# Supplementary Material on Reputational Cheap Talk

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In Ottaviani and Sørensen, henceforth OS, (2004b), we have formulated a model of strategic communication by an expert concerned about being perceived to be well informed. In that model, the expert observes a private signal informative about the state of the world. The amount of information about the state contained in this signal is parametrized by the expert's ability, assumed for simplicity to be unknown to the expert. The expert then sends a message to an evaluator, who uses it in conjunction with the ex-post realization of the state to update the belief about the expert's ability. Under general conditions on the distribution of the expert's private signal, we have shown that the expert does not wish to truthfully reveal the signal observed. In equilibrium, professional experts can credibly communicate only part of their information.

OS (2004a) have provided a further characterization of the equilibrium when the expert's signal is assumed to be *multiplicative linear*, a natural generalization of the binary signal experiment that allows for continuously varying intensity. In that setting, OS (2004a) have shown that no more than two messages can be effectively reported in equilibrium. In addition, when the prior distribution on the state is concentrated enough on a particular state, there exists no informative equilibrium. As a result, when experts speak in sequence, learning of the fixed state stops after a finite number of rounds. Reputational herding then obtains.

This note collects some supplementary material on the analysis of reputational cheap talk games. Section 1 provides an illustration of the direction of the incentives, studied more generally in Section 4 of OS (2004b). We then perform two robustness checks of the

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results obtained by OS (2004a) in the context of the multiplicative linear model (2.1), which corresponds to Example 1C of OS (2004b). Section 2 considers an interim reputational cheap talk model in which the evaluator does not observe the ex-post realization of the state. Section 3 allows the expert to be directly concerned about the accuracy of the decision made.

### 1. Optimal Deviation in Linear Model

In this section, assume that the distribution of the signal  $s \in \mathbb{R}$  conditional on the state  $x \in \mathbb{R}$  and ability  $t \in [0, 1]$  is linear in t,

$$f(s|x,t) = tg(s|x) + (1-t)h(s),$$
(1.1)

being a mixture between an informative and an uninformative experiment. Better experts are more likely to receive a signal drawn from the informative g(s|x) rather than the uninformative h(s). As shown by OS (2004a), with this linear signal structure a more able expert receives better information in the sense of Blackwell. The prior on the state is denoted by q(x) and the evaluator's prior on ability is p(t). State x and ability t are stochastically independent.

As shown in the proof of OS's Proposition 1, it is without loss of generality to let the expert have payoff

$$V(m|s) = \int_X \frac{\hat{g}(m|x) - \hat{h}(m)}{\hat{f}(m|x)} q(x|s) \, dx.$$
(1.2)

With  $\hat{f}(m|x,t) = t\hat{g}(m|x) + (1-t)\hat{h}(m)$ , the higher  $\hat{g}(m|x)$  is relatively to  $\hat{h}(m)$ , the higher is the expert's reputation, for this corresponds to higher weight on the t term and lower weight on the 1-t term. The result that  $W(m|x) = \left(\hat{g}(m|x) - \hat{h}(m)\right)/\hat{f}(m|x)$  clearly reflects this.

In the following we assume that S, X are subsets of  $\mathbb{R}$ . Assume that both g(s|x)and h(s) are twice continuously differentiable and that g satisfies the MLRP in (s, x). A necessary and sufficient condition for a linear signal structure to satisfy the MLRP in (s, x) for all t is  $g_{sx}h > g_xh_s$ . This follows from the observation  $f_{sx}f - f_sf_x = t^2(g_{sx}g - g_sg_x) + t(1-t)(g_{sx}h - g_xh_s)$ . This MLRP assumption, satisfied by model (2.1) below, is maintained throughout the paper.

A particular signal realization  $\tilde{s}$  is *neutral* about the state, if  $g(\tilde{s}|x)$  is constant in x. An expert who receives a neutral signal has posterior beliefs  $q(x|\tilde{s}) = q(x)$ : the signal is not informative about x since  $f(\tilde{s}|x,t)$  is independent of x. We now revisit the impossibility of truthtelling within the linear model. In response to naive beliefs, the ideal signal an expert wishes to send is different from the one observed. With a few restrictions on the model, we can predict that the direction of the deviation is towards the neutral signal:

**Proposition 1 (Best Deviation).** Assume  $g_{sx} > 0$  and that signal  $\tilde{s}$  is neutral. Assume that any signal is uninformative about ability (p(t|s) = p(t) for all s). The best deviation against naive beliefs is to report a signal s' strictly in between the neutral signal  $\tilde{s}$  and the signal actually possessed s.

**Proof.** Observe that any sender who reports truthfully has expected value Ev:

$$V(s|s) = \int_{T} v(t) \int_{X} p(t|s, x) q(x|s) \, dx \, dt = \int_{T} v(t) p(t|s) \, dt = Ev(t).$$

Now, fix  $s > \tilde{s}$  without loss of generality. We argue that the sender with s can profitably deviate to any signal  $s' \in (\tilde{s}, s)$ . Reporting s' gives the expected reputational value

$$V(s'|s) = \int_X \int_T v(t)p(t|s', x) \, dt \, q(x|s) \, dx$$

to be compared with the truthtelling value V(s|s). We will argue that V(s'|s) > V(s|s) for any  $s' \in (\tilde{s}, s)$ , as this proves the incentive to deviate from s to s'. Since V(s'|s') = V(s|s), we can equivalently show V(s'|s) > V(s'|s').

Our comparison rests on two facts. First, since s > s', q(x|s) first-order stochastically dominates q(x|s'). Second, with the signal  $s' > \tilde{s}$ , the higher is the state of the world, the more favorable the updated reputation, so that

$$W(s'|x) = \int_T v(t)p(t|s', x) dt$$

is an increasing function of x. This follows from Milgrom's (1981) Proposition 1 because: x is good news for t when  $s' > \tilde{s}$ ,

$$\frac{p(t'|s',x')}{p(t|s',x')} = \frac{f\left(s'|x',t'\right)}{f\left(s'|x',t\right)} \frac{p(t')}{p(t)} > \frac{f\left(s'|x,t'\right)}{f\left(s'|x,t\right)} \frac{p(t')}{p(t)} = \frac{p(t'|s',x)}{p(t|s',x)}$$

for t' > t and x' > x, as a consequence of Lemma 1 proved below; and v(t) is increasing. Combining the two facts we reach the desired

$$V(s'|s) = \int_X \int_T v(t)p(t|s', x) \, dtq(x|s) \, dx > \int_X \int_T v(t)p(t|s', x) \, dtq(x|s') \, dx = V(s'|s').$$

Finally, we show that deviating to any s' outside the interval  $(\tilde{s}, s)$  is not profitable. First, for  $s' \ge s$ , the first fact is reversed, since s' is better news than s for x. This in turn reverses the final inequality, making the deviation unattractive. Second, consider  $s' \leq \tilde{s}$ . The second fact above is reversed, as higher x is worse news about ability when  $s' \leq \tilde{s}$ . We can conclude that the best deviation is to some  $s' \in (\tilde{s}, s)$ .

In the previous proof we have used the following result:

**Lemma 1.** Consider the linear model with  $g_{sx} > 0$  and neutral signal  $\tilde{s}$ . Let t' > t and x' > x. Then

$$\frac{f(s|x',t')}{f(s|x',t)} > \frac{f(s|x,t')}{f(s|x,t)}$$
(1.3)

for all  $s > \tilde{s}$ .

**Proof.** Substituting f(s|x,t) = tg(s|x) + (1-t)h(s), (1.3) is equivalent to

$$\frac{t'g(s|x') + (1-t')h(s)}{tg(s|x') + (1-t)h(s)} > \frac{t'g(s|x) + (1-t')h(s)}{tg(s|x) + (1-t)h(s)}$$

or

$$(t'-t) \left[ g(s|x') - g(s|x) \right] > 0, \tag{1.4}$$

for t' > t, x' > x, and  $s > \tilde{s}$ . Notice that  $g_x(\tilde{s}|x) = 0$  for all x and  $g_{sx} > 0$  imply that  $g_x(s|x) > 0$  for  $s > \tilde{s}$ , so that (1.4) holds.

Proposition 1 requires a stronger assumption (that signals are uninformative about ability) than Proposition 1 of OS (2004b), but is valid for the more general class of linear models. Its logic relies on the following three observations. First, higher realizations of the state x are better news about ability when signal s' such that  $s' > \tilde{s}$  is understood to have been reported. Second, the sender with s such that s > s' believes more in higher realizations of x the sender with s'. Third, the sender who reports truthfully is expecting the same value Ev regardless of the signal actually observed. Therefore, the sender with s has a higher expected reputational payoff from reporting  $s' \in (\tilde{s}, s)$  compared to that of the sender with signal s', itself equal to the truthtelling value Ev.

### 2. Interim Evaluation

From now on, we focus on the multiplicative linear experiment, according to which the signal  $s \in S = [-1, 1]$  has conditional density

$$f(s|x,t) = tg(s|x) + (1-t)h(s) = t\frac{1+sx}{2} + (1-t)\frac{1}{2} = \frac{1}{2}(1+stx),$$
(2.1)

where the expert's ability is  $t \in T = [0, 1]$  and the state  $x \in X = [-1, 1]$ .

In the baseline model, we have assumed that the evaluator observes the state of the world. This section considers what happens when the evaluator only observes the message sent by the expert, but does not have access to any additional information on the state of the world. Denoting the evaluator's conjecture of the expert's mixed strategy by  $\hat{\mu}(m|s)$ , we have  $\hat{f}(m|t) = \int_{S} \hat{\mu}(m|s)f(s|t) ds = (1 + E[s|m]tEx)\hat{\mu}(m)/2$ , so that  $p(t|m) = \hat{f}(m|t)p(t)/\hat{f}(m) = (1 + E[s|m]tEx)p(t)/(1 + E[s|m]EtEx))$ . The interim reputational payoff from sending m is

$$\int_{T} v(t)p(t|m) dt = E[v(t)] + (E[tv(t)] - E[v(t)]Et) \frac{E[s|m]Ex}{1 + E[s|m]EtEx}.$$
(2.2)

Exactly like in the partian model of Crawford and Sobel (1982), the receiver's (evaluation) action is based exclusively on the message reported by the sender. When the evaluator does not receive any information about the state in addition to the message sent by the adviser, the sorting condition is not satisfied. No information can then be communicated in equilibrium:<sup>1</sup>

**Proposition 2 (Interim Reputation).** In the interim model with multiplicative linear experiment with  $Ex \neq 0$ , there is no informative equilibrium, even allowing for mixed strategies.

**Proof.** Any two messages m and m' sent with positive probability in equilibrium must give the same payoff, as otherwise the message with lowest payoff would not be sent. The indifference condition V(m) = V(m') is equivalent to E[s|m]Ex = E[s|m']Ex, and since  $Ex \neq 0$  the messages satisfy E[s|m] = E[s|m']. Now all messages sent in equilibrium have the same average signal, and this common average must be zero, since 0 is the overall average signal. Then no message conveys any information about t or x.

## 3. Mixed Incentives

This section extends the model to allow the expert to be concerned about the accuracy of the decision made on the basis of the message reported. This is done by adding a second

<sup>&</sup>lt;sup>1</sup>This model also relates to Brandenburger and Polak's (1996) analysis of investment decisions by privately informed managers who are concerned with current share price. The current share price in turn reflects the information inferred by the stock market from the manager's observable investment behavior. Our interim model can be seen as a continuous-signal reputational-objective analogue of their model. In their binary-signal model there is no pure-strategy informative equilibrium other than for a degenerate prior on the state (their Proposition 1), but there is an informative mixed-strategy equilibrium for a set of non-degenerate priors on the state (Proposition 2). Their mixed strategy equilibrium has the property that all messages are equally attractive to the sender. Yet, their messages convey some information about the state of the world, something impossible in our reputational context. See also Heidhues and Lagerlöf (2003).

receiver who uses the expert's message to make a decision under uncertainty about the state of the world.

For simplicity, we assume that this second receiver must estimate the state x and suffers a loss proportional to the mean squared error. In our reputational cheap talk framework, both receivers (the decision maker and the evaluator) cannot commit not to use in an ex-post optimal way all the information gleaned from the message.<sup>2</sup> Conditional on m, the posterior belief on the state is

$$q(x|m) = \frac{\hat{f}(m|x)}{\hat{f}(m)}q(x) = \frac{1 + mxEt}{1 + mExEt}q(x), \qquad (3.1)$$

where as before *m* represents the average signal of the message, according to the expert's conjectured strategy. Due to the sequential rationality requirement, the decision maker chooses the estimate  $y \in \mathbb{R}$  that minimizes  $E\left[(y-x)^2 | m\right]$ , where the conditional expectation refers to the belief q(x|m). As is well known, the solution is y = E[x|m].

We assume that the expert has a stake in the decision marker's problem and cares about both the reputational payoff and a fraction of the quadratic estimation loss. This means, that when the expert chooses m, he must now maximize the augmented

$$V(m|s) = E\left[\beta W(m|x) - (1 - \beta) (E(x|m) - x)^{2} |s\right].$$

Here,  $\beta \in [0, 1]$  is a parameter that measures the weight of the reputational payoff in the expert's payoff function.

If the evaluator conjectures truthful reporting, the estimate will be E(x|m), which minimizes  $E[(y-x)^2|m]$ . Thus the estimation problem per se does not give the expert any reason to bias the message away from truthtelling. Since the derivative  $V_m(s|s)$ is independent of  $\beta$ , the deviation incentives characterized in OS's (2004a) Proposition 2 carry over to this setting. In addition, V(m|s) is supermodular in (m, s), so that equilibria must have interval messages:

**Proposition 3.** For any  $\beta < 1$ , the incentives to deviate from truthtelling are as stated in OS (2004a) Proposition 2 and equilibria must use interval messages.

**Proof.** Since the estimate E(x|m) minimizes  $E[(y-x)^2|m]$  in y, and since E(x|m) is a differentiable function of m, the derivative of  $E[(y(m)-x)^2|s]$  with respect to m

<sup>&</sup>lt;sup>2</sup>Prendergast and Stole (1996) instead identify the second decision maker with the expert, to whom the decision is delegated. In their reputational signalling framework, the expert's payoff therefore depends directly on the message sent. This difference explains why their equilibrium is fully-revealing, while ours is coarse.

is zero at s. Thus  $V_m(s|s)$  is independent of  $\beta$ , and therefore still satisfies OS's (2004a) Proposition 2.

By the MLRP, E(x|m) is increasing in m. The quadratic loss function  $L(y|x) = -(y-x)^2$  has a positive cross partial derivative,  $L_{yx}(y|x) = 2$ . Together, these facts imply that  $-\beta (E(x|m) - x)^2$  has a positive cross partial derivative in (m, x). Proceeding as in the proof of OS's (2004a) Proposition 1 we conclude that V(m|s) is still strictly supermodular in (m, s). By the argument used in OS's (2004a) Proposition 3, equilibria must have interval messages.

Clearly, truthful reporting of s would be an equilibrium in the absence of reputational concerns ( $\beta = 0$ ). We now show that if reputational concerns are important enough, the number of equilibrium messages cannot exceed two, as in the extreme case with  $\beta = 1$  (OS's (2004a) Proposition 4). The proof relies on a continuity argument.

**Proposition 4.** For any non-degenerate prior belief with  $Ex \neq 0$ , all informative equilibria are binary for  $\beta < 1$  sufficiently large. If instead Ex = 0, the maximal number of equilibrium messages is three when  $\beta$  is large.

**Proof.** We begin a Lemma derived from an extension of the indifference conditions from proof of OS's (2004a) Proposition 4 to the present case.

**Lemma 2.** Let any non-degenerate prior on the state be given. For every  $\varepsilon > 0$  there exists a  $\beta^* < 1$  such that for all  $\beta \in (\beta^*, 1]$  the two following properties hold: (i) If there are three equilibrium messages, the two extreme messages are of length less than  $\varepsilon$ . (ii) If there are four of more equilibrium messages, they are all of length less than  $\varepsilon$ .

**Proof.** Consider any three adjacent equilibrium messages, defined by the end-points  $a \leq b \leq c \leq d$ . The messages are denoted m = (a + b)/2, m' = (b + c)/2, and m'' = (c + d)/2, and since messages are distinct we have m < m' < m''. Denote also y = E[x|m], y' = E[x|m'], and y'' = E[x|m'']. Recall  $V(m|s) = E[\beta W(m|x) - (1 - \beta)(E[x|m] - x)^2|s]$ , and observe  $E[(y - x)^2|s] = y(y - 2E[x|s]) + E[x^2|s]$ . Using this with  $W(m|x) = \frac{mx}{1+mxEt}$  and (3.1), we can arrange the indifference condition V(m|b) = V(m'|b) as

$$\beta \int_{X} \left( \frac{mx}{1 + mxEt} - \frac{m'x}{1 + m'xEt} \right) q(x|b) dx$$
  
=  $(1 - \beta) (y(y - 2E[x|b]) - y'(y' - 2E[x|b])) = (1 - \beta) (y - y') (y + y' - 2E[x|b])$   
=  $(1 - \beta) \left( \frac{Ex + mE[x^2]Et}{1 + mExEt} - \frac{Ex + m'E[x^2]Et}{1 + m'ExEt} \right) (y + y' - 2E[x|b]).$ 

Re-arranging both sides, we get

$$\beta \frac{m-m'}{1+bExEt} \int_X \frac{x(1+bxEt)}{(1+mxEt)(1+m'xEt)} q(x) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)(1+m'ExEt)} \left(y+y'-2E\left[x|b\right]\right) \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)Et}{(1+mExEt)} \left(y+y'-2E\left[x|b\right]} \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)}{(1+mExEt)} \left(y+y'-2E\left[x|b\right]} \, dx = (1-\beta) \frac{(m-m') \left(E[x^2] - (Ex)^2\right)}{(1+mExEt)}$$

Eliminating  $m - m' \neq 0$ , this condition can be rewritten as

$$\int_{X} \frac{x(1+bxEt)}{(1+mxEt)(1+m'xEt)} q(x) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x|b]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)(1+m'ExEt)} \left(y + y' - 2E[x]\right) \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)} \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)} \left(y + y' - 2E[x]\right)} \, dx = \frac{1-\beta}{\beta} \frac{\left(E[x^2] - (Ex)^2\right)Et(1+bExEt)}{(1+mExEt)} \, dx = \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta}{\beta} \frac{1-\beta$$

Since  $Et \in (0,1)$ , and  $b, m, m', Ex, y, y', E[x|b] \in [-1,1]$ , and  $E[x^2] - (Ex)^2 \in [0,1]$ , we finally obtain the bound

$$\left| \int_{X} \frac{x \left(1 + bxEt\right)}{\left(1 + mxEt\right) \left(1 + m'xEt\right)} q\left(x\right) \, dx \right| < \frac{1 - \beta}{\beta} \frac{Et \left(1 + Et\right) 4}{\left(1 - Et\right)^{2}}.$$

The indifference V(m'|c) = V(m''|c) gives the analogous bound

$$\left| \int_{X} \frac{x \left( 1 + cxEt \right)}{\left( 1 + m'xEt \right) \left( 1 + m''xEt \right)} q\left( x \right) \, dx \right| < \frac{1 - \beta}{\beta} \frac{Et \left( 1 + Et \right) 4}{\left( 1 - Et \right)^2}.$$

The triangle inequality gives that the difference of the integrals satisfies

$$\left| \int_{X} \left[ \frac{1 + bxEt}{1 + mxEt} - \frac{1 + cxEt}{1 + m''xEt} \right] \frac{x}{1 + m'xEt} q(x) \, dx \right| < \frac{1 - \beta}{\beta} \frac{Et \, (1 + Et) \, 8}{\left(1 - Et\right)^2},$$

or equivalently,

$$\left| \int_{X} \left[ \frac{(b-m)xEt}{1+mxEt} + \frac{(m''-c)xEt}{1+m''xEt} \right] \frac{x}{1+m'xEt} q(x) \, dx \right| < \frac{1-\beta}{\beta} \frac{Et \, (1+Et) \, 8}{(1-Et)^2}. \tag{3.2}$$

Further, since 1 + mxEt,  $1 + m'xEt \in (0, 1 + Et]$ ,

$$\int_{X} \frac{(b-m)xEt}{1+mxEt} \frac{x}{1+m'xEt} q(x) \, dx > (b-m) \frac{EtEx^2}{(1+Et)^2}$$

and likewise for the term with m'' - c, so (3.2) finally implies

$$(b-m) + (m''-c) < \frac{1-\beta}{\beta} \frac{(1+Et)^3 8}{(1-Et)^2 Ex^2}$$

For  $\beta$  sufficiently close to 1 we force each of b - m and m'' - c below  $\varepsilon/2$ . By definitions of m and m'' this also forces the interval lengths b - a and d - c below  $\varepsilon$ . Thus, whenever we have three adjacent messages, the two extreme ones must be of length less than  $\varepsilon$ . Extending this fact left and right, we obtain the Lemma.

We can now conclude that equilibria with four or more messages are impossible, once  $\beta$  is sufficiently large. Let any non-degenerate prior distribution q(x) be given. By Proposition 3, there exists at most one point  $\hat{s}$  that solves  $V_m(s|s) = 0$ . Consider now any other

signal  $s \neq \hat{s}, 1, -1$ . Suppose without loss of generality that  $V_m(s|s) > 0$ . Since V is nicely behaved (concave in m, and continuously differentiable), there exists some  $\varepsilon > 0$  such that V(m|s) is increasing in m on the interval  $(s, s + \varepsilon)$ . By the Lemma, once  $\beta$  is sufficiently large, all message intervals are of length at most  $\varepsilon/2$ . Then the equilibrium message to be sent at s is at most  $\varepsilon/4$  above s, and there is a next adjacent equilibrium message at most  $3\varepsilon/4$  above s which is more attractive. Thus, incentive compatibility fails.

Now, fix a non-degenerate prior distribution such that  $Ex \neq 0$ . In the limiting case when  $\beta = 1$ , we see from

$$\int_{X} \frac{x (1 + axEt)}{[2 + (a - 1)xEt] [2 + (a + 1)xEt]} q(x) \, dx = 0$$

with a = 1 that  $V(1|1) \neq V(0|1)$ . By continuity of the payoff function, there exists  $\hat{\beta} < 1$ such that  $|V(\mu''|\sigma) - V(\mu'|\sigma)| > 0$  for all  $\beta, \mu'', \sigma \in (\hat{\beta}, 1]$  and  $\mu' \in (\hat{\beta} - 1, 1 - \hat{\beta})$ . Let  $\varepsilon = 1 - \hat{\beta}$  and apply the Lemma. If the equilibrium has three messages, the two extreme messages are of length at most  $\varepsilon$  once  $\beta$  exceeds some  $\beta^*$ . Then the middle message satisfies  $m' \in (\hat{\beta} - 1, 1 - \hat{\beta})$  and the top message satisfies  $m'' \in (\hat{\beta}, 1]$ , and the higher indifference point also satisfies  $c \in (\hat{\beta}, 1]$ . But for all  $\beta > \max(\hat{\beta}, \beta^*)$  we then have  $V(m''|c) \neq V(m'|c)$ , in violation of the equilibrium conditions.

Finally, assume Ex = 0. In the purely reputational case  $(\beta = 1)$ , there exists a threemessage equilibrium with messages -1, 0, 1. As  $\beta$  departs slightly from 1 and we add weight to the estimation objective, it becomes clear that the above indifference conditions are changed to strict preferences V(-1|-1) > V(0|-1) and V(1|1) > V(0|1), so the three message equilibrium becomes non degenerate.

In order for equilibria to be necessarily binary in the case with  $Ex \neq 0$ , the weight on the statistical payoff has to be bounded away from zero, with a bound that is not uniform with respect to Ex. In particular, this condition is harder to satisfy as Ex approaches zero.

#### 4. References

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