efficiency of Sports and Lottery Betting Markets,” in *Handbook of Finance*, edited by R.A. Jarrow, V. Maksimovic, and W. T. Ziemba (North Holland Press), 1995.

Holden, Craig and Avindhar Subrahmanyam, “Long-Lived Private Information and Imperfect Competition,” *Journal of Finance*, 1992, 47(1), 247–270.

Hurley, William and Lawrence McDonough, “Note on the Hayek Hypothesis and the Favorite-Longshot Bias in Parimutuel Betting,” *American Economic Review*, 1995, 85(4), 949–955.

Isaacs, Rufus, “Optimal Horse Race Bets,” *American Mathematical Monthly*, 1953, 60(5), 310–315.

Jullien, Bruno and Bernard Salanié, “Empirical Evidence on the Preferences of Racetrack Bettors”, 2002, CREST and IDEI, mimeo.

Koessler, Frédéric and Anthony Ziegelmeyer, “Parimutuel Betting under Asymmetric Information,” 2002, THEMA, Université de Cergy-Pontoise, mimeo.

Kyle, Albert, “Continuous Auctions and Insider Trading,” *Econometrica*, 1985, 53(6), 1315–1335.

Manski, Charles F., “Interpreting the Predictions of Prediction Markets,” 2004, Northwestern University, mimeo.

Matsumura, Toshihiro, “Quantity-Setting Oligopoly with Endogenous Sequencing,” *International Journal of Industrial Organization*, 1999, 17, 289–296.

Medrano, Luis A. and Xavier Vives, “Strategic Behavior and Price Discovery,” *Rand Journal of Economics*, 2001, 32(2), 221–248.

National Thoroughbred Racing Association, Wagering Systems Task Force, “Declining Purses and Track Commissions in Thoroughbred Racing: Causes and Solutions,” 2004, mimeo at www.hbpa.org/resources/Ntrataskreportsep04.pdf.

Ottaviani, Marco and Peter Norman Sørensen, "Noise, Information and the Favorite-Longshot Bias," 2004, London Business School and University of Copenhagen, mimeo.

Ottaviani, Marco and Peter Norman Sørensen, "Parimutuel versus Fixed-Odds Markets," 2005, London Business School and University of Copenhagen, mimeo.

Plott, Charles, Jörgen Wit and Winston C. Yang, "Parimutuel Betting Markets as Information Aggregation Devices: Experimental Results," *Economic Theory*, 2003, 22(2), 311–351.

Potters, Jan and Jörgen Wit, "Bets and Bids: Favorite-Longshot Bias and the Winner's Curse," 1996, CentER Discussion Paper 9604, Tilburg University.

Roth, Al and Axel Ockenfels, "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, 2002, 92(4), 1093–1103.

Sauer, Raymond D., "The Economics of Wagering Markets," *Journal of Economic Literature*, 1998, 36(4), 2021–2064.

Schnytzer, Adi and Yuval Shilony, "On the Timing of Insider Trades in a Betting Market," *European Journal of Finance*, 2002, 8(2), 176–186.

Shin, Hyun Song, "Optimal Betting Odds Against Insider Traders," *Economic Journal*, 1991, 101(408), 1179–1185.

Shin, Hyun Song, "Prices of State Contingent Claims with Insider Traders and the Favourite-Longshot Bias," *Economic Journal*, 1992, 102(411), 426–435.

Snowberg, Erik and Justin Wolfers, "Explaining the Favorite-Longshot Bias: Is it Risk-Love or Misperceptions?" 2005, Stanford GSB and Wharton School, mimeo.

Thaler, Richard H. and William T. Ziemba, "Anomalies: Parimutuel Betting Markets: Racetracks and Lotteries," *Journal of Economic Perspectives*, 1988, 2(2), 161–174.

Watanabe, Takahiro, "A Parimutuel System with Two Horses and a Continuum of Bettors," *Journal of Mathematical Economics*, 1997, 28(2), 85–100.

Weitzman, Martin, "Utility Analysis and Group Behavior: An Empirical Analysis," *Journal of Political Economy*, 1965, 73(1), 18–26.

Wolfers, Martin and Eric Zitzewitz, "Prediction Markets," *Journal of Economic Perspectives*, 2004, 18(2), 107–126.

Appendix A: Proofs

Proof of Proposition 1. This result follows directly from Proposition 3 of Matsumura (1999), once we verify the three conditions about two-stage betting games with exogenous timing and arbitrary pre-existing bets. To analyze this two-stage game, suppose that I_1 bettors are leaders while I_2 are followers. Let $m^1(x)$ and $m^2(x)$ denote the total amounts bet by the leaders and followers, respectively. The three conditions are:

1. There exists a pure strategy equilibrium and the equilibrium is unique.

2. If the number of followers is one, this follower strictly prefers the Cournot outcome to the follower's outcome.

3. If the number of leaders is one, this leader strictly prefers the leader's outcome to the Cournot outcome.

The verification of these conditions involves straightforward computations available on request from the authors. □

Proof of Proposition 2. Given a set of strategies of the continuum of opponents, every bettor can compute $m(y|x)$ and $W(x|x) > 0$. The bettor's expected returns satisfy the following monotonicity properties: $U(A|p) = pW(A|A) - 1$ is strictly increasing in p, $U(0|p) = 0$ is constant, and $U(B|p) = (1-p)W(B|B) - 1$ is strictly decreasing. It follows that every bettor has the same optimal response, characterized by the thresholds $\hat{p}_A$ and $\hat{p}_B$, with $\hat{p}_A \geq \hat{p}_B$. Since $U(A|0) = U(B|1) = -1 < 0 = U(0|p)$, we have $\hat{p}_A > 0$ and $\hat{p}_B < 1$. The amounts bet are $m(A|A) = I[1 - G(\hat{p}_A|A)]$, $m(A|B) = I[1 - G(\hat{p}_A|B)]$, $m(B|A) = IG(\hat{p}_B|A)$, and $m(B|B) = IG(\hat{p}_B|B)$. Now, if $\hat{p}_A = 1$, then $W(A|A) = (1-\tau)[n(A) + n(B) + m(B|A)]/n(A) \geq (1-\tau)[n(A) + n(B)]/n(A) > 1$ by assumption, so that $U(A|1) > U(0|1)$, contradicting optimality of the threshold $\hat{p}_A = 1$. Thus $\hat{p}_A < 1$, and a similar argument establishes $\hat{p}_B > 0$.

Next, we show that $\hat{p}_B < \hat{p}_A$. Suppose to the contrary that $\hat{p}_B = \hat{p}_A$. Indifference yields $\hat{p}_A W(A|A) = (1 - \hat{p}_A) W(B|B)$, solved by $\hat{p}_A = W(B|B) / [W(A|A) + W(B|B)] \in (0,1)$. Since abstention is not preferred, $\hat{p}_A W(A|A) \geq 1$, or equivalently $1 \geq 1/W(A|A) + 1/W(B|B)$. This is $1 - \tau \geq [n(A) + m(A|A)] / [n(A) + n(B) + m(A|A) + m(B|A)] +$

$[n(B)+m(B|B)]/[n(A)+n(B)+m(A|B)+m(B|B)]$. We now show that this fails since the right-hand side strictly exceeds 1, which is equivalent to $m(A|A)\,m(B|B) - m(B|A)\,m(A|B) > n(A)[m(B|A)-m(B|B)] + n(B)[m(A|B)-m(A|A)]$. This inequality follows since the left-hand side is positive and the right-hand side negative, by $m(A|A) > m(A|B) > 0$ and $m(B|B) > m(B|A) > 0$. These facts follow from the expressions for $m(y|x)$ and the property that $G(\hat{p}_A|A) < G(\hat{p}_A|B)$.

We now have $0 < \hat{p}_B < \hat{p}_A < 1$, and optimality of the threshold rule implies the indifference conditions $U(A|\hat{p}_A) = 0$ and $U(B|\hat{p}_B) = 0$. Simple algebra results in the conditions (5) and (6). Rewrite (5) as

$$G(\hat{p}_B|A) = \frac{n(A)}{I(1-\tau)\hat{p}_A} - \frac{n(A)+n(B)}{I} + \left[\frac{1}{(1-\tau)\hat{p}_A} - 1\right][1 - G(\hat{p}_A|A)]. \tag{7}$$

The right-hand side is a continuous function of $\hat{p}_A \in (0,1)$. It tends to infinity as $\hat{p}_A$ tends to 0 and is equal to $n(A)/[I(1-\tau)] - [n(A)+n(B)]/I < 0$ at $\hat{p}_A = 1$ by assumption (4). Moreover, its derivative is $-(n(A)+I[1-G(\hat{p}_A|A)])/[I(1-\tau)\hat{p}_A^2] - (1/[(1-\tau)\hat{p}_A]-1)\,g(\hat{p}_A|A) < 0$. It follows that for every $\hat{p}_B \in [0,1]$ there is a unique solution $\hat{p}_A \in (0,1)$ to equation (5). Thus equation (5) defines an implicit function in the space $(\hat{p}_B, \hat{p}_A) \in [0,1]^2$. By the implicit function theorem, this function is downward sloping with

$$\left.\frac{d\hat{p}_A}{d\hat{p}_B}\right|_{(5)} = -\frac{g(\hat{p}_B|A)}{\frac{n(A)+I[1-G(\hat{p}_A|A)]}{I(1-\tau)\hat{p}_A^2} + \left[\frac{1}{(1-\tau)\hat{p}_A} - 1\right] g(\hat{p}_A|A)} < 0.$$

Likewise, for every $\hat{p}_A \in [0,1]$ there is a unique solution $\hat{p}_B \in (0,1)$ to (6), and the set of solutions defines a downward sloping curve in the space of $(\hat{p}_B, \hat{p}_A) \in [0,1]^2$ with

$$\left.\frac{d\hat{p}_B}{d\hat{p}_A}\right|_{(6)} = -\frac{g(\hat{p}_A|B)}{\frac{n(B)+IG(\hat{p}_B|B)}{I(1-\tau)(1-\hat{p}_B)^2} + \left[\frac{1}{(1-\tau)(1-\hat{p}_B)} - 1\right] g(\hat{p}_B|B)} < 0.$$

Existence follows from the fact that the curve defined by (5) traverses continuously from the left side $\{0\} \times (0,1) \subseteq [0,1]^2$ to the right side $\{1\} \times (0,1) \subseteq [0,1]^2$ of the $[0,1]^2$ square, while the curve defined by (6) traverses continuously from the bottom $(0,1) \times \{0\} \subseteq [0,1]^2$ to the top $(0,1) \times \{1\} \subseteq [0,1]^2$. See Figure 1.

Uniqueness follows from the fact that the (6)-curve is steeper than the (5)-curve wherever they cross. Namely,

$$\left.\frac{d\hat{p}_A}{d\hat{p}_B}\right|_{(5)} \left.\frac{d\hat{p}_B}{d\hat{p}_A}\right|_{(6)} < 1.$$

This inequality is equivalent to

$$1<\left[\frac{n(A)+I(1-G(\hat{p}_A|A))}{I(1-\tau)\hat{p}_A^2 g(\hat{p}_A|B)}+\frac{1-(1-\tau)\hat{p}_A}{(1-\tau)\hat{p}_A}\frac{g(\hat{p}_A|A)}{g(\hat{p}_A|B)}\right]\left[\frac{n(B)+IG(\hat{p}_B|B)}{I(1-\tau)(1-\hat{p}_B)^2 g(\hat{p}_B|A)}+\frac{1-(1-\tau)(1-\hat{p}_B)}{(1-\tau)(1-\hat{p}_B)}\frac{g(\hat{p}_B|B)}{g(\hat{p}_B|A)}\right],$$

where all the terms are positive. The inequality therefore holds since

$$\begin{aligned} & \frac{1-(1-\tau)\hat{p}_A}{(1-\tau)\hat{p}_A}\frac{g(\hat{p}_A|A)}{g(\hat{p}_A|B)}\frac{1-(1-\tau)(1-\hat{p}_B)}{(1-\tau)(1-\hat{p}_B)}\frac{g(\hat{p}_B|B)}{g(\hat{p}_B|A)} \\ = & \frac{1-(1-\tau)\hat{p}_A}{(1-\tau)\hat{p}_A}\frac{\hat{p}_A}{1-\hat{p}_A}\frac{1-(1-\tau)(1-\hat{p}_B)}{(1-\tau)(1-\hat{p}_B)}\frac{1-\hat{p}_B}{1-(1-\hat{p}_B)}\geq 1 \end{aligned}$$

where we used $g(p|A)/g(p|B)=[p/(1-p)][(1-q)/q]$, $\hat{p}_A\geq(1-\tau)\hat{p}_A$, and $1-\hat{p}_B\geq(1-\tau)(1-\hat{p}_B)$. □

Proof of Proposition 3. (i) First, we show that in equilibrium no information about x can be revealed by the observable history of past bets before T. Namely, the insiders' bets placed in any period $t<T$ must satisfy $m(A|A)=m(A|B)$ and $m(B|A)=m(B|B)$. Otherwise, the true state x would be fully revealed in the following period $t+1$ and all the remaining bettors would bet on x, either until there are no more insiders or until $W(x|x)=1$. Every bettor at t with an interior belief $p\in(0,1)$ would strictly profit from postponing the bet, because either the bet is on the wrong outcome giving a loss of -1, or the bet returns the same as when postponing. This profitable deviation contradicts equilibrium.

We have therefore established that the game must proceed deterministically until period T, since the amounts bet by the insiders are independent of the true state. When the game reaches period T, the prior belief is still q. Now, the total amounts placed by insiders before and at time T must satisfy the characterization of the simultaneous equilibrium given in Proposition 2. For individual rationality in the last period again implies that there are thresholds $\check{p}_B,\check{p}_A$, such that individuals who did not bet before the final period will bet on B if $p<\check{p}_B$, bet on A if $p>\check{p}_A$, and otherwise abstain. Anyone who bet on horse B before time T must also have $p\leq\check{p}_B$, for the game proceeded deterministically, and at belief $\check{p}_B$ there is zero expected return to the bet on B. Likewise, early bettors on A must have $p\geq\check{p}_A$. But then we have established that the equilibrium must be in common threshold strategies, and Proposition 2 characterized the only such equilibrium.

(ii) The following strategy profile constitutes such a perfect Bayesian equilibrium. After any history, all remaining bettors postpone betting to the last period and play then the simultaneous Bayes-Nash equilibrium. The first observation of early insider bets makes beliefs shift to certainty that the most-bet horse is the winner; as long as both horses have received equal amount of insider bets, beliefs are unchanged.

To show that this is an equilibrium, consider any public history at time $t < T$. No individual player ever loses from postponing to the last period, for an earlier bet from the marginal player is too small to influence the beliefs of others through the tote board. Thus, the behavior of all other players is unaffected.

(iii) Suppose that bettors with private beliefs in the positive-measure set $K \subseteq [0,1]$ bet on horse A at some period $t < T$. Since these bettors should eventually bet as proved in (i), then $K \subseteq [\hat{p}_A, 1]$. In addition, these bets cannot reveal any information, so that $m(A|B) = I \Pr(K|B) = I \Pr(K|A) = m(A|A)$. But this equality is impossible, unless q belongs to the convex hull of K. To see this note that if $\hat{p}_A \geq q$, we have

$$\Pr(K|B) = \int_K g(p|B)\, dp = \int_K \frac{1-p}{p} \frac{q}{1-q} g(p|A)\, dp < \frac{1-\hat{p}_A}{\hat{p}_A} \frac{q}{1-q} \int_K g(p|A)\, dp \leq \Pr(K|A),$$

leading to a contradiction. Likewise, there cannot be a positive measure of betting on horse B. $\square$

Appendix B: Details Not Submitted for Publication

Omitted Details for the Proof of Proposition 1. We verify Matsumura's three conditions.

(1) Consider the simultaneous game of the followers. We concentrate on the case where $q(1-\tau) > [n(A)+m^1]/[n(A)+n(B)+m^1]$, for otherwise the leaders have bet irrationally too much. It is standard to show that there is a unique equilibrium of this simultaneous Cournot game. All followers bet the same amount m^2/I_2, determined by the first order condition

$$q(1-\tau)\left[\frac{n(A)+n(B)+m^1+m^2}{n(A)+m^1+m^2} - \frac{n(B)m^2/I_2}{[n(A)+m^1+m^2]^2}\right] = 1. \tag{8}$$

In equilibrium, $q(1-\tau) > [n(A)+m^1+m^2]/[n(A)+n(B)+m^1+m^2]$ so that all bettors earn a positive profit.

The leaders play a simultaneous betting game, taking into account how m^2 will respond to m^1. In equilibrium, all bets belong to the open range $(0, M)$, where M is the competitive amount defined by $q(1-\tau) = [n(A)+M]/[n(A)+n(B)+M]$. The necessary first order condition for an arbitrary leader's amount m_i^1 is then

$$q(1-\tau)\left[\frac{n(A)+n(B)+m^1+m^2}{n(A)+m^1+m^2} - \frac{n(B)m_i^1}{[n(A)+m^1+m^2]^2}\left(1+\left.\frac{dm^2}{dm^1}\right|_{(8)}\right)\right] = 1.$$

Notice the implication that $1 + dm^2/dm^1|_{(8)} > 0$, since otherwise the left-hand side would exceed $q(1-\tau)[n(A)+n(B)+m^1+m^2]/[n(A)+m^1+m^2]$ which strictly exceeds 1 when betting is profitable. Thus, holding m^1 fixed, the left-hand side is a strictly decreasing function of m_i^1. It follows that all leaders optimally respond with the same quantity $m_i^1 = m^1/I_1$, so that the first order condition reduces to

$$q(1-\tau)\left[\frac{n(A)+n(B)+m^1+m^2}{n(A)+m^1+m^2} - \frac{n(B)m^1/I_1}{[n(A)+m^1+m^2]^2}\left(1+\left.\frac{dm^2}{dm^1}\right|_{(8)}\right)\right] = 1. \tag{9}$$

It is now straightforward, if tedious, to verify that there exists a unique solution (m^1, m^2) to equations (8) and (9). We focus on the case where $I_1, I_2 > 0$ since it is otherwise the simpler simultaneous game.

To this end, we analyze (8) closer. For any $m^1 \in [0, M]$ there is a unique $m^2 \in [0, M]$ that solves (8). At $m^1 = 0$ the solution is $m_0^2 < M$, the inequality being due to the negative term of the LHS. At $m^1 = M$ we note that $m^2 = 0$. Thus, (8) defines a curve in (m^1, m^2)-space travelling from $(0, m_0^2)$ to $(M, 0)$, and this curve is continuous by the implicit function theorem. Implicit differentiation of (8) gives

$$\left.\frac{dm^2}{dm^1}\right|_{(8)} = -\frac{I_2\left(n\left(A\right)+m^1\right)+\left(I_2-2\right)m^2}{\left(I_2+1\right)\left(n\left(A\right)+m^1\right)+\left(I_2-1\right)m^2}.$$

from which we easily verify $1 + dm^2/dm^1|_{(8)} > 0$.

Likewise, we now analyze (9) more closely. The explicit expression for $dm^2/dm^1|_{(8)}$ allows us to rewrite (9) as

$$q\left(1-\tau\right)\left[\frac{n(A)+n(B)+m^1+m^2}{n(A)+m^1+m^2} - \frac{n(B)m^1/I_1}{[n(A)+m^1+m^2][(I_2+1)(n(A)+m^1)+(I_2-1)m^2]}\right] = 1.$$

For any $m^2 \in [0, M]$ there is a unique $m^1 \in [0, M]$ that solves (9). At $m^2 = 0$ the solution is $m_0^1 < M$, the inequality being due to the negative term of the LHS. At $m^2 = M$ we note that $m^1 = 0$. Thus, (9) defines a curve in (m^1, m^2)-space travelling from $(m_0^1, 0)$ to $(0, M)$, continuous by the implicit function theorem. Implicit differentiation of the rewritten (9) gives

$$\left.\frac{dm^2}{dm^1}\right|_{(9)} = -\frac{I_1\left[(I_2+1)\left(n(A)+m^1\right)+(I_2-1)m^2\right]^2+\left[n(A)+m^2\right]\left[(I_2+1)\left(n(A)+m^1\right)+(I_2-1)m^2\right]-m^1\left[n(A)+m^1+m^2\right](I_2+1)}{I_1[(I_2+1)(n(A)+m^1)+(I_2-1)m^2]^2-m^1[(I_2+1)(n(A)+m^1)+(I_2-1)m^2]-m^1[n(A)+m^1+m^2](I_2-1)}.$$

Here, the denominator is $\left(I_1\left(I_2+1\right)^2-2I_2\right)m^1\left(n(A)+m^1\right)+\left(I_1\left(I_2+1\right)^2\right)n(A)\left(n(A)+m^1\right)+I_1\left(I_2-1\right)^2\left(m^2\right)^2+2I_1\left(I_2^2-1\right)n(A)\,m^2+\left(2I_1\left(I_2^2-1\right)-2\left(I_2-1\right)\right)m^1m^2 > 0$since $I_1\left(I_2+1\right)^2 > 2I_2$ and $2I_1\left(I_2^2-1\right) \geq 2I_2-2$.

A drawing of the continuous curves shows that there exists an intersection. To prove uniqueness, we verify that at any intersection, $dm^2/dm^1|_{(8)} > dm^2/dm^1|_{(9)}$. Simple algebra reduces this to $\left[\left(I_1+1\right)\left(I_2+1\right)^2-\left(I_2+3\right)\left(I_2+1\right)-\left(I_2-1\right)\right]m^1\left(n\left(A\right)+m^1\right)+\left(I_1+1\right)\left(I_2+1\right)^2 n\left(A\right)\left(n\left(A\right)+m^1\right)+\left(I_1+1\right)\left(I_2-1\right)^2\left(m^2\right)^2+2\left(I_1+1\right)\left(I_2^2-1\right)n\left(A\right)m^2+\left[2\left(I_1+1\right)\left(I_2^2-1\right)-\left(I_2+4\right)\left(I_2-1\right)\right]m^1m^2 > 0$. This usually holds since $\left(I_1+1\right)\left(I_2+1\right)^2 \geq \left(I_2+3\right)\left(I_2+1\right)+\left(I_2-1\right)$ and $\left(I_1+1\right)\left(2I_2+2\right) > I_2+4$ when $I_1, I_2 \geq 1$. The only exception is where $I_1 = I_2 = 1$ and $n\left(A\right) = 0$. In this very special case, (8) and (9) imply $m^2 = m^1$. Equation (9) then states $1/\left[q\left(1-\tau\right)\right] = \left[n\left(B\right)/2+2m^1\right]/2m^1$ which has a unique solution for m^1.

(2) The equilibrium amount m^C of the simultaneous Cournot game with I players is the solution in m^2 of equation (8) with $m^1 = 0$, i.e.

$$q\left(1-\tau\right)\left(1+\frac{n(B)\left[n(A)+\frac{I-1}{I}m^C\right]}{\left[n(A)+m^C\right]^2}\right) = 1. \quad (10)$$

When there is one follower, denote by m^L the total amount bet by the $n-1$ leaders, and m^F the amount of the follower. From (8) we have

$$q\left(1-\tau\right)\left(1+\frac{n(B)\left[n(A)+m^L\right]}{\left[n(A)+m^L+m^F\right]^2}\right)=1, \tag{11}$$

and from (9) we have

$$q\left(1-\tau\right)\left(1+\frac{n(B)\left[n(A)+m^F+\frac{I-2}{I-1}m^L-\frac{1}{I-1}m^L\left.\frac{dm^F}{dm^L}\right|_{(11)}\right]}{\left[n(A)+m^L+m^F\right]^2}\right)=1. \tag{12}$$

In this Cournot setting any bettor is better off in equilibrium if and only if the remaining players reduce their total amount. Thus, (2) follows once we verify that $m^L>\frac{I-1}{I}m^C$. If $m^F<m^L/\left(I-1\right)$, then using (10) and (11),

$$\frac{n(A)+m^L}{\left(a_{-1}+\frac{I}{I-1}m^L\right)^2}<\frac{n(A)+m^L}{\left(n(A)+m^L+m^F\right)^2}=\frac{n(A)+\frac{I-1}{I}m^C}{\left(n(A)+m^C\right)^2}$$

and it follows that $m^L>\frac{I-1}{I}m^C$ as desired. We therefore prove that $m^F<m^L/\left(I-1\right)$. Equations (12) and (11) imply that $m^F=\left(1+dm^F/dm^L\big|_{(11)}\right)m^L/\left(I-1\right)$ so we need only verify that $dm^F/dm^L|_{(11)}<0$. This follows directly from (11).

(3) Note that the leader is weakly better off than in the Cournot game, since the leader could choose to bet the same amount as in the Cournot game. To verify that the leader is strictly better off, it is enough to verify the leader changes decision. If not, the others would still produce a total of $\frac{I-1}{I}m^C$. The leader's first order condition, derived from (9), would then be

$$q\left(1-\tau\right)\left(1+\frac{n(B)\left[n(A)+\frac{I-1}{I}m^C-\left.\frac{dm^2}{dm^1}\right|_{(8)}\frac{m^C}{I}\right]}{\left(n(A)+m^C\right)^2}\right)=1,$$

in contradiction with (10). □